# THE NEW ZEALAND MATHEMATICAL SOCIETY

# NEWSLETTER

Philoſophiæ Naturalis

PRINCIPIA

MATHEMATICA.

DEFINITIONES.

DEFINITIO I.

*Quantitas materiæ eſt menſura ejuſdem orta ex illius denſitate et magnitudine conjunctim.*

AER denſitate duplicata, in ſpatio etiam duplicato, fit quadruplus; in triplicato ſextuplus. Idem intellige de nive & pulveribus per compreſſionem vel liquefactionem condenſatis. Et par eſt ratio corporum omnium, quæ per cauſas quaſcunque diverſimode condenſantur. Medii interea, ſi quod fuerit, interſtitia partium libere pervadentis, hic nullam rationem habeo.

IS MATHEMATICS THE 'LATIN' FOR THIS GENERATION?

CENTREFOLD

PROFESSOR DESMOND SAWYER

CONTENTS



Number 32 December 1984 ISBN 0110-0025

# Editorial

This issue features two articles of general interest. The first on the role of mathematics was an invited address delivered by Dr Lindsay Johnson at the Nineteenth Mathematics Colloquium last May. The second on the future of applied mathematics was a Faculty of Science open lecture given at the University of Otago by Professor Bill Bonnor, the William Evans Visiting Professor.

As many members will know, the Mathematics Department at Otago is shortly to loose two professors. Both Professor Bill Davidson (see Centrefold Newsletter 30) and the Department Chairman, Professor Desmond Sawyer, are retiring at the end of this year. Desmond's contribution to Mathematics in New Zealand has been considerable, so he will be greatly missed not only by his colleagues at Otago but by the wider mathematical community. It is thus appropriate that he is the subject of the Centrefold this issue.

Recently the constitution of the society was amended to change the presidential term to a two year period, with some consequential changes to other positions. As a result of a directive from the NZMS council meeting of last December the amended constitution has been published in this issue.

Finally a reminder that items of news, notices, articles of general interest, suggestions for centrefolds, letters to the editor on current issues are always welcome and may be sent to the editor or one of the honorary correspondents. Copy date for the next issue is 15 March 1985.

*John Curran*
Editor

# Notices

## AUSTRALIAN MATHEMATICAL SOCIETY LECTURE SERIES

Cambridge University Press announces the establishment of a new series, The Australian Mathematical Society Lecture Series. It is envisaged that the series will operate at the frontiers either of mathematics itself or of its teaching, and therefore will contain both monographs and textbooks suitable for graduates and/or undergraduates. The series will be listed in Cambridge catalogues and marketed throughout the world, with special attention to the American market. It will differ from some other learned series in mathematics in that authors will receive royalties.

Authors with manuscripts or projects which they consider might be suitable for the series are invited to contact the series editor responsible for the relevant subject. Given an appropriate author response and a flow of books of a suitable standard, we are aiming for an output of about six books a year. Potential authors are advised that to keep the prices of books in the series relatively low, we shall require final typescripts in camera-ready form.

Subject Editors: Applied Mathematics:

Professor C.J. Thompson, Department of Mathematics, University of Melbourne.

Pure Mathematics:

Professor A.J. van der Poorten, School of Mathematics and Physics, Macquarie University.

Statistics:

Professor C.C. Heyde, Department of Statistics, University of Melbourne.

Editor in Chief: Dr S.A. Morris, Department of Mathematics, La Trobe University.

## AUSTRALIAN MATHEMATICAL SOCIETY

The AMS has informed the NZMS of the following editorial changes of the Australian mathematics journals:

The new editor of the Bulletin of the Australian Mathematical Society is Dr S. Oates-Williams, Department of Mathematics, University of Queensland, St. Lucia, Queenland 4067, Australia. The new editor of the Australian Mathematical Society Gazette is Dr G. Cohen, Mathematics Department, The N.S.W. Institute of Technology, P.O. Box 123, Broadway, Sydney N.S.W. 2007. The new editor of the Journal of the Australian Mathematical Society [Series B] is Professor E. Tuck, Department of Mathematics, University of Adelaide, Adelaide 5000 S.A. Australia.

## RSNZ NOTICES

Vol. 112 of the Proceedings of the Royal Society of New Zealand will be ready for posting early in December. Members of the NZMS are reminded that copies are available at the special Member Bodies rate of $7.50 (normal price $18.00) on direct application to the Executive Officer, Royal Scoeity of N.Z., Private Bag, Wellington.

The Royal Society has also been notified that DSIR wishes to continue with the arrangement that their journals be sold to NZMS members at a 50% discount. The discount offered means that the subscription rate per volume will be $18.00 plus $2.00 postage. There is no time restriction for this offer; however they have indicated that subscriptions should be received by March 1985, when the journals begin arriving.

The Science Information Publishing Centre have asked that we inform all members that they plan to discontinue Journal of Science and start Journal of Technology. The contents of the new journal will be some of what would have been in Science, but also other more technological papers. Science ends with Vol. 27(4), Technology begins at Vol. 1(1).

## BRITISH MATHEMATICAL COLLOQUIUM

The 37th British Mathematical Colloquium will be held in the University of Cambridge on 2nd, 3rd and 4th April 1985. The Principal speakers will be D. Zagier (Bonn), C.L. Fefferman (Princeton), and M. Gromov (Paris).

Speakers from the U.K. include T.O. Hawkes, E.C. Lance, M. Rees, A.D. Gardiner, C.R. Leedham-Green, K.J. Falconer, R.A. Bailey, W. Parry, D.B.A. Epstein, B.Birch, J.M. Hammersley, M. Paterson, R. Brown, J.S. Pym, and C. Hooley.

There will be a discussion of Information Technology presented by C.A.R. Hoare, F.R.S., and a display of Computer Aided Teaching of Applied Mathematics.

The registration fee is £12 for those paying their bill in full by January 31st 1985; thereafter the fee is £18. For research students of no more than 3 years standing these amounts are halved. The cost of accommodation and meals for the full period is £108.

Application forms and further information are available from the Colloquium secretary Dr R.C. Mason, Department of Pure Mathematics and Mathematical Statistics, 16 Mill Lane, Cambridge CB2 1SB.

## THE LONDON MATHEMATICAL SOCIETY

The Society was established in 1865 for the promotion and extension of mathematical knowledge. Members of the New Zealand Mathematical Society are very welcome to join the London Mathematical Society under the Reciprocity Agreement, which enables certain formalities to be dispensed with and a reduced subscription to be applied. For the year 1983/84 the reduced subscription is £3.00 (US$6.25).

Members of the New Zealand Mathematical Society may commence or continue their reciprocity membership of the London Mathematical Society during periods of temporary residence in the United Kingdom. They may then take advantage of all the facilities offered to London Mathematical Society members, such as use of the Society's extensive Mathematical Library at University College, London. They are also welcome to attend Society meetings, which take place about seven times a year.

Members of the New Zealand Mathematical Society resident outside the United Kindom may be chiefly interested in subscribing to the periodicals of the L.M.S. These are the Bulletin, the Journal and the Proceedings of the London Mathematical Society. One volume of the Bulletin is published per year, and two volumes each of the Journal and Proceedings.

The London Mathematical Society also publishes a series of Monographs, a series of Lecture Notes, and the proceedings of its Instructional Conferences. Members may purchase these books at a discount of 25%.

Full particulars of the activities of the L.M.S. and application forms for reciprocity membership may be obtained from the Administrative Assistant of the L.M.S., Burlington House, Piccadilly, London W1V 0NL, United Kingdom.

# Local News

## AUCKLAND UNIVERSITY

### DEPARTMENT OF MATHEMATICS & STATISTICS

Visiting Professor George P.H. Styan arrived in the Department in September from McGill University, Montreal. George will be with us for approximately 6 months.

Dr Chris and Eileen Wild had their first child, a daughter, to be named Catherine.

Ken Ashton underwent minor surgery and made a good recovery.

Assistant lecturer, Noel Wass, departed at the end of August when the tenure of his appointment terminated, for further study at University of Michigan.

Dr Stuart Scott has recently been appointed an Honorary Research Fellow within the Department.

#### Seminars

Dr Jocelyn Dale (Applied Maths Division, DSIR Auckland), *'A statistical model for bivariate ordered responses'.*
Dr R. Goldblatt (Auckland University and Victoria University of Wellington), *'Unprovable truths of arithmetic'.*
Professor D.B. Gauld (Auckland University), *'Topological museum'.*
Mr Chris King (Auckland University), *'The origin of life as a stable product of symmetry-breaking'.*
Professor G.A.F. Seber (Auckland University), *'Application of blood genetics to paternity testing and forensic medicine'.*
Professor E.P. Cass (University of Western Ontario), *'Nörlund matrices as bounded operators on $l_p$'.*
Mr Jørgen Harmse (Advanced Course of Study student, Auckland University), *'An introduction to degree theory'.*
Dr Simon Fitzpatrick (Auckland University), *'The Radon-Nikodym property for Banach spaces'.*
Dr Ron King (University of Southampton), *'Symmetric and supersymmetric functions and their connection with Lie algebras and superalgebras'.*
Professor N.J. Pullman (Queen's University, Kingston, Canada), *'Rank preserving operators on matrices over semirings'.*
Professor M.V. Subbarao (University of Alberta), *'Addition chains'.*
Professor T.W. Anderson (Stanford), *'Aspects of multivariate analysis'.*
Professor R.L.E. Schwarzenberger (University of Warwick), *'Colour symmetry'.*
Professor H. Whitney (Institute for Advanced Study, New Jersey), *'Letting research come naturally'*, *'Geometric origins of the cohomology of groups'* and *'How can we help students regain their natural power?'.*
Dr I.R. Porteous (University of Liverpool), *'Curves and surfaces in 3-space - a singular look'.*
Mr Mark Norwood (Advanced Course of Study student, Auckland University), *'Harmonic analysis'.*
Mr Matthew Bell (Advanced Course of Study student, Auckland University), *'Normal families'.*
Sherry Fraser and Molly Whitely (University of California, Berkeley), *'The equals project'.*
Mr Bruce Pollock (Advanced Course of Study student, Auckland University), *'Distribution, Fourier transforms and the Garding-Wightman axioms for a quantum field theory'.*
Dr Bruce Calvert (Auckland University), *'The fundamental theorem of projective geometry'.*
Dr Harold Henderson (Ruakura Agricultural Research Centre), *'Johann Georg Zehfuss and Leopold Kronecker: Their matrix product and associated relationships with some statistical applications'.*
Mr John Maindonald (Applied Maths Division, DSIR), *'The structuring of statistical calculations: Patterns that pervade'.*

*I.L.R.*

### DEPARTMENT OF COMPUTER SCIENCE

Mr John Hosking, who has been a Junior Lecturer and then a temporary part-time lecturer, has now been appointed as a Lecturer.

Peter Gibbons has returned from leave at the University of Toronto, and Kevin Burrage has now gone there on leave.

Seminars

Professor John Hine (VUW), *'The role of names in operating systems'* and *'The development of the personal computer: a university perspective'*.
Professor Philip Rabinowitz (Weizmann Institute, Rehovot), *'Numerical solution of weakly singular linear Fredholm integral equations of the second kind - a survey'*. (3 lectures) *'The SEAC - personal reminiscences'*.
Rick Mugridge (University of Auckland), *'Expert systems: applied artificial intelligence'*.
Dr Murray S. Klamkin (University of Alberta), *'Patterns in problem-solving'* and *'Mathematics competitions around the world'*. (Jointly with the Department of Mathematics & Statistics, and the Auckland Mathematical Association.)
Dr Bruce Hutton (University of Auckland), *'Re-parsing of programs in an interactive environment'*.
Professor Rob Goldblatt (VUW), *'Fixed-point induction, self-application and the lambda-calculus'*.
Cao Liming (China Institute of Mining, Xuzhou), *'The travelling salesman problem'*.
Graeme Macferson (Defence Scientific Establishment), *'Computing activities at the Defence Research Establishment'*.
G.W. Blanchard (School of Engineering, University of Auckland), *'The use of microcomputers in process and industrial control'*.
Professor E.V. Krishnamurthy (University of Waikato), *'New computer arithmetic systems'*.
Dr Arndt Bode (Friedrich-Alexander-Universität, Erlangen-Nürnberg), *'The impact of VLSI technology on computer architecture'*.
Professor Phil Cox (Technical University of Nova Scotia), *'Surface deduction: a uniform mechanism for logic programming'*.

*G.J.T.*

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS

Professor Ian Collins has been elected to the Congress Committee of the International Union of Theoretical and Applied Mechanics.

In September Professor Collins presented an invited keynote address at the First International Conference on the Technology of Plasticity, Tokyo, Japan.

Dr Robert McKibbin has been awarded the 1984 Hamilton Prize by the Royal Society of New Zealand.

Seminars (1984)

Professor Peter Oxley (School of Mechanical and Industrial Engineering, University of New South Wales), *'Plastic deformation of a metal surface in sliding contact with a hard wedge: its relation to friction and wear'*.
Dr Chris Calladine, FRS (Department of Engineering, University of Cambridge, U.K.), *'Construction and operation of bacterial flagella'*.
Dr Iain Duff (Atomic Energy Research Establishment, Harwell, U.K.), *'Direct methods for solving sparse systems of linear equations'*.
Dr David Ryan (T.A.M. Department, University of Auckland), *'Scheduling problems and conditions for natural integrality'*.
Mr Ian McCrae (T.A.M. Department, University of Auckland), *'Mechanics of the McMurdo Ice Shelf, Antarctica'*.
Drs Bill and Kath Dowsland (Swansea University, U.K.), *'Two dimensional packing problems'*.
Dr Brian Mace (Department of Mechanical Engineering, University of Auckland), *'Vibration of periodic structures'*.
Dr Brian Graham (University of Saskatchewan, Saskatoon, Canada), *'The single breath diffusing capacity of the lungs for carbon monoxide'*.
Dr Mike Osborne (Department of Statistics, I.A.S., A.N.V., Canberra), *'Exponential data fitting'*.
Professor Bruce Morton (Department of Mathematics, Monash University), *'Vortex wakes behind surface mounted obstacles'*.
Professor John Duncan (McMaster University, Hamilton, Ontario, Canada), *'Plastic bending of strain aged sheet metal'*.
Mr Ross Vennell (Woods Hole Oceanographic Institute, M.I.T.), *'Oceanic wind driven circulation'*.
Dr Bob Milner (Department of Civil Engineering, Chisholm Institute of Technology, Melbourne), *'Planning and organisational structure in Academe'*.
Mr Stephen Taylor (Department of Mathematics and Statistics, University of Auckland), *'The thermal instability of a layer of fluid'*.
Mr Stanley Wing (T.A.M., University of Auckland), *'An application of the shooting method to the geothermal well test equations;*.

Mr Mike Frisby (T.A.M., University of Auckland), *'A finite element model for pre-failure dilatancy in a rock mass'.*
Professor Ted Buchwald (University of New South Wales), *'The stress-concentration for a double cantilever beam'.*
Professor Adrian Bejan (Duke University, Durham, North Carolina), *'Natural convection in porous media'.*

*D.M.R.*

## WAIKATO UNIVERSITY

Professor Zualauf (after several weeks on leave in Singapore), John Turner and Mark Schroder all took part in ICME V. John's poster paper showed our first steps in CAI: I just opened my eyes to the sights of Adelaide and my ears to the welcome sound of commonsense.

To our regret, Bronwyn Beder has resigned, and so will Peter Hill, if he can get a visa to allow him to join the Hebron School, Ootacamund, in India.

Professor Willard Miller Jr visited, in July and August, to collaborate with Ernie Kalnins and to act as examiner-in-chief for Greg Reid's D.Phil. thesis, *'Variable separation for the heat and Schrödinger equations'*. Then, Greg left for South Dakota State University as an Assistant Professor.

Shortly after, John Turner successfully defended his D.Phil. thesis, *'A study of knot-graphs'*. At the age of 56, he had considered euthanasia as a way of busting his twist-spectra, but (luckily for us) the Academic Calendar did not permit this. However, one near-Geordian (that is, Leedsy) knot remains as secure as when it was tied: just two weeks later, John and Barbara celebrated their Silver Wedding.

Kevin Broughan is breeding cadmice in his office. "We have recently installed a 68000 based microprocessor, a Cadmus 9790 with 3 mb memory, 140 mb fixed disk and a high-resolution graphics screen. The disk-drive chirps; hence its name TUI. Mainly, it will house MACSYMA, the large LISP-based symbolic manipulation system that regularly brings a VAX to its knees. TUI will run NAG, Fortran and graphics of course, and should provide a nice environment for mathematical modeling and other 'compumath' activities and research. A (free) beta-test 68020 Cadmus running somewhat faster (14 mhz, rather than 10) is due in March 1985."

Hassler Whitney's morning here began with the talk noted below, to which several other groups came: Roger Osborne and others involved in the 'Learning in science project', with representatives from Education, Computer Science, and science and maths lecturers from the teachers college next door. Animated discussion followed, right through morning tea and lunch ...

Seminars

G. Reid (Waikato), *'R-separation for head and Schrödinger equations'.*
Dr Carl Pierchala (Texas Tech . U. Health Sciences Centre), *'Determination of sample size when investigating new scientific hypotheses'.* (To the Waikato Statistics Group)
Professor W. Miller Jr (Minnesota), *'Mathematics in America'.* (Public lecture)
Professor P. Rabinowitz (visiting Auckland from Israel), *'Software for numerical integration'.*
Professor H. Whitney (I.A.S., Princeton), *'How can we help students regain their natural powers'.* (The NZMS Visiting Lecturer)

*M.S.*

## MASSEY UNIVERSITY

The latest round of promotions saw Howard Edwards and Doug Stirling given senior lectureships - congratulations to them both.

August saw a minor exodus to Australia. Hugh Morton went to the 7th Australian Statistical Conference in Brisbane, where he enjoyed the opportunity to meet other researchers in biological applications of statistics. Gordon Knight and Mike Carter formed the Massey contingent to ICME V in Adelaide. Gordon's paper on the mathematics on Maori art was very well received.

Chin Dieu Lai spent October at the National University of Singapore. A particular aim of his visit was attendance at a workshop on time series and its applications, which he found well organised and enjoyable; an outstanding feature was that there were more invited addresses than contributed papers. Mathematics is a very popular subject at the National University of Singapore, Chin Dieu tells us; the department has more than 50 full-time academic staff, who incidentally enjoy salaries a great deal higher than those of their New Zealand counterparts!

Apart from benefits to our own staff, ICME V produced spinoff during September in the shape of visitors from Pretoria, Haifa, Liverpool and Princeton, who all appear in our list of seminars.

Seminars

Hugh Morton, *'The 1984 TVNZ election prediction system'.*
Professor Philip Rabinowitz (Weizmann Institute), *'A survey of software for evaluation of multiple integrals'.*
Professor S. Avital (Haifa), *'What constitutes good teaching'.*
Dr Ian Porteous (Liverpool), *'Mathematics education on Merseyside: Mathematics competition in schools'.*
Professor H.S.P. Grasser (Pretoria), *'Teaching problem-solving at a university specialising in distance education'.*
Professor H. Whitney (IAS, Princeton), *'Letting research come naturally'* and *'How can we help students regain their natural powers'.*
Janet McCall, *'The Hadamard conjecture and on infinite lattice of points'.*
Bruce Aubertin, *'A hitchhiker's guide to* $R^d$, *or how to walk in space without getting lost'.*

*M.R.C.*

## VICTORIA UNIVERSITY

Wilf Malcolm is to leave us after 21 years as a staff member in Mathematics at Victoria, not to mention 6 as a student here before that! He is to be Waikato University's new Vice-Chancellor; we congratulate him, and also the Waikato Council on its good sense, but we are very sorry to lose him, especially as it means at least the temporary loss of a position in the VUW Mathematics Department. Rob Goldblatt's personal Chair is to be transmuted into the Chair of Pure Mathematics vacated by Wilf, and Rob's established 'lectureship' is to be frozen for a year.

Congratulations to Megan Clark, Shirley Pledger and Ken Russell on their promotion to Senior Lecturers, and also to Shirley and Ken Pledger on the birth of their second son, Lloyd James.

James Graham-Eagle has recently rejoined us for a year's Postdoctoral Fellowship, working on non-linear partial differential equations, having originally been a student here and recently finished his Oxford D.Phil.

We were very happy to have Marston Conder with us this year from Auckland; he and Rob Goldblatt were doing each other's jobs. We gather that Marston was happy too and would recommend such exchanges.

We are sorry to lose Roger Young who has been a Teaching Assistant this year. He has nearly finished his Ph.D. supervised by Jim Ansell who is due back in January from extended research leave.

When Desmond Sawyer was here in November for the annual Honours assessing exercise with Otago's and our scripts we took the opportunity to have a Departmental Lunch in one of Wellington's better smorgasborderies and wish him well in his imminent retirement. That the assessment system has worked so well is due largely to Desmond's hard and conscientious work.

Overseas visitors recently have included Professor Ted Buchwald from New South Wales, who was in NZ visiting the Oceanographic Institute DSIR but who was persuaded to give us a seminar too, on some ingenious methods for Wiener-Hopf factorizations needed in elasticity theory, and Drs Hilary and John Orkendon who are about to arrive as I write this, to tell us and AMD, DSIR about their experiences helping non-University people with applied mathematical problems in Oxford (their home) and Melbourne (where they have been visiting CSIRO). There will be an Applied Mathematics workshop on Monday 3 December where they will be speaking, as will 10 from NZ; not only from VUW and AMD, DSIR, but also Canterbury University, Geophysics Division DSIR, Wallaceville and the Ministry of Energy.

*J.F.H.*

# DSIR

## APPLIED MATHEMATICS DIVISION

Philipa Dafey is joining the Lincoln substation after graduating from Massey University.

Tim Ball has been seconded for a year to become the founding program director of the W. Edwards Deming Institute of New Zealand. The institute has Deming as patron, and has been established as a non-profit making body funded solely from its membership and activity fees to provide education and consultancy to top management for continuing quality improvement within member organizations.

Malcolm Grant has visited Italy for one week and is about to visit the Philipines and Kenya for two weeks for discussions on geothermal. An inhouse DSIR conference in September discussed geothermal modelling at Wairakei.

Joint AMD/VUW seminars by Professors Willard Miller (Minnesota) and Rabinowitz (Israel) were well attended and received. John Law of the National Institute of Agricultural Biology, England gave a seminar on the Coordinated Variety Trail Computer Package.

*G.W.*

# CANTERBURY UNIVERSITY

In August, David Robinson spent three weeks in Australia. He attended the Australasian Combinatorics Conference in Perth, and the International Conference on Mathematical Education in Adelaide. At the latter, he chaired a session on local mathematics competitions.

Robert Bull, with his family, left for a month's visit to Japan, where he will give seminars in Tokyo and Hiroshima, and present two papers to the annual Logic Symposium in Hakone. He intends to complement these activities with a holiday in Kyoto.

Three members of the staff have either gone or will soon go on study leave. Roy Kerr has left for a year at Queen Mary College, London, David Robinson leaves for a month in Australia followed by five months in England, and Frank Gair leaves in January for twelve months in the USA and England.

Those recently returned from one year's study leave are: Brian Woods, who divided his time between the Centre for Vulcanology at the University of Oregon, and the University of British Columbia; Derrick Breach, who visited many universities in the USA, Canada and Europe; and Bill Taylor who was at Westfield College, University of London.

Two members of the staff are currently each giving a series of seminars. Neil Watson is dealing with *'Classical potential theory and Brownian motion'*, and Graham Wood with *'Elements of continuous multi-variate analysis'*.

## Seminars

Dr Ian Porteous (University of Liverpool), *'The geometry of curves and surfaces in three-dimensional space - a singular look'*.
Dr Hemion (University of Bielefeld, West Germany), *'Topology, space and computers'*.
Dr Bill Dowsland (University of Swansea), *'Two- and three-dimensional packing problems'*.
Professor A.J. Ellis (University of Hong Kong), *'Infinite-dimensional simplexes'*.
Dr Mark Anderson (New Zealand Post Office), *'Post Office videotex and packet switching'*.
Mrs Inessa Levi (who is completing a doctorate here), *'Normal transformation semigroups and their automorphisms'*.
Dr Roger Smith (Monash University), *'A theoretical and observational study of cold fronts'*.
Professor Ted Buchwald (University of New South Wales), *'Tides and other long period oscillations on the east Australian continental shelf'*.
Mr Frank Gair (Canterbury), *'The design of mortgages and the need for indexation'*. (Presented to the Department of Economics.)

*R.S.L.*

## OTAGO UNIVERSITY

Professor Ivor Francis will be on "Special Leave" in Auckland next year in order to set up THE W. EDWARDS DEMING INSTITUTE OF NEW ZEALAND. This Institute "is being established to promote improved quality and productivity in New Zealand's manufacturing and service industries and in government, and to provide the means whereby this improvement can be achieved, namely the philosophy and methods of W. Edwards Deming, the renowned American consultant". The Management Committee consists of Professor Francis (President), Mr Tim Ball of the Applied Mathematics Division, DSIR (Director of Programmes) and Mr K. Fink-Jensen, Chairman of the Department of Marketing at the University of Otago (Director of Business).

Dr John Curran has recently been promoted to Senior Lecturer.

The name of our Department has been officially changed to the "Department of Mathematics and Statistics", and the "Biometrics Unit" will be called the "Statistics Unit".

The Department had some visitors who had attended ICME V in Adelaide. They were Dr Schmuel M. Avital (Science Education Department of the Israel Institute of Technology), Dr Porteous of the University of Liverpool and Professor Hassler Whitney of the Institute of Advanced Study at Princeton who was the 1984 NZMS VISITING LECTURER. At a meeting of the Otago Mathematics Association Mr Henry Stoddart (HOD of Mathematics at Otago Boys High School) spoke about ICME V, and Dr Porteous talked about the use of mathematical contests in secondary schools.

Seminars

Professor D.B. Sawyer, *'The Markoff spectrum'*.
Professor Hassler Whitney (Instutute for Advanced Study, Princeton), *'Letting research come naturally'* and *'Geometric origins of the cohomology of groups'*.
Dr I. Porteous (the University of Liverpool), *'The birth of umbilics'*.
Stephen Haslett (Victoria University, Institute of Statistics and Operations Research), *'Biomass estimation for the Brine Shrimp ARTEMIA at Lake Grassmere, Marlborough'*.
Professor W. Davidson, *'Projective methods in Mathematics'*.
Dr Wouter Schaap (Department of Mineral Technology), *'The problem of evaluation and/or optimal extraction of low-grade orebodies'*.

Professors Davidson and Sawyer also gave "farewell seminars" in the Physics Department. Professor Davidson spoke on *'Recent research in relativity and cosmology'* and Professor Sawyer spoke on *'Number theory: Is it Mathematical Physics?'*.

*G.O.*

## UNIVERSITY OF PAPUA NEW GUINEA

Raka Taviri has returned from postgraduate studies at the University of New England and will be chairman in 1985. Om Ahuja has arrived from the University of Khartoum as Senior Lecturer in Analysis. Janelle Humphreys and Jasmin Lucero have been appointed as tutors.

Billy Kaleva has resigned to return to the North Solomons.

The department has cooperated with Demography and Economics in developing two new programmes in Quantitative Methods: a Certificate and a Diploma. The Elements of Computing course has been very popular with 70 internal and 100 external enrolments.

Seminars

Fred Dewa (University of the South Pacific), *'Computer education in Fiji'*.
Len Raj (University of the South Pacific), *'Mathematics education in the South Pacific'*.
Donald Joyce (University of PNG), *'Reflections on ICME V'*.
Kathleen Collard (PNG University of Technology), *'Another catastrophe'*.

*D.C.J.*

## THIRD AUSTRALASIAN MATHEMATICS CONVENTION

The NZMS has received no official notification of the dates of the third Australasian Mathematics Convention. However, according to the minutes of the 55th council of the Australian Mathematics Society (AMS Gazette, Vol. 10 No. 2), this conference is to be held at the University of New South Wales from 13-17 May 1985, with Professor Gavin Brown as chairman of the Organising Committee.

# Feature Article

## IS MATHEMATICS THE 'LATIN' FOR THIS GENERATION?

L.C. JOHNSTON
VICTORIA UNIVERSITY

*An invited lecture at the 19th N.Z. Mathematics Colloquium, Wellington, May, 1984*

INTRODUCTION

Why should mathematicians concern themselves with issues of 'Mathematics Education'? After all, the study of mathematics education does not require mathematical skills. In this small country there are hundreds of thousands of children in schools learning mathematics; they are spending a substantial proportion of their time of formal education on this subject. By no stretch of the imagination can the return on this investment of pupils' time and teacher resources be regarded as satisfactory at the moment; even the results of official studies confirm this. The system will be changed in the next decade; it is necessary for mathematicians to influence this change if it is to be 'mathematics' which pupils study and not some loosely related activity.

This paper will give a personal view of the issues raised by the question in the title. It will give a view of 'Mathematics Education' as a subject and a consequent view of the state of mathematics education in New Zealand. It is very difficult to be original in this area; every mathematics teacher has strong views on our education system, what they like about it and what they would like changed. What can be done in a paper of this kind is to set a context so that issues can be discussed with some hope of clarifying areas of consensus and points of disagreement.

'Mathematics Education' as a subject is relatively new and there is some doubt in my mind as to whether or not mathematics is sufficiently different to other areas of study for it to warrant a special subject status; many problems of mathematics education have equivalent problems in other subjects. What is different about mathematics is the way in which it serves other areas of study and the consequent importance it is generally seen to have. It is regarded as so important that almost all secondary pupils take mathematics for most of their schooling. Whether or not the learning theory of mathematics is significantly different from that of other areas of study is a question I do not intend to even try to answer.

Research work which comes into the area of mathematics education can be divided into two kinds; the first is the study of the process of teaching and learning; it involves theories of what goes on in a pupil's mind and of course such theories are very difficult to test. I believe, in fact, that there will never be a sense in which such a theory can even be verified. A comparison between theories can only be made as far as their helpfulness is concerned. The second area of research is the area called 'technology of education'; it concerns the management of teaching, curriculum design, assessment procedures, and the constraints put on teachers and learners of mathematics by the society in which they work. This area is one which has the same difficulties as the social sciences when it comes to getting 'hard' results. The methods of statistical inference are seldom appropriate (even though they are often used) because in an educational situation there are too many uncontrollable variables. Descriptive statistical methods have a central place but they are very limited in their ability to establish cause and effect relationships. Results are usually subject to a number of different interpretations.

It appears then that the two main areas of research are fraught with difficulty. Griffiths and Howson in their book "Mathematics Society and Curricula" write:

> "In noting the absence of a science of learning and thus, a fortiori of a science of mathematical education, we might, however, claim that our prime aim at the present time is to develope a sound technology of education."

Perhaps our problem is in expecting a science of learning; a study should not need to be 'scientific' to be respectable and it should not need to be quantified and use computers to have its results taken seriously.

PRINCIPLES OF MATHEMATICS LEARNING

In spite of the difficulties of research in both areas of mathematics education there are some principles of learning which have been demonstrated to be helpful and would be accepted as clearly true by experienced teachers. Such principles can be backed up by observation even though the way they affect individuals' minds can only be surmised. I will list some of these principles and in doing so, recognise that I am not presenting anything new; in fact all possible principles of learning have probably been stated centuries ago. The purpose of mathematics education studies then, is not to discover new principles of learning but to present principles in a way that is relevant to the present time and to present a theoretical structure of how our system is working so that we can reach common ground enabling us to understand and discuss important issues; results of research into learning theories can then be fitted into this structure and a basis is laid for good decisions to be made both in wider areas such as the administration of examinations and the design of the curriculum and in the personal but equally important area of classroom teaching.

Without such a structure it is too easy to be persuaded by an argument which ignores important issues and facts. One of the features of educational thought through the centuries is that great educators have been successful and influential with diametrically opposed methods and philosophies.

I have chosen the following principles of learning to discuss in this paper.

1. Skills are learned by repetitive practice.
2. Concepts are learned by examples.
3. Memory is aided by continual (spaced) recall.
4. Learning is faster and more effective if pupils are strongly motivated; preferably by the subject matter rather than by external motivations such as examinations, career prospects or threats of punishment.
5. Information on the nature of a good performance aids learning.
6. Continual, helpful feedback on the progress of learning is very helpful.
7. Different students learn at different rates, are at different stages of development, and have various motivating forces.
8. A teacher must have a good knowledge of the subject he or she is teaching.

These principles can be demonstrated experimentally even though the mechanism by which the mind of the learner benefits through their application cannot be known. It is principles of this kind that can be helpful to teachers and to educational administrators. It is instructive to examine how the managers of our system measure up in the light of these principles. I put forward for discussion the following facts and opinions.

(a) Most syllabuses are so full that the vast majority of pupils cannot both complete the syllabus and practice skills,; in the event most (teachers) opt for completing as much of the syllabus as possible. (cf 1)

(b) Many text books introduce concepts by definition. (cf 2)

(c) School texts are not designed to encourage continual spaced recall of skills and content that have been previously learned. University courses tend to study topics in short periods of time before leaving them to the following year or later. (cf 3)

(d) For many of our secondary school pupils the only motivations for learning mathematics are an external examination and a supposed need for mathematics in their employment. (cf 4)

(e) Standard of performance is irrelevant in a system which will automatically fail one half of the pupils irrespective of how high a level they reach and, by the same token, automatically pass one half of the pupils no matter how low a level they have reached. (cf 5)

(f) Good teachers do give pupils helpful feedback.

(g) We have one syllabus for all pupils irrespective of ability, background, level of development, and interests. We have a report on the 'core curriculum' which suggests that 70% of the time of all pupils would be spent on core syllabuses no matter how able they are. (cf 7)

(h) Personal observation indicates that a knowledge of mathematics is a positive hindrance to acceptance into teachers college. (cf 8)

These are the responses of our educational managers to universally accepted principles of learning; it appears that the inertia of the bureaucracy is such that our educational administrators can accept and defend a system which is not only unhelpful to pupils but is totally against principles which would be accepted by those same administrators.

On the other side of mathematics education, the teacher learner side, it is heartening to realise that many teachers behave in a much more rational way and by superhuman efforts succeed in teaching in spite of the constraints our system imposes on them.

In New Zealand then, the urgent need in mathematics education is not for new learning theories, it is not for a detailed knowledge of the mechanism by which mathematics is learned, it is for an educational technology which takes account of principles which are already known and accepted.

## A STRUCTURE OF MATHEMATICS EDUCATION

Mathematicians tend to be very suspicious of experts in educational technology. There is some justification for this; they have seen statistics 'used' in ways which are quite unjustified to support supposed research findings but more often their criticism is based on the use of 'jargon' by the educationists. Mathematicians usually try to express results in the most elegant concise way possible; very often educational writings do not conform to this ideal. However my experience has been that behind the words there are ideas well worth considering. Two such words are 'aims' and 'objectives'. I have often heard them labelled as jargon but it is still quite obvious that before making decisions either as teachers or managers we must know what we are trying to do.

What are the possibilities? What can mathematics education do for a pupil? The following analysis is put forward for discussion.

### MATHEMATICS EDUCATION

- (A) Teach content of the kind which is needed in everyday life.
- (B) Teach content which has specialised uses.
- (C) Teach general skills such as ability to discover patterns and relationships, develop concepts, draw logical conclusions, express thoughts precisely and concisely, form generalisation, solve problems, thing abstractly.
- (D) Teach skills which are needed by specialists who use mathematical techniques.

This analysis provokes the question: "Which of these possibilities do we want for our pupils?" and of course taking into account principle number eight the response must be "for what purpose and for which pupils?".

On the surface some answers are clear enough. Everyone needs to learn the content and skills which are needed in everyday life. (A)

Pupils who will use particular areas of mathematics in their employment to learn other subjects, to learn more mathematics or in their recreation need to learn the content and skills involved. (B) & (C)

Mathematicians need the general skills. (D)

This is what is absolutely essential; it is not necessarily what is desirable. The points which must be decided are:

(i) to what extent the general skills (D) should be learned by all pupils (through Mathematics);

(ii) how the specialised material is to be chosen given that there are too many possible different requirements to suit all individuals.

One answer which is becoming increasingly fashionable is that all pupils should learn the essential content and skills (A) but that only those who choose to or who intend to use mathematics at a higher (professional) level should do more; they should learn a comprehensive coverage of content and skills. Children who will eventually need a limited amount of specialised material in trades can learn this later when the need is clear and their motivation is stronger.

An opposing view is that since it takes time and practice to learn skills they are best learned at school before they are needed and that the majority of pupils who will never use any mathematics (other than (A) ) should be learning mathematics in order to develop general skills (D) . Certainly, if one considers Piaget's levels of development (which are widely accepted even by opponents of Piaget's detailed theories) it is at least possible on the available evidence, and in my opinion it is very likely that the movement of a child through

Piaget's levels depends very much upon teaching. A child who is not presented with challenges will not make progress. These challenges do not have to be mathematical but mathematics is unique in that the skills used to solve particular problems can be clearly delineated. The fact that mathematics education is not being used in this way at present is not a reason for eliminating it; it is a reason for altering the system so that teachers are not bound by constraints which prevent them from teaching general skills.

In my opinion the general skills should be the first priority (after list A ) in our mathematics education. Of course, they cannot be taught in a vacuum apart from particular content but our most important aim should be to teach the skills. Most pupils will never need to solve a quadratic equation after leaving school but being able to do so may enable them to solve problems which practice worthwhile skills. The pupils who must be able to solve equations efficiently, to do algebraic manipulation correctly and quickly, and to remember formulae are the more able ones who will learn tertiary level mathematics not those who will never use mathematics after leaving school. For most pupils we must see mathematics as a means to an end; the end of developing skills.

The proponents of the view that only the essential every day mathematics should be taught to any but the more able students may well reply that to talk of teaching general skills begs the question of whether or not such a thing is possible; they may state that skills learned <u>through mathematics</u> can be used only to <u>do mathematics</u>. This reply raises the very important and difficult issue of 'transfer of learning'.

## TRANSFER OF LEARNING

The concept of transfer of learning has been part of educational thought since the first century or earlier. Indeed the concept as it was understood in post Renaissance England dates back to the ancient Greeks. In its modern form "transfer of learning" is taken to mean the use of skills learned in one area to solve a problem in a different area. The question of whether or not such transfer is possible is related to the question of whether individual characteristics are developed by the environment or whether they are inherited. There is no doubt that humans have the ability to solve new problems; the skills they use to do this must be either learned or inherited. If they are learned then the solving of new problems exhibits "transfer of learning".

Quintilian, a teacher and writer of Rome, wrote in the first century:

> "As regards geometry, it is granted that portions of this science are of value for the instruction of children; for admittedly it exercises their minds, sharpens their wits, and generates quickness of perception. But it is considered that the value of geometry resides in the process of learning, and not as with other sciences in the knowledge they acquired."

A more recent commonly quoted writer on this theme is John Locke (1632-1704); his concern was "the education of a gentleman". He wrote:

> "I have mentioned mathematics as a way to settle in the mind a habit of reasoning closely and in train; not that I think it necessary that all men should be deep mathematicians, but that having got the way of reasoning which that study necessarily brings the mind to, they might be able to <u>transfer it</u> to other parts of knowledge as they have occasion."

Closely associated with the notion of transfer as we may understand it today was the notion of formal discipline. To quote John Locke again, he wrote:

> "As the strength of the body lies chiefly in being able to endure hardships so also does that of the mind."

Clearly implicit here is the theory that the mind is capable of general training which can later be applied in other directions. The association between a healthy body and a healthy mind dates from the ancient Greeks and centuries later was translated in Western Europe into a philosophy that physical hardships and mental discipline "trained the mind"; that the content of education was far less important than the way it was learned. This philosophy of education has been interpreted as saying that it matters little what a child learns as long as he does not like it.

It is the notion of training of the mind which became associated with the teaching of Latin and Greek. To quote from a text by John Brubacher:

> "Making the method of acquiring knowledge rather than the content of knowledge itself the aim of education led to the formulation of the aim of education as mental discipline. This has been a most persistent aim, especially in periods of cultural lag. Perennial as were many of truths embedded in the Latin and Greek classics, numerous authors no longer spoke with authority to the people of the seventeenth and eighteenth centuries. Nonetheless they remained in the curriculum of the schools by sheer weight of inertia of social prestige, to say nothing of the formalism and obscurantism of the schools and universities. Hence, a new reason or purpose had to be found for their continued study. This new purpose or aim was the 'discipline of the mind'. Education was to aim not so much at storing the memory with information as at exercising and improving the analytical and organisational facilities of the mind. Brought to a fine edge, these faculties could later cut into any field of human knowledge with equal incisiveness. Discipline of the mind, furthermore, was to be matched by discipline of the body."

The psychological theory which supported the notion of training the mind was 'faculty psychology' - a theory which persisted into the twentieth century; particularly among laymen and teachers. The mind was thought to be divided into various faculties such as judgment, memory, imagination, and observation. Thus, study of a subject which exercised these 'faculties' was supposed to sharpen them so that they were improved wherever they were used. Memorisation of a large vocabulary in the learning of a language was expected to improve memory in general. I do not intend to discuss the truth or otherwise of this theory; but wish to report on its influence on the school curriculum. In a paper published in 1901 E.L. Thorndike and R.S. Woodworth reported on experiments which showed that the extent of transfer of training of the mind in this sense to other fields was extremely limited - much less than had been popularly believed. These results shook confidence in the theory of transfer and cast serious doubt on the validity of faculty psychology. In doing so it took away the last excuse for retaining Latin and Greek as a major part of school curricula. Nevertheless, fifty years later some universities retained Latin as a compulsory subject for Arts degrees and the old "training of the mind" view still persisted. It is interesting to note that in Renaissance times the teaching of Classical languages was a means to the end of understanding the Greek and Roman cultures. Through the centuries, as the cultures become less vital to the societies of the times, the curriculum came to include only the languages and later the emphasis was mainly on grammar. At the present time classical studies courses are again centred on the cultures.

## LATIN AND MATHEMATICS

There are many similarities between the position of mathematics in the curriculum now and the position of Latin in the last century. I wish to exclude the useful everyday mathematics from this discussion which could be regarded as independent of much of the present curriculum.

1. Mathematics is regarded as an essential part of an education.
2. Mathematics is a passport to some types of employment in which it will never be used.
3. Mathematics has been regarded as a subject unsuited to women.
4. A serious consideration of the value of teaching mathematics leads one to the idea of 'transfer of learning'.
5. Mathematics has been a traditional part of school and university curricula in Western Europe since the twelfth century.

Latin is no longer taught in many New Zealand secondary schools and the size of university classes is only a fraction of what it was a few decades ago. This may be appropriate (I am not qualified to pass judgement on that) but I believe the decline would have been less dramatic and classical studies would at present be valued more highly if Latin had not persisted in the curriculum for reasons which were later decided to be false.

The lesson for mathematicians is that we should not make false claims for our subject. We should recognise the fact that the vast majority of students will never use mathematics after leaving school (except for the basic everyday useful component).

What, then is the value of mathematics in the curriculum and can the 'transfer of learning' theory justify its inclusion? It is I believe, obvious, that some transfer of learning does take place and this assertion is backed up by educational research. For example the skill of proving a uniqueness theorem in one branch of mathematics is likely to 'transer' to another branch fairly readily; practice at discovering a general formula for the terms of a sequence will clearly be helpfull in discussing formulae for different sequences but at the other extreme it is not so clear that the process of memorising a large number of mathematical formulae will be helpful in remembering an appointment next week. What appears likely and again is backed up by limited research is that the extent of transfer of a skill between tasks depends on how closely the tasks are related. To put this another way, the more specific the skill the more likely it is to transfer. A general skill such as "good judgement" would be applied in very diverse situations and so cannot be expected to transfer. An intermediate case would be "ability to reason logically". It is not clear that this skill, practised in geometry would transfer to politics. It is more hopeful however that if it is practised in a wide variety of problems of a 'real' kind, expressed in words rather than mathematics, that transfer would occur.

The results of research as well as common sense indicate that 'transfer of learning' does occur; such transfers may be limited but on the other hand the logical result of supposing that no transfer occurs is that human beings, like computers, can only perform exactly as they have been programmed and that attempts to educate them are doomed to failure.

The lesson for designers of mathematics curricula is that the aim of teaching transferable skills should be foremost in their thinking and courses should be designed appropriately. Mathematical content as such is of no value to most pupils (only the minority who will use mathematics need any particular content); it is of course, necessary if problems are to be solved mathematically but it should be seen as a means to the end of acquiring skills.

In my view three kinds of mathematics courses are needed:

(a) A course for all pupils which teaches essential everyday mathematical skills and content.

(b) A course (not necessarily for all pupils) which teaches problem solving at appropriate levels in a wide range of real situations; this course to have no fixed content and to be appropriate to the ability and level of the pupils involved.

(c) A course which teaches mathematical content and skills (including mechanical skills) to pupils who will use mathematics at a higher level and to those who will continue the development of this basic part of our culture.

The design of such a curriculum should not be beyond the powers of human ingenuity but I fear that the response of our administrators may be "how would it be examined?"

# Conferences

*Compiled by Dr M.R. Carter, Massey University*

***1985***

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|---|---|
| January 7-18 (Berkeley, California) | *Workshop on Three-Manifolds*<br>Details from Mathematical Sciences Research Institute, 2223 Fulton Street, Room 603, Berkeley, California 94720, U.S.A. |
| January 10-19 (Santa Barbara, California) | *Conference on Solitons and Coherent Structures*<br>Details from The Director, Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, U.S.A. |
| January 21-24 (Houston, Texas) | *SEG-SIAM-SPE Conference on Mathematical and Computational Methods in Seismic Exploration and Reservoir Modeling*<br>Details from Society for Industrial and Applied Mathematics, 117 South 17th Street, Suite 1405, Philadelphia, Pennsylvania 19103, U.S.A. |
| February 3-7 (Launceston, Tasmania) | *Australian Applied Mathematics Conference*<br>Details from Dr D.F. Paget, Department of Mathematics, University of Tasmania, GPO Box 252C, Hobart, Tasmania 7001, Australia. |
| February 18-21 (Kuwait) | *Conference on Mathematical Analysis and its Applications*<br>Details from Department of Mathematics, Kuwait University, P.O. Box 5969, Kuwait, Kuwait. |
| March 26-28 (Rome) | *M.C. Escher: An Interdisciplinary Congress*<br>Details from M. Emmer, Dip. to di Matematica, Ist. "G. Castelnuovo", Universitá "La Sapienza", Piazzale A. Moro, 00100 Rome, Italy. |
| April 17-19 (New York) | *Symposium on Complexity of Approximately Solved Problems*<br>Details from Computer Science Department, Columbia University, New York, New York 10027, U.S.A. |

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|---|---|
| May 16-21 (Berkeley, California) | *Workshop on Differential Geometry* Details from Mathematical Sciences Research Institute, 2223 Fulton Street, Room 603, Berkeley, California 94720, U.S.A. |
| May 20-23 (Pau, France) | *15th Journées de Statistique* Details from Laboratoire de Statistique et Probabilités, Dépt. de Math., Faculté des Sciences, Avenue de l'Université, 64000 Pau, France. |
| May 20-24 (Auckland) | *Pacific Statistical Congress* Details from the Committee Secretary, Pacific Statistical Congress, Department of Mathematics, University of Otago, P.O. Box 56, Dunedin, New Zealand. |
| May 23-June 1 (Berkeley, California) | *Workshop on Four-Manifolds and Geometry* Details from Mathematical Sciences Research Institute, 2223 Fulton Street, Room 603, Berkeley, California 94720, U.S.A. |
| May 26-June 2 (Jachranka, Poland) | *International Conference on Functional-Differential Systems and Related Topics IV* Details from Danuta Przeworska-Rolewicz, Mathematical Institute, Polish Academy of Sciences, Sniadeckich 8, 00-950 Warsaw, Poland. |
| June 5-14 (Berkeley, California) | *Conference on Geometry and Operator Algebras* Details from Mathematical Sciences Research Institute, 2223 Fulton Street, Room 603, Berkeley, California 94720, U.S.A. |
| June 19-21 (Dublin) | *Fourth International Conference on the Numerical Analysis of Semiconductor Devices and Integrated Circuits* Details from NASECODE Organising Committee, Doole Press Limited, PO Box 5, 51 Sandycove Road, Dun Laoghaire, Co. Dublin, Ireland. |
| June 24-28 (Tübinegen, West Germany) | *Aspects of Positivity in Functional Analysis* Details from Mathematisches Institut der Universität Tübingen, Auf der Morgenstelle 10, D-7400 Tübingen, Federal Republic of Germany. |
| July 16-19 (Exeter) | *Second International Conference on the Teaching of Mathematical Modelling* Details from Sally Williams, Conference Secretary, University of Exeter, St. Lukes, Exeter, EX1 2LU, England. |
| July 31-August 8 (Berkeley, California) | *Symposium on the Transmission of Mathematical Science* Details from J. Dhombres, UER de Mathématiques, 2 rue de la Houssinière, F-44072 Nantes Cedex, France. |
| September 2-12 (Castelvecchio-Pascoli, Italy) | *NATO Advanced Study Institute: Advances in Microlocal Analysis* Details from H.G. Garnir, Department of Mathematics, University of Liege, 15, avenue des Tilleuls, B-4000 Liège, Belgium. |
| September 16-20 (Dusseldorf, West Germany) | *Sixth International Meeting on Clinical Biostatistics* Details from Dr R.A. Dixon, University of Sheffield Medical School, Beech Hill Road, Sheffield S10 2RX, United Kingdom. |
| September 17-19 (Madrid) | *International Symposium on Numerical Analysis* Details from Carlos Vega, Faculty of Computer Sciences, Polytechnical University of Madrid, Carretera de Valencia, km 7, Madrid 31, Spain. |
| November 18-22 (Hamilton, New Zealand) | *International Symposium on Single and Multiphase Flow through Heterogeneous Permeable Materials* Details from IUTAM Symposium, Physics and Engineering Laboratory, D.S.I.R., Private Bag, Lower Hutt, New Zealand. |

***1986***

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|---|---|
| July 27-August 1 (Seattle, Washington) | *Thirteenth International Biometric Conference* Details from Gerald van Belle, Dept. of Biostatistics, University of Washington, Seattle, Washington 98195, U.S.A. |
| August 3-11 (Berkeley, California) | *International Congress of Mathematicians* Details from ICM-86, P.O. Box 6887, Providence, Rhode Island 02940, U.S.A. |
| August 11-16 (Victoria, British Columbia) | *Second International Conference on Teaching Statistics (ICOTS 2)* Details from Professor T. Lietaer, University Extension Conference Office, University of Victoria, P.O. Box 1700, Victoria, British Columbia, Canada B8W 2Y2. |

# Problems

Sub-edited by A. Zulauf, University of Waikato

*PROPOSALS of problems should be sent to the sub-editor and should be accompanied by solutions and/or relevant references, comments, etc.*

*SOLUTIONS should be sent to the sub-editor within three months from the publication of each problem. If you discover that a problem has already been mentioned or solved in the literature, please send full details to the sub-editor.*

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Problem 15 *(3816547890 and all that)*

For any base $b$ $(b \geqslant 2)$, let us say that the positive integer $n$ has the property $P(b)$ if and only if its $b$-ary representation $n = a_{b-1}a_{b-2} \cdots a_1a_{0(b)}$ contains each of the $b$ $b$-ary digits exactly once, and $k$ is a divisor of $a_{b-1}a_{b-2} \cdots a_{b-k(b)}$ for $k = 1, 2, 3, \ldots, b$. (For instance, 3816547290 has property $P(10)$ since it contains each of the ten denary digits exactly once, and $1|3$, $2|38$, $3|381$, $4|3816$, ... .)

Further, let $N(b)$ be the number of positive integers with property $P(b)$ .

D.B. Gauld of Auckland suggests the following problem.

(i) Prove that $N(10) = 1$;

(ii) Determine $N(b)$ for $b = 12, 14, 16, 18, \ldots$

(See also the mini-problem below. The point of the rather elementary problem 15(i) is, of course, to find a short and elegant proof. Problem 15(ii) appears to be much more difficult, and partial solutions would be welcome.

*(Sub-editor)*

Mini-problem:
*(3816547890 and some of that)*

With the notation of problem 15 above, prove

(i) that $N(b) = 0$ if $b$ is odd,

(ii) that all the integers with property $P(b)$, $2 \leq b \leq 8$, are

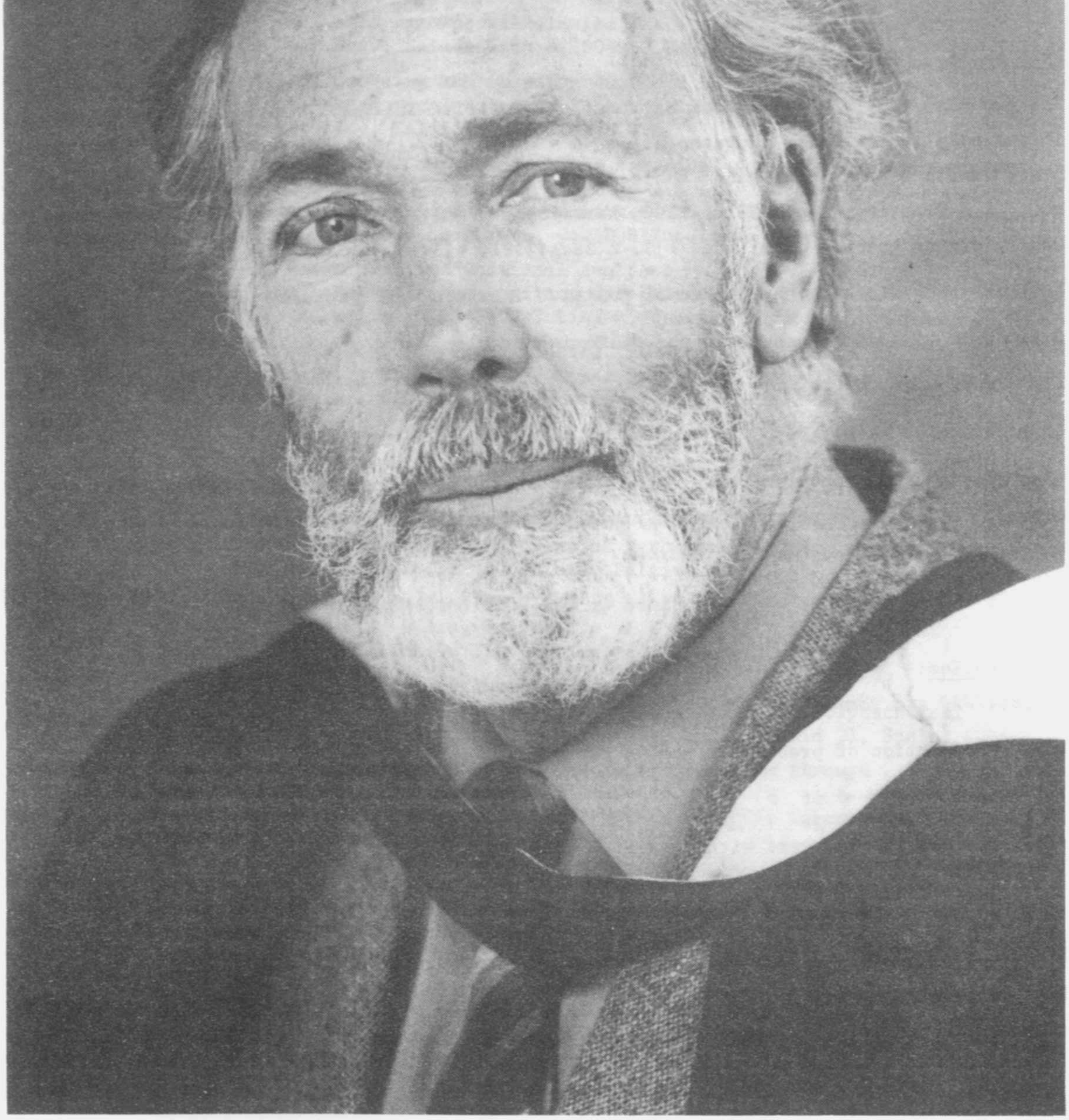
$10_{(2)}$, $3210_{(4)}$, $1230_{(4)}$, $54321_{(6)}$, $143250_{(6)}$, $56743210_{(8)}$, $32541670_{(8)}$ and $52347610_{(8)}$

Problem 12 revisited:

D.B. Gauld of Auckland, whilst checking whether $e^{\pi\sqrt{163}}$ is nearly an integer, accidentally discovered that $(\pi\sqrt{163})^e = 22806{\cdot}9992$ is nearly an integer.

Problems 8a and 13 still open

These problems have not, so far, elicited any response. The deadline has been extended, and solutions or partial solutions are still welcome.



# Centrefold

## PROFESSOR DESMOND SAWYER

The retirement of Desmond Sawyer at the end of this year marks a watershed for the Otago University Mathematics Department. As department head for twenty three of the last twenty eight years (broken by a five year spell at Waikato in the sixties) Desmond has had a major influence on the present day shape of the department; through times of uncertainty, rapid growth and latterly, retrenchment, his firm leadership has been the main stabilizing factor.

Desmond Sawyer was born in Sheffield in 1924 and educated at Manchester Grammer School and then St John's College, Cambridge. As with others of his generation, university studies were interrupted by the war; after two terms in 1943 he spent three years in the army serving with the Royal Artillery and then as a survey officer with the Royal Indian Artillery. He returned to Cambridge for two years after the war, gaining a Distinction in Part III of the Mathematical Tripos and the Adams Prize. Later in 1948, newly married, he accepted a lectureship in mathematics at Otago University (under Professor R.M. Gabriel).

There were only four members of staff at that time; teaching loads were both diverse and heavy while opportunities for research were limited. After 3½ years in this first appointment Desmond left New Zealand and took a position at the University College of the Gold Coast with Professor Hugh Blaney who had similar mathematical interests in convexity and the geometry of numbers. As far as research was concerned the next two years in Africa proved to be a productive period, though the Sawyers were pleased to return again to Dunedin in 1954. Here promotion to a readership came quickly, at the beginning of 1957, but almost simultaneously events took another turn and Desmond found himself acting head of department following the death of Professor Gabriel. Within a few months, at the age of thirty three, he was appointed to the chair.

During the next seven years at Otago his interest and involvement in administrative matters grew steadily. For a time he was chairman of the Otago branch of the A.U.T. He served on the Academic Board of the University of New Zealand until its dissolution in 1961 and also on its Entrance Board; subsequently he was a member of the new Universities Entrance Board for several years. He served also on the boards of local high schools and his three year term as Dean of the Faculty of Science coincided with the phasing in of reforms in the degree structure including the new Honours degree.

At the beginning of 1963 Desmond was appointed by the University Grants Committee to the Academic Advisory Committee for the University of Waikato and became intrigued by its possibilities. This led the following year to an invitation to become Professor of Mathematics and Deputy Vice-Chancellor at the new university, where he remained for five years. Over this period he became deeply involved with many aspects of university development, in particular, playing an active role in the formation of the School of Education and acting as its Dean during the initial stages.

Desmond returned to Otago University in January 1970, now as Professor of Pure Mathematics and Chairman of the Department. The rapid growth in student numbers and influx of new staff in the early seventies led to a rebuilding of the department and the consequent growing pains placed a heavy burden on the chairman's shoulders. Nevertheless, he continued to participate fully in all aspects of the mathematical life of the department, carrying the same teaching load as other members of staff throughout and playing an active role in the various series of staff seminars which have occurred over the years. Among his colleagues he is respected most for his keen geometric insight, which none can challenge, and for a certain flair for concise expression of difficult matters, which penetrates to the heart of a problem. He has plainly set himself very high standards in every aspect of his work and earned the respect of his staff by his own example.

At the end of this year Desmond and his wife Pam will move to Wanaka where they have had a holiday home for several years. All his friends and colleagues will join with me in wishing them a long and happy retirement.

*David Hill*

# Book Reviews

AN INTRODUCTION TO NUMERICAL METHODS WITH PASCAL, by L.V. Atkinson and P.J. Harley. Addison-Wesley, London (1983), 300 pages.

This text is intended for undergraduate students of numerical analysis, and it is based on the authors' experience of teaching such courses at the University of Sheffield.

Chapter 1: A summary of Pascal The language Pascal is summarized in Chapter 1, and thereafter numerical algorithms are exemplified by Pascal procedures. The programming is generally of fairly high quality and the programs are neatly type-set. (Note that $\pi$ is best generated as 4 * arctan(1) which assures adequate accuracy, and not declared to 6 or 9 figures as on pages 8 and 158.) In Chapter 4, matrices are represented not by doubly-subscripted arrays of reals, but by records of singly-subscripted arrays, which are stored as files. Such a representation might perhaps improve efficiency of execution for large matrices with some compilers and some hardware systems - but any possible gain is offset by the increased difficulty of writing and understanding matrix computations with such a non-standard representation.

It is stated that computers store information and perform arithmetic upon binary digits. Such statements are misleading, since some computers operate in various bases other than 2 (including 10, -2, 3 and 8); and a high-level language such as Pascal is independent of the machine-level representation (apart from the details of real arithmetic).

Chapter 2: Rounding errors No indication is given that normalized floating-point numbers require special representation for zero. The discussion of round-off does not point out that the mantissa sometimes needs to be shifted down after rounding.

Chapter 3. Non-linear algebraic equations The standard methods for solving a single non-linear equation are presented, with an indication of the extension of the Newton-Raphson method to pairs of equations.

Chapter 4. Linear algebraic equations Eigenvalues are used on pages 122 and 123, but they are not defined until the next chapter. The account of iterative methods (on p. 134) is very muddled, with a confused restriction to matrices with non-negative elements. Theorem 4.21 states that the SOR matrix (with parameter $\omega$) has spectral radius less than 1 "only if $0 < \omega < 2$"; whereas W. Kahan proved that this holds for all matrices to which SOR can be applied. Theorem 4.22 asserts various relations between the spectral radii of the Jacobi, Gauss-Seidel and SOR methods; without any indication that those results hold only for consistently-ordered matrices with Property A. The important topic of iterative refinement of solutions of linear algebraic equations is not treated.

Chapter 5: Eigenvalues and eigenvectors The fundamental Definition 5.1 is muddled, and it does not indicate any relation between an eigenvalue and the existence of an eigenvector. The restriction to real matrices is pointless, since the theory is most naturally developed for complex matrices. The discussion of normalization of eigenvectors (p.141) overlooks the ambiguity of sign of the normalized vector and hence the assertion (on p.146) that the scaling factor $\mu_{k+1}$ converges to the dominant eigenvalue is false. The account of the power method is restricted to the case of a simple dominant eigenvalue, with the false assertion (p.144) that the method might not converge for a multiple dominant eigenvalue. The scaling of the vectors which is proposed on p.145 does not work if the eigenvector has approximately dominant elements with opposite signs. The section on the shifted inverse power method gives no warning of the problems arising from near-singularity of the shifted matrix. In the account of Jacobi's method for eigenvalues nothing is said about convergence of the cyclic Jacobi and threshold Jacobi methods. Givens's method is presented with no indication that it is numerically unstable.

Chapter 6: Discrete function approximation Theorem 6.3 states the standard expression for the error of polynomial interpolation, but falsely restricts the formula to be valid only for interpolating between the extreme interpolation points. That error is compounded by the explicit denial (on p.183) that anything can be said about the error of extrapolation. Theorem 6.4 indicates the advantages of taking interpolation points at the roots (sic!) of a Chebyshev polynomial; but without any indication that the interval for interpolation needs then to be normalized to [-1, +1]. In the discussion of least-squares approximation by polynomials the convenience of orthogonal polynomials is indicated - but nothing is said about how to construct them, or about their advantages when changing the degree of the approximating polynomial. Theorem 6.9 states the standard Chebyshev alternation property characterising minimax approximation by polynomials; but does not illustrate it by the very important case of the Chebyshev polynomials (which are not defined in this text). The exchange algorithm is presented very confusingly.

Chapter 7: Differentiation and integration The account of Romberg's method claims (on p.227) that the Romberg estimates of a definite integral are equal to those of corresponding Newton-Cotes formulae; which is false (except for Romberg's columns 0, 1 and 2). Theorem 7.4 proves that any representation of a polynomial as a linear combination of orthogonal polynomials is unique - but nothing is said about whether such a representation exists for a general polynomial, or how to construct it. Theorem 7.8 states the standard expression for error of weighted Gaussian quadrature in terms of the corresponding orthogonal polynomials; but those are mistakenly called monic Chebyshev polynomials of the first kind.

Chapter 8: Ordinary differential equations This chapter presents descriptions of linear multi-step methods and Runge-Kutta methods, and their stability, consistency and convergence.

The mathematical parts of the text state many results without proof, and frequently without even a reference to a proof. Addison-Wesley Publishing Company have published this book in their International Computer Science Series. The manifold errors listed here suggest that the two persons who are named as editors of that Series cannot have examined the manuscript of this book with adequate attention, when they accepted it for publication.

*G.J. Gee*

NUMERICAL METHODS FOR THE PERSONAL COMPUTER, by Terry E. Shoup, Prentice-Hall, Englewood Cliffs (1983), 238 pages.

The Preface claims that "the text and its accompanying software should provide a package suitable for teaching numerical methods courses as universities, and for individual scientific and engineering personnel". There are lots of glossy pictures, illustrating various items of computer hardware available on the American market in the latter part of 1982.

In Chapter 1, the Introduction, the sketchy history is restricted solely to the USA. A typical 8-bit microprocessor is stated (p.5) to be "capable of calculations providing nine-significant-digit accuracy", without any explanation of what is meant here by "capable".

In the following chapters various numerical methods are presented, without proof or adequate references to proofs. They are accompanied by clumsy flow-charts and by programs written in some undefined version of BASIC. The programs are each specific to one particular problem, with all coefficients written into the program. Such programs could have been made very much more general by using functions and sub-routines, and by typing in data as input from the keyboard.

In Chapter 2, on roots of equations, the classification of equations (p.18) is absurd. The explanation of Newton's method is very muddled, with a false assertion that Newton's method does not converge to a multiple root. The definition of polynomial equations (p.28) restricts them implicitly to be monic, since the leading coefficient is taken as 1. The assertion (p.29) that quadratic and cubic equations can be solved directly, but that equations of higher "order" (rather than "degree") require indirect methods is false, since quartic equations can be solved algebraically; and no warning is given that even for cubic equations the direct solution is practically useless. The long program for Bairstow's method is badly unstructured. Polynomials are evaluated by summing the powers of $x$ multiplied by the coefficients, with no mention of nested multiplication! Quadratic equations are solved by evaluating both -P + SQR(D) and -P-SQR(D), without any warning of possible loss of significant figures. The methods recommended (p.38) for estimating roots of equations are absurd.

In Chapter 3, on roots of simultaneous equations, the process of back substitution, used for triangular matrices on p.44, is not defined. The Gauss-Jordan method for linear equations is said to be better than Gaussian elimination, without any comparison of their computational costs. The program for the Gauss-Jordan method always performs row-interchange, even when no pivotal interchange is required; and it operates on all rows and all columns of the matrix at each stage, rather than on the appropriate smaller submatrix. The discussion of matrix inversion contains no indication that the actual inversion of a matrix is very rarely advisable. Cholesky's method is falsely said (p.53) to be "also known as Crout's method". The program for factorizing A as LU is made unnecessarily obscure by over-writing A with L and U.

In the section on iterative methods for systems of linear equations, the definitions of the Gauss-Seidel and Successive Over-Relaxation methods are very obscure. The program for performing Successive Over-Relaxation is recommended to connoisseurs of unconscious humour. It could well be preserved as an "awful warning", demonstrating several types of fault which should never be committed in programming. In the section on nonlinear simultaneous equations the account of fixed-point iteration is very muddled, and the discussion of convergence is hopelessly confused. The "parameter perturbation procedure" is utter nonsense.

In Chapter 5, on eigenvalue problems, the assertion about unique solutions is wrong (p.74), since eigenvectors have arbitrary scaling. The statement that "if the eigenvalues of a matrix are distinct then the eigenvectors will be orthogonal" (p.76) is false for non-symmetric matrices. The recommended scaling of an eigenvector "by dividing each element by the sum of the squares of the other elements" (p.76) is ridiculous. The treatment of the power method is very muddled. The discussion of smallest eigenvalues specifies iteration with the inverse of the matrix, rather than factorization of the matrix and back substitution for each successive vector. In the treatment of Jacobi's method the sign is determined as $(x/y) \;/\; |(x/y)|$, where $x = 2a_{kl}$ and $y = a_{kk} - a_{ll}$. That is not only very cumbersome; but it fails when $x$ or $y$ is zero. The account of Givens's method (mis-spelt as Given's method) gives no warning that it is less stable numerically than Householder's method. The account of Sturm sequences for eigenvalues of symmetric tridiagonal matrices does not consider the important case of zero elements adjacent to the diagonal. The very brief mention of the QR method (p.97) is followed by a very long program (pp. 98-103) which is so badly written as to be practically incomprehensible.

The book continues for another 135 pages. However, enough has been said to show that the programming is of a quality which would not be acceptable from a first-year student and that the author does not possess a depth of knowledge in the subject presented. So much of what is said, and what is not said, indicates the author's lack of familiarity with even the most rudimentary aspects of numerical analysis.

Not recommended.

*G.J. Tee*

SÉMINAIRE D' ALGÈBRE PAUL DUBREIL ET MARIE-PAULE MALLIAVIN. Proceedings, Paris 1982, edited by M.-P. Malliavin, Springer-Verlag, New York (1983), 339 pages, US price approx. $15.20.

This annual seminar on algebra was founded in 1947 and for many years was run under the direction of Paul Dubreil. The main aims of the seminar are to provide a means for the dissemination of recent theories which are not yet widely known and to give algebraists an opportunity to report on recent progress. This volume presents articles by fifteen algebraists from various countries and we will now briefly describe the contents of these.

The general purpose of the first article, "Lectures on invariants, representations and Lie algebras in systems and control theory" by M. Hazewinkel, is to explain to pure mathematicians some of the many mathematical problems (and their solutions) which arise in systems and control theory. This is followed by T.A. Springer's "Series de Poincaré dans la théorie des invariants" which describes the Poincaré series of a finitely generated graded algebra and how this notion is useful in the theory of invariants of reductive groups.

G. Falting's "On the cohomology of locally symmetric hermitian spaces" comes next. This looks at the representation of torsion-free discrete subgroups of certain linear semi-simple Lie groups using cohomology groups. Then follows an article by A. van den Essen on the torsion-free part of the cokernel of certain differential operators acting on modules over derived rings of polynomials on power series rings over a field.

The next two articles, "The regular representations of the tame hereditary algebras" by V. Dlab and "Separating tubular series" both deal with the representation of finite-dimensional algebras, a part of ring theory where there has been considerable activity in the last ten years following on from the pioneering work by M. Auslander and I. Reiten.

More representation theory follows, but now of Lie algebras, in G.B. Seligman's "Higher even Clifford algebras" where it is used to describe certain nondegenerate hermitian or antihermitian forms on finite-dimensional vector spaces over finite-dimensional involutorial division algebras. "Modifications monomiales" by J. Bartijn and J.R. Strooker explains how many questions of a homological nature in algebraic geometry can be answered depending on the existence of certain Cohen-Macaulay modules over local Noetherian rings and it discusses the construction of pre-Cohen-Macaulay modules and their flat dimension. More on local Noetherian rings follows in J. Lescot's paper "La series de Bass d'un produit fibre d'anneaux locaux" while the next article by A. Verschoren surveys his work with F. van Oystaeyen in the theory of relative invariants for commutative rings using localization at idempotent kernel functors.

Given a ring R and a group G of a ring automorphisms of G, the associated fixed ring is the subring of elements of R which remain fixed under each element of G. J. Alev's paper investigates the prime ideals in the fixed ring when G is finite, using several dimension techniques. The notion of a Krull ring is particularly important in commutative ring theory and several attempts have been made to generalise it to the non-commutative situation. An article here by M. Chamarie presents such a generalisation and studies the resulting modules.

The next paper, "Localization of crossed-modules" by P. Hilton introduces the concept of a crossed-module to establish a connection between nilpotent group theory and nilpotent homotopy theory via the fundamental group. The final in the series of articles is "The division ring of fractions of a group ring" by R.L. Suider. In it he gives the state-of-the-art on four questions in the theory of group rings, namely, for k a field and G a group, (1) when is k[G] an Ore domain, (2) if G is torsion free, is k[G] a domain (the infamous zero divisor conjecture), (3) if k[G] is a domain, can it be embedded in a division ring, and (4) if G is torsion free, are the only units of k[G] trivial?

*J. Clark*

GEOMETRICAL METHODS OF NONLINEAR ANALYSIS, by M.A. Krasnosel'skiĭ and P.P. Zabreĭko, translated by C.C. Fenske, Springer-Verlag, Berlin (1984), XIX, 409 pages. Approx. US$51.50.

Readers of English will be delighted to see this translation of

Геометрические методы нелинейного анализа

reviewed by J. Danes in Mathematical Reviews 48 #17976 (1979).

Krasnoselskii's 1956 book "Topological Methods in the Theory of Nonlinear Integral Equations", translated into English in 1964, has been widely used as a text to learn from, or a fruitful reference, by many who are interested in aspects of nonlinear analysis. This new book partly updates this material, and partly develops other methods. Although there are a number of rather separate fields of nonlinear analysis, a method that appears in many areas is that of finding solutions of equations by computing the rotation, or topological degree, of a mapping. It is this method that forms the backbone of the book. The interest is not so much in the topological problems, but in giving results in a functional analytic setting which are then shown to have applications to nonlinear differential equations.

This book has been written in a fairly solid style, that is, proofs are rather condensed or even omitted, and consequently there are a great number of results covered. The reader is assumed to know basic functional analysis, real analysis, differential equations, and topology.

Christian Fenske, who has worked in this subject area, has produced a very readable version, adding a handful of footnotes, listing the references, and correcting some inaccuracies.

Chapter 1 deals with vector fields in $\mathbb{R}^n$ and studies their rotation. For example they show a nonsingular coercive potential has index 1. Chapter 2 extends this theory to compact fields in Banach spaces. This is extended again to special classes of noncompact fields in Chapter 4.

Chapter 3 deals with equality of various rotations, such as the rotation of $I - AB$ and $I - BA$. Chapter 5 could be said to apply the preliminary material to give fixed point theorems such as Schauder's. Other lines of attack, such as using contraction mappings and monotone (i.e. order preserving) mappings in a partially ordered space, are used in conjunction with the basic method.

Chapter 6 gives conditions for more than one solution of an equation. Chapter 7 is concerned with construction and convergence of approximating solutions. Chapter 8 shows how the solution changes when the equation is slightly changed, using a variety of methods of implicit function type.

In this subject there tend to be a variety of methods used, which militates against a unified theory. For example there are four different settings of the problem of multiple solutions, studied in Chapter 6. Against this tendency to variety the authors have sensibly limited themselves at various points and imposed a good deal of organization on their diverse material. To sum up, this is an impressive book which will be valuable to mathematicians with an interest in nonlinear problems.

*Bruce Calvert*

THE THEORY OF MENTAL ARITHMETIC, Dr Stephen W. Taylor. The Mothers' University Series. An Octaray Publication.

In a previous Newsletter I reviewed the same author's "The Science of Mental Arithmetic". I took this volume to be an explanation of the methods there set out as mechanical procedures. This expectation was not fulfilled.

The book indeed deals with 'mental arithmetic', but the emphasis is on the 'mental', his concern with the theory of the mind, in a philosophical sense. One of the few names he mentions with approval is Hegel. He writes much of 'absolute philosophy' and 'absolute science'. The former of these is described in his own words:

> "The absolute philosophy sees contradiction as the law of the deeper realm, that everything logically is composed of moments, which both complement and oppose each other. In this way it discovers a foundation where otherwise none might be thought to exist."

Criticism of this book in terms of philosophy is outside my competence, but might be undertaken from the standpoint of the work done over the past century in the foundations of mathematics.

The first half of the book is really devoted to an attack on what Dr Taylor believes to be the current state of arithmetic. Perhaps he was badly taught as a child, but he seems to have no conception of arithmetic beyond that, of its place in the theory of numbers or the general study of algebra.

He introduces circle arithmetic (congruence arithmetic) using his own language. He claims that there is no valid explanation for $(-1) \times (-1) = 1$ in conventional arithmetic, translates it into circle arithmetics of various sizes, and demonstrates (without the attention to detail I would have thought he needed if conventional arithmetic is as bad as he says) that the formula holds. He then claims that it works in every circle. "We do not have to heave every stone in the world to prove the law of gravity". O yes, Dr Taylor, you do, if you want to match the standard of proof demanded by mathematics.

This is his basic problem, looking at the book from the standpoint of the mathematician. He has no conception of mathematical proof, nor of the need to make proper formal definitions. Nor does he use any algebra, which would make things so much clearer to the mathematician, and without which it is so difficult to extract the rules which he exemplifies in the worked examples in the second half of the book.

As a system of arithmetic adapted for mental use, I have no doubt that it works. It depends more on the framework of conventional mathematics than the author seems to realise, and I doubt whether it would be so easy to teach it to children as the initial method as he supposes.

*David Robinson*

LEARNING HIGHER MATHEMATICS, by Lev S. Pontryagin, translated by Edwin Hewitt, Springer-Verlag (1984), 304 pages, US price approx. $29.10.

This is an extraordinary book. It represents the first two of four volumes, that in Pontryagin's words, "set forth the foundations of higher mathematics in a form that would have been accessible to myself as a lad". The first part is an account of co-ordinate geometry, mainly in two dimensions but with supplements to each chapter to do with three dimensions, and the second part is an introduction to analysis that begins with series and finishes with Cauchy's theorem for analytic functions and some of its consequences. Doubtless Pontryagin was not your average lad. The exposition is measured but never laboured; results are unfolded slowly as one might uncover something of great value. Nowhere is the reader fobbed off with the glib motivational phrase or the enthusiastic assertion that, really, for one reason or another it was all to be expected anyway. One has the sense that here is an authentic expression by one of its masters of an old and honourable art.

As Hewitt says in his preface, the book "cannot be said to be easy reading", but it should be within the range of good undergraduates. There are no exercises in the book and it has no delineated sequence of definitions and theorems but I think it does not pretend to be a text in that sense. In fact its intentions are summarised in a title which, though at first sight awkward, is in the end, by Pontryagin's intimate, almost conversational style, rendered perfectly apt.

*Peter Fenton*

ANALYTIC FUNCTIONS, Blazejewko 1982, edited by J. Lawrynowicz, Springer-Verlag (1983), 494 pages, US price approx. $24.10.

This is a selection of papers delivered to the 8th Conference on Analytic Functions held at Blazejewko, Poland, in 1982. According to the Foreword the theme of the volume is the use of "extremal methods and their applications to various branches of complex analysis", though there are inevitably exceptions. Thirty four papers are represented and the whole is rounded off with four collections of problems that arose during the conference, on functions of one complex variable, functions of several complex variables, quasiconformal mapping and finally analysis on complex manifolds.

*Peter Fenton*

## SPRINGER-VERLAG PUBLICATIONS

The following Springer-Verlag publications are available for review in the NZMS Newsletter. Interested members should contact John Clark, Mathematics Department, University of Otago.

Lecture Notes in Mathematics

Padé Approximations and its Applications Proceedings of a Conference held at Bad Honnef, Germany March 7-10, 1983, 264 pages.

Lie Group Representations III Proceedings of the Special Year held at the University of Maryland, College Park 1982-1983, 454 pages.

Janssen, A.J.E.M.; Steen, P. van der . Integration Theory, 224 pages.

Ruppert, W. Compact Semitopological Semigroups: An Intrinsic Theory, 260 pages.

Probability Theory on Vector Spaces III Proceedings of a Conference held in Lublin, Poland, 373 pages.

Undergraduate Texts in Mathematics

Dixmier, J. General Topology, 140 pages.

Grundlehren der mathematischen Wisenschaften

Rham, G. de Differentiable Manifolds.Forms, Currents, Harmonic Forms, 167 pages.

Tata Institute Lectures on Mathematics

Maass, H. Modular Functions of one Complex Variable, 266 pages.

Vogel, W. Lectures on Results on Bezout's Theorem, 136 pages.

Watanabe, S. Lectures on Stochastic Differential Equations and Malliavin Calculus, 118 pages.

Universitext

Stroock, D.W. An Introduction to the Theory of Large Deviations, 196 pages.

Mathematical Sciences Research Institute Publications

Freed, D.S.; Uhlenbeck, K.K. Instantons and Four-Manifolds, 232 pages.

Problem Books in Mathematics

George, C. Exercises in Integration, 550 pages.

Springer Series in Soviet Mathematics

Fuks, D.B.; Rokhlin, V.A. Beginner's Course in Topology.Geometric Chapters, 519 pages.

## STOP PRESS

Garry Tee informs us that Rob Goldblatt's text on Topoi has been published in Russian translation by "Mir" Press, Moscow, in an edition of 5700 copies. So anyone who missed out on the North-Holland edition (1979) can get this 1983 edition for a mere 3 rubl 30 kopek!

# Feature Article

## THE FUTURE OF APPLIED MATHEMATICS

W.B. BONNOR

QUEEN ELIZABETH COLLEGE, LONDON

*A Faculty of Science Open Lecture delivered at the University of Otago by the William Evans Visiting Professor.*

Mathematics, at least in the Commonwealth, has traditionally consisted of two parts, pure and applied. Anyone who has done mathematics at school has some idea of what pure mathematics is about. A.N. Whitehead defined it as "the science concerned with the logical deduction of consequences from general premises". It is normally done purely for the fun of the thing, any application to the real world being accidental, or for purposes of illustration.

What applied mathematics is will be the subject of this lecture. From what is often taught under this heading in the higher forms of schools, one might think that it means mathematics applied to physics but this would be a mistake. What I shall do is to look back quickly through the history of mathematics and pick out those parts of it which I think most experts would classify as applied mathematics.

The known history of mathematics beings with Babylonians a few centuries before 2000 B.C. In geometry they achieved little beyond some elementary mensuration. For $\pi$ (ratio circumference of circle/diameter) they took the value 3 (the same approximation was adopted by the Jews, as recorded in the Bible). In arithmetic, however, the Babylonians were extremely advanced, being able to handle numbers up to 1 million, and using not only the decimal scale as we do, but also the sexagesimal scale, in which 60 is the base.

The first serious work in geometry was undertaken by the Egyptians about 2000 B.C. The subject was forced on them by the need to revalue land for tax purposes after the constant floodings of the Nile. Their work was severely practical, and proofs were never given. They knew the areas of many common figures, and also that a triangle with sides measuring 3, 4 and 5 units must contain a right angle. Their approximation to $\pi$ was 256/81 (= 3.16), somewhat better than the Babylonians and Jews. They also had some elementary notions of trigonometry, as indeed they would need to have had for the construction of the pyramids.

The geometry of the Egyptians may be called applied mathematics in that it was mathematics applied to surveying, or as the name suggests, the measurement of the earth. It became pure mathematics under the influence of the Greeks, who developed it as an abstract discipline during the period 600 B.C. - A.D. 400. From that time applied mathematicians paid little attention to geometry until their interest in the geometry of four dimensions was aroused by Einstein. The Greeks insisted on logical proof of their propositions, and their methods have been a pattern for geometers ever since. To the Greeks must be attributed the notion, still widely believed, that the best mathematics must be completely devoid of practical use.

This view was held not only by Plato, but also, strangely enough, by the greatest applied mathematician of antiquity, Archimedes of Sicily. Archimedes discovered the laws of statics, which is the science of bodies and fluids in equilibrium. In particular he understood the theory of the lever, as illustrated by his famous remark "Give me a place to stand on and I will move the earth". In spite of his contempt for useful science, Archimedes was often prevailed upon to show his undoubted genius as an engineer. One such occasion was when Hieron, the king of Sicily, had built a ship so heavy that it could not be got off the stocks. By means of systems of cogwheels and an endless screw Archimedes was able, so the story goes, to launch the ship single-handed. Later, Hieron sought his help in repelling a Roman invasion under Marcellus, and Archimedes prepared a series of fearsome catapults which made Marcellus retreat. One might say that Archimedes was the first Scientific Civil Servant, albeit on a part-time basis.

Astronomy is part of applied mathematics, and here there was much activity among the ancients. Their astronomical work, in so far as it was correct, was kinematical, and not dynamical. That is to say, they were able to predict the motions of the heavenly bodies by past experience, but had no correct idea of the cause of these motions.

Although in pure mathematics medieval times brought important advances, notably in the introduction of arabic numerals and methods of calculation, in applied mathematics nearly 2000 years elapsed after Archemedes before significant advances took place.

It is generally thought that modern applied mathematics began with the work of Newton, but this is not so. Before Newton started his investigations into mathematical physics several mathematicians were applying their skills to a different practical subject, namely the theory of probability.

I shall spend a few minutes describing the early history of this theory which is, of course, the basis for statistics. It had its origins in gambling, particularly by dice-throwing. The idea that the probability of throwing say, a 3, is 1 in 6 was current in the sixteenth century, and was first clearly stated by Cardano (1501-1576) the Italian mathematician whose name is associated with the solution of the cubic equation.

Galileo (1564-1642) was apparently ordered by his patron (the Grand Duke of Tuscany) in 1623 to produce some thoughts on dice games. He considered the following problem. Three dice are thrown: what are the relative chances of obtaining totals of 9 and of 10? Galileo noted that the number of 3-partitions of 9 is the same as that of 10, namely

(621), (531), (522), (432), (441), (333) make 9,
and (631), (622), (541), (532), (433), (442) make 10:

yet it was known empirically that the probability of throwing 9 is less than that of throwing 10. Why? Galileo explained that (621) can turn up in six ways, i.e. as 621, 612, 216, 261, 126, 162, (531) can turn up in six ways, (432) can turn up in six ways, (522) can turn up in three ways, i.e. 522, 252, 225, (441) can turn up in three ways, (333) can turn up one way, so 9 can occur as a result of 25 throws of three dice. Hence the probability of throwing a 9 is $\frac{25}{216}$. A similar calculation shows that the probability of throwing a 10 is $\frac{27}{216}$. The difference in probabilities is $\frac{27}{216} - \frac{25}{216} = \frac{1}{108}$. This slight difference was apparently well known to gamblers at the time.

Another name prominent in the early history of probability was that of the French lawyer and amateur mathematician, Fermat. Unlike Cardano, who was a very unpleasant character and at least partially mad, and Galileo, who was tough-minded and aggressive, Fermat was the completely urbane gentleman about whom nothing unfavourable is known. His most famous work was, of course, in the theory of numbers. The following problem was proposed to Fermat by Blaise Pascal, mathematician, philosopher and religious neurotic: a gambler undertakes to throw a six with a die in 8 throws. He makes three throws without success. What proportion of the stake should he have returned to him if he agrees to give up his fourth throw. Fermat solved the problem, and in the correspondence shows up as a jump or two ahead of Pascal in his understanding of the subject.

The English at this time made no contribution to probability theory, possibly because they did not use dice for gambling very much. However, Samuel Pepys, prattling his way into history as usual, asked Isaac Newton in 1693 to solve a problem which, he said, had been put to him by a Mr. Smith (thought to be fictitious, and invented because Pepys did not wish to reveal his own interest in gambling). The problem was as follows:

A has 6 dice in a box with which to throw a 6,
B has 12 dice in a box with which to throw two 6s,
C has 18 dice in a box with which to throw three 6s.

Who has the best chance? This could certainly have been solved by Fermat many years before (possibly even by Galileo). After about three weeks Newton sent Pepys the answer. A has a better chance than B, who has a better chance than C. The calculations are formidable, and the results are

A's chance $\frac{31031}{46656} \sim 0.665$

B's chance $\frac{1346704211}{2176782336} \sim 0.619$

C's chance was not stated by Newton.

The foundations of modern mathematical physics were laid by Isaac Newton (1642-1727) during the years 1664-5. Newton had been awarded his B.A. degree at Cambridge in January 1664, and had retired to his home in Woolsthorpe when the university was closed because of the Great Plague. By the time he returned in 1667 he had laid the foundations of mechanics, invented the calculus, discovered the law of gravitation, and shown experimentally that white light is composed of the spectrum of colours. In 1669, at the age of 26, he became Lucasian Professor of Mathematics, the previous holder of the Chair, Isaac Barrow, who had been Newton's teacher, having resigned it in favour of his pupil.

In mechanics Newton's achievement consisted of two quite separate parts. One part relates the forces acting on a particle to its motion. Aristotle had thought that the velocity of a body was caused by the force acting on it. It was Newton's great insight to realise that it is not the *velocity* of the body which is caused by the force, but the *acceleration*. It is this fact, contained in Newton's second law of motion, which is the keystone of dynamics.

Newton's second great contribution to mechanics was the discovery of the law of gravitation - that the gravitational force between two particles is proportional to their masses and inversely proportional to the square of the distance between them. The importance of this is that it is the force of gravitation which causes the acceleration of the planets, and a combination of this law and the second law of motion enables the orbits to be calculated with tremendous accuracy. Good astronomical predictions had of course been made before Newton's time, but, as I said previously, these were on purely kinematical grounds. Newton's theory was the first to supply a satisfactory cause for planetary motion.

The mechanics of Newton is founded on the idea of the particle. It is particles which obey Newton's first two laws of motion and his law of gravitation. Newtonian theory sees particles moving against a background of absolute space. Space plays no active part in Newtonian mechanics. This passive role is very different from its function in the field theories I shall refer to later.

Such was Newton's genius that it took applied mathematicians 200 years to work out in detail the results of his system of mechanics. Diligent among the early developers were the Bernoullis, a remarkable Swiss family of which no fewer than eight were distinguished mathematicians. For applied mathematics the most important were Jacob and Daniel.

Jacob Bernoulli (1654-1705) was one of the first to apply Leibniz's form of the calculus to problems of applied mathematics. He took the first steps in the creation of a theory of elasticity, and solved a number of famous problems, such as that of the catenary (the shape of a rope hanging freely under gravity) and the brachistochrone (the smooth curve between two points $A$ and $B$ at different levels such that the time of descent of a particle following it is least). Perhaps his most important work was in the theory of probability in which he was one of the first to use freely permutations and combinations.

Daniel Bernoulli (1700-82) has been called the founder of mathematical physics. Some of his most important work was in the flow of fluids, but he worked also on the theory of vibrating strings and made an attempt to create a kinetic theory of gases. He too interested himself in the theory of probability, and applied it to problems in insurance.

The next great applied mathematician was also a Swiss - Leonhard Euler (1707-83). Euler had an enormous output of both pure and applied mathematics; he was blind in one eye at 28, and lost the sight of the other when 70. Even this however did not stop his copious flow of memoirs. He invented no great theories but had a genius for problem-solving, at a time when the theories of Newton and the development of the calculus by the Bernoullis had made a great many problems ripe for solution. He did fundamental work on the motion of rigid bodies, and he established the general equations of fluid dynamics. He wrote on the theory of sound and on the tides. He could never resist any problem which occurred to him: on reading Virgil's lines "The anchor drops, the rushing keel is stayed" he was led to enquire what precisely was the motion of a ship after anchor was dropped!

Newton's mechanics was founded on laws tremendously powerful yet beautifully simple. However, the mathematics which he used to apply the laws to the world was very cumbersome indeed. Part of the task of mathematical simplification had been undertaken by Euler. The perfection of the mathematical techniques was accomplished by Joseph Louis Lagrange (1726-1813), one of the greatest mathematicians of all times.

At the same time as Lagrange was re-formulating Newton's work, mechanics was finding its most ambitious application in the hands of Laplace (1749-1827). In his famous book "Mécanique Céleste" Laplace worked out in amazing detail the motions of all the observed bodies of the solar system, taking account of their interactions. The fundamental enquiry was whether or not the solar system is stable, and Laplace came to the conclusion that it is.

Laplace worked also in the theory of probability and made important developments in the theory of errors invented by Gauss. However, of greater significance was his invention of the potential function of gravitation theory, and his demonstration that it satisfies the famous partial differential equation that bears his name. This function was the forerunner of the potential in electricity and magnetism, and played an important part in the growth of the conception of field of which I shall speak later.

E.T. Bell chooses for the three greatest mathematicians of all time Archimedes, Newton and Karl Friedrich Gauss (1777-1855). Gauss's work in applied mathematics does not compare with the epoch-making pure mathematics he created, in spite of the fact that he laboured for 20 years in astronomy, carrying out fantastic calculations on the orbits of two minor planets Ceres and Pallas. He would not have been able to do this work had he not been an arithmetical prodigy with an amazing memory and incredible speed of mental calculation. It is said that if he felt too lazy to reach for a set of logarithm tables he would work out in his head the logarithm he wanted.

His astronomical calculation brought him fame but was not of the inventive standard of his other work. In applied mathematics he will always be remembered for a theorem which bears his name and is concerned with the flux of force issuing from electric charges. This theorem was of the utmost importance in the development of the conception of physical fields, which had received powerful treatment in the hands of the non-mathematical physicist Faraday, and achieved its full implementation in the work of Maxwell.

The electromagnetic theory of Maxwell (1831-79) was revolutionary because it abandoned the old idea of action at a distance, and substituted for it that of a field of force pervading space. This idea has suffered many vicissitudes since Maxwell's time when it was supposed that the field needed a medium in which to act - called the ether. The ether was recognised to be an unnecessary hypothesis following the introduction of special relativity by Einstein in 1905. The conception of the field has, however, remained and grown in importance. It is now accepted that fields of force of many types can exist and act in empty space, and that no medium is necessary. As such they have become essential to modern physics in electromagnetism and general relativity, and also in the theory of fundamental particles.

It is fair to say that the two ideas, one of particle introduced by Newton, and the other of field introduced by Maxwell, are at the root of theoretical physics today.

At this point - towards the end of the nineteenth century - a new type of figure appears on the scene I have been describing. I mean the theoretical physicist - men like H.A. Lorentz, Einstein and later Schrödinger and Heisenberg.

Meanwhile other branches of applied mathematics had been starting up. The one that has had the most revolutionary results was automatic computing. The pioneer was Charles Babbage (1791-1871). His first idea, conceived in 1812, was to build a simple special-purpose machine to produce tables of functions. Tables of functions, such as logarithms, sines and cosines, have long been important in computational mathematics. Babbage proposed to calculate functions by what is known as a table of differences, and to do this he designed what he called a difference engine. This was a purely mechanical device, in which each digit was to be recorded by the position of a wheel capable of being set in one of 10 positions. Addition was to be accomplished by gearing.

By 1822 Babbage had built a small version of the difference engine capable of calculating quadratic functions to eight figures. The government then, as now, realised the potential importance of automatic computing, and gave financial support. However, after a while Babbage lost interest in the difference engine because he had conceived what he called his analytical engine.

Textile manufacturers had used punched cards to guide the weaving of patterns. Babbage (c.1832) saw that this principle could be used to encode other kinds of information, and he made it the basis of the analytical engine. This was to be capable of all basic arithmetical operations. The latter were to be performed on what Babbage called the mill, to which data would be transferred as required. The transfer of data, and the operations to be performed on them, were to be controlled by two sets of cards - the cards of the variables, and the operations cards. The sets of cards would today be called a program library.

Babbage was assisted by the Countess of Lovelace, Lord Byron's daughter. She was a very beautiful and talented woman and she discovered many of the basic principles of programming.

The analytical engine was never built and for one reason and another Babbage died an embittered man. Sadly, his work had almost no influence on twentieth century computers.

The inspiration for modern computing came through punched cards. Herman Hollerith round 1870-1880 designed machines using the punched cards of textile technology to encode information of the U.S. census returns. These ideas were taken up industrially in the U.S., especially by I.B.M.

The theory of computing was worked out in the thirties by Church, Gödel and Turing. Turing invented a mathematical model for the ideal computing machine, and was able to deduce the limitations such a machine must have.

The first electronic calculator was ENIAC, constructed in the 1940s. John von Neumann became associated with it in 1944, and he developed the stored programme, (as distinct from a mechanical input) now used in all computers. von Neumann is an important name in modern applied mathematics, and I shall refer to him again later.

In recent years there have been many applications of mathematics in unexpected areas. Take, for instance, the communication of information.

Information theory was started by the electrical engineer Shannon in 1948. It attempts to measure the amount of information contained in a message, and deals with the transmission of information and loss by random noise in the process of transmission. The definition of information is the key to the whole subject, and Shannon's great contribution was to produce a definition which enabled a mathematical formulation of information to be given. Information theory has important applications in electrical engineering, physics and even linguistics.

I could refer to many more examples, mathematics applied to engineering, business, traffic flow, geography, and so on. As a very simple example I should like to describe very briefly an application to athletics.

The table shows the world running records (as at December 1977). We are looking for a relation between the times T, for men, and the distance D. The more fundamental approach is to set up a mathematical model of an athlete: this would require dynamical equations of motion and a study of oxygen balance. It has been attempted. However, let me here take a purely empirical attitude, based simply on the data available.

If we plot D against T we get a diagram like the one shown (Fig. 1). The points do not,

**TABLE 1. World running records**

| Distance (m) D | Time (s) Men T | Time (s) Women |
|---|---|---|
| 100 | 9.9 | 10.8 |
| 200 | 19.8 | 22.1 |
| 400 | 43.8 | 49.9 |
| 800 | 103.4 | 114.9 |
| 1000 | 113.9 | - |
| 1500 | 212.2 | 236.0 |
| 2000 | 291.4 | - |
| 3000 | 455.2 | 507.2 |
| 5000 | 792.9 | - |
| 10000 | 1650.5 | - |
| 20000 | 3444.2 | - |
| 25000 | 4456.8 | - |
| 30000 | 5490.4 | - |

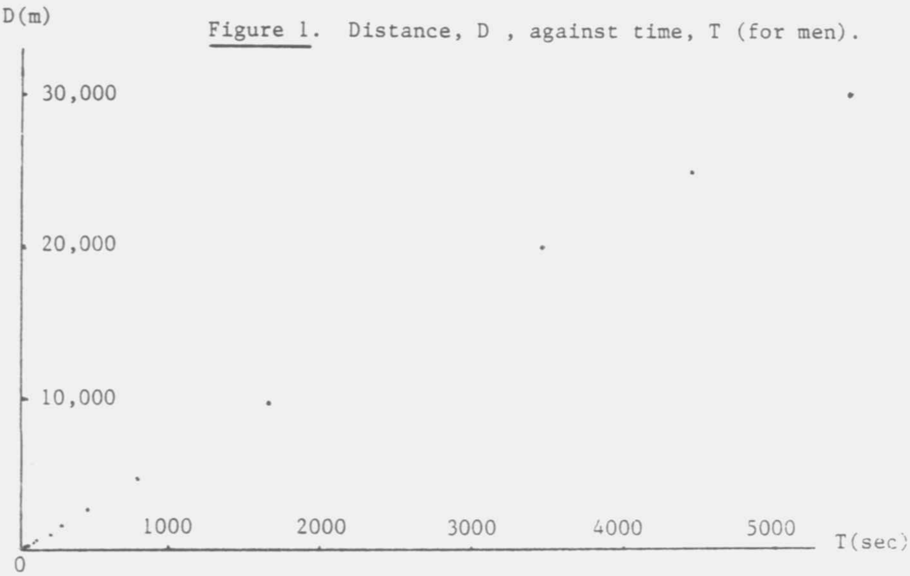



Figure 1. Distance, D , against time, T (for men).

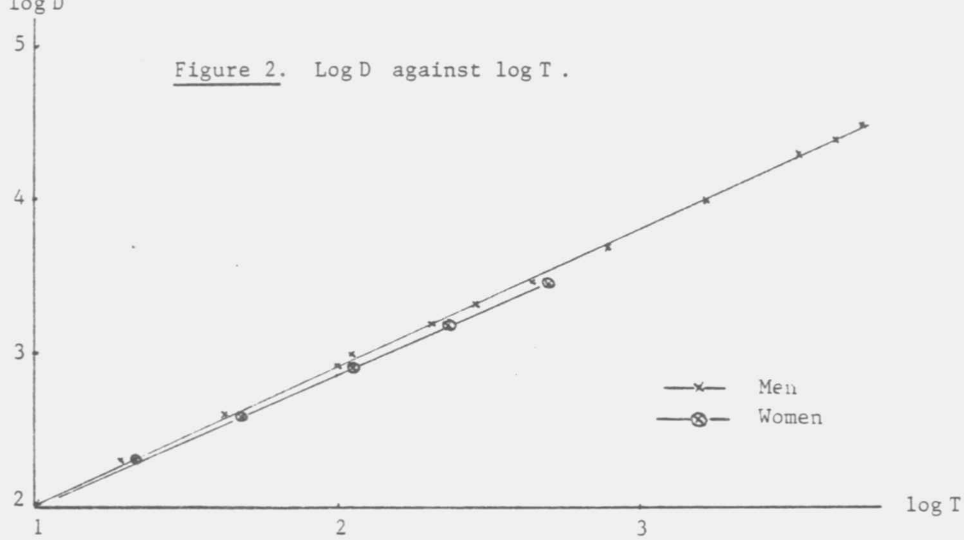



Figure 2. Log D against log T .

of course, lie on a straight line, because that would mean that the speeds were the same over all distances. Moreover, it is hard to represent the data on one graph because the points for shorter races bunch together. A better method is to plot log D against log T: the result is shown in the next figure (Fig. 2). A straight line fits the data very well, both for men and women. The slope of the line is about 0.9 so

$$\log D = 0.9 \log T + \text{constant},$$

or

$$D = kT^{0.9}$$

where k is a constant. Thus for the record performances $D/T^{0.9}$ is roughly constant, irrespective of the length of the race. It comes out to be about 12.7. One can think of this as a sort of handicapped speed of record-breaking athletes. For others it will, of course, be less. This theory can be used in two ways:

(i) to determine a runner's best distance; one looks for his best $D/T^{0.9}$ for various distances D;

(ii) to compare a runner over distance $D_1$ with a different runner over distance $D_2$.

As an example of (ii) we find that the best performance in the Table is the one at 200m (by Don Quarrie of Jamaica).

As a simple example of a more fundamental approach consider weight-lifting. We set up a very primitive theoretical model. We assume that the maximum weight L which can be lifted by a weight-lifter of bodyweight W is proportional to the cross sectional area A of his muscle, say $L = k_1 A$ ($k_1$ is a constant). We next assume that A is proportional to a typical body length, say $\ell$, squared, so $A = k_2\ell^2$. Moreover, the body-weight is proportional to $\ell^3$: $W = k_3\ell^3$. We find

$$L = K W^{2/3},$$

where K is another constant, so $L/W^{2/3}$ should be constant for weightlifters. This is a fair approximation, and has sometimes been used for handicapping.

The most important future applications of mathematics could be in the field of biology. Until recently mathematics in biology turned up during the application of physical laws, such as diffusion, in living forms: or in the use of statistics. However in recent years a new, and revolutionary approach has come into being. This is called catastrophe theory, and it started off as pure mathematics, discovered by a Frenchman, R. Thom. The essential idea of catastrophe theory is that phenomena do not always take place continuously. Continuity is nearly always assumed in applied mathematics - because the differential and integral calculus need this assumption. Yet many phenomena in the real world show discontinuities, and this happens to an important extent in biology, for instance in the theory of evolution. Catastrophe theory may be able to deal better with these matters.

It will be clear from what I have said that applied mathematics is a very varied subject, which is constantly changing its growing points. This makes it a particularly difficult discipline to teach. There is a constant danger that it will ossify. The teacher, himself knowing only one or two of its aspects, may offer the student no more than a travesty of applied mathematics.

The resistance of applied mathematics teachers to new ideas is illustrated by a story about the famous Cambridge mathematician Todhunter in the nineteenth century. It is recorded that Maxwell invited Todhunter to see some experiments in conical refraction. "No" said Todhunter, "I have been teaching conical refraction in physical optics for many years, and it might upset me." Maxwell then suggested that Todhunter's pupils might like to see the experiments; whereupon Todhunter is said to have replied, "If a young man will not believe his tutor, a gentleman, and often in Holy Orders, I fail to see what can be gained by practical demonstration."

For a long period in the twentieth century applied mathematics in schools and even in undergraduate courses meant no more than mathematics applied to certain branches of physics, particularly to mechanics. Many of you will be familiar with the strange former world of school applied mathematics, lumbered with frictionless planes, perfectly rough spheres, rigid ladders and so on. Some of this is worth teaching but it is totally wrong to leave the student with the idea that this is all applied mathematics is.

In the last twenty years the teaching of the subject has begun to change, and pupils - or at least some of them - are learning that applied mathematics can treat a range of every-day activity, such as traffic flow and sport. This is done by constructing a mathematical model of the activity. Inevitably this is an abstraction, sometimes a very drastic one (like the mathematical model of the weightlifter), but the pay-off is that on it one can use the power of mathematical analysis. (Of course the planes and spheres of old-fashioned applied mathematics are models, too).

What the teacher ought to do, in my opinion, is to accustom the pupil to the idea that applied mathematics is exactly what its name implies: the application of mathematics to any area where it can be helpful or interesting. It is certainly not confined to mechanics, or mathematical physics, or statistics.

Indeed, what I am suggesting, and this is the central part of my thesis, is that applied mathematics should be an attitude of mind, or an activity, rather than a subject area. The great applied mathematicians like Euler or von Neumann have made contributions in many different subject areas. von Neumann's name is renowned in quantum mechanics, in computing, in the theory of automata, and in the theory of games and economics. He is probably the greatest applied mathematician of the twentieth century (though he is famous for some pure mathematics too). He was educated in Hungary and emigrated to U.S.

Another feature of the history I have described has been that although the applied mathematician creates new subjects, such as Newtonian mechanics or electromagnetism or probability, eventually these become disciplines in their own rights, with specialised practitioners, like theoretical physicists, statisticians or computer people. Thus applied mathematics should be a sort of intellectual midwife, bringing new branches of science into the world and eventually letting them grow up on their own.

I have been speaking about applied mathematics as a research discipline. However most of our students will not go into research, and indeed will not become professional mathematicians. What then is the role of applied mathematics in the education of somebody doing a degree in mathematics, but who will eventually take up some other career?

University education in the Commonwealth, in whatever faculty, is fairly specialised. For most graduates, what they study at the university is likely to bear little relation to their subsequent professional careers, or to the conduct of their later lives. Yet in some way their university education is supposed to fit them better for what they do afterwards. For example the Civil Service Commission in Britain has believed for years that a man with a first-class degree in, say, Sanskrit or philosophy or mathematics, is quite likely to make a satisfactory member of the Administrative Class. It is widely believed that the detailed study of one, or a limited number of highly specialised subjects is a good training for a subsequent career in quite another field of knowledge.

I admit I do not know how far this is true, or even what the evidence for it is. But if we accept it, we may ask what features of mathematics are likely to make it a valuable degree subject for those who will not become professional mathematicians.

We have no way of giving a definite answer to this question, but a plausible one, expounded by Professor Bondi, is the following. An excuse for teaching mathematics to future non-professionals lies in the number of ideas we can confront them with. These ideas provide an element of intellectual shock, which is a preparation for shocks they will get later. To this extent we may hope that mathematics is a good training for such people.

If we look at applied mathematics courses from this point of view, we ought to put as many profound ideas into them as the students can reasonably assimilate, without too much preoccupation with details, and perhaps without so many of the intricate exercises which we so like to set for examination questions.

Applied mathematics in a degree course is important too in demonstrating the power of mathematics in the real world. When one comes to think of it, it is amazing that a few marks on paper can predict, with the accuracy of a fraction of a second, an eclipse of the sun one hundred years hence. Every university student of mathematics should be confronted with this startling fact. For this reason I am completely against degrees in pure mathematics only, which are offered in some universities in the U.S.

Let me draw together the threads of arguments I have used in this lecture. Applied mathematics has always been a prominent ingredient in mathematics. To begin with, in pre-classical times, all mathematics was applied mathematics as it consisted of counting and land-surveying. Pure mathematics - the wholly disinterested study - began with the Greeks. Probably the next important and sustained effort in applied mathematics was the theory of probability which arose in the sixteenth century because of gambling. The first golden age of applied mathematics was initiated by Newton about 1664. It was then clear that both the kinematics and dynamics of the heavens were amenable to mathematical treatment. For a long time applied mathematics consisted mainly of statistics and mechanics, but in the late nineteenth and early twentieth centuries, the latter developed into mathematical physics.

The twentieth century may be called the second golden age of applied mathematics. Mathematics has extended its scope beyond statistics and mechanics, into many aspects of human activity.

I have argued that applied mathematics is not so much a subject as a mental outlook in which the practitioner is seeking to use mathematics in any field where it can profitably be employed. It has sometimes happened that after a particular subject area has been created by the applied mathematician it has been taken over by experts specialising in it.

I have said that the teaching of such a varied and changing discipline is a particular challenge. Many teachers even at university level, having toiled to master one branch of applied mathematics, shrink from the effort of learning other branches. Thus there is a danger that teaching of the subject will consist of the regurgitation of well-established topics. To counter this, a determined effort must be made to see that the new applications of mathematics are brought into the syllabus.

Lastly, I want to suggest that applied mathematicians should be militant advocates of their subject. Because it is not recognised in some parts of the world, and because some schools now find difficulty in teaching it, or do not wish to do so, some applied mathematicians are on the defensive. They should not be. Their subject has a most distinguished history and should continue to bear important fruits in the future.

# Secretarial

## CONSTITUTION OF THE NEW ZEALAND MATHEMATICAL SOCIETY INCORPORATED

ARTICLE I: NAME

The name of this organization shall be the New Zealand Mathematical Society Incorporated (hereinafter referred to as "the Society").

ARTICLE II: OBJECTS

The purposes for which the Society shall be established are

(1) To promote the development, application and dissemination of mathematical knowledge within New Zealand.

(2) To assist mathematicians in New Zealand to maintain effective cooperation with one another and with mathematicians and mathematical societies in other countries.

The Society shall be administered with these ends in view and not for the purpose of financial gain for its members.

ARTICLE III: MEMBERS

The membership of the Society shall consist of three classes of members - ordinary, honorary, and institutional members. Ordinary membership shall be open to any person interested in the objects of the Society. Election to ordinary membership shall be by vote of Council (Article V) upon written application and upon payment of the annual subscription. However, a member of a Society with which the New Zealand Mathematical Society maintains a reciprocity agreement shall, upon application to Council, be admitted as and remain an ordinary member of the New Zealand Mathematical Society at a reduced subscription, provided that he is not normally a resident in New Zealand. An honorary member shall be any person of distinction in the field of mathematics or any other person whose work or whose services to the Society are judged by the Council to merit election to honorary membership. There is no subscription for honorary members. Institutional membership may be granted by the Council to Institutions, Associations, business enterprises and other organizations interested in the objects of the Society.

The Annual General Meeting shall set the subscription for the following financial year (1 January to 31 December) for ordinary members, which shall be payable in advance. The subscription for Institutional members will be determined by Council in each case.

Resignations from membership of the Society shall be made in writing. Any person more than two years in arrears in subscription is no longer a member of the Society.

ARTICLE IV: BRANCHES

With the approval of the Council (Article V), regional branches may be formed from members of the Society normally residing in a particular region. Each such regional Branch shall elect annually a Convenor and a Secretary (who may be the same person) and other officers from among its members. The persons so elected shall constitute the Committee of the Branch, and shall arrange meetings, including an Annual General Meeting, and otherwise conduct the business of the Branch.

Each Branch may send a delegate to each meeting of the Council. Delegates shall be allowed to speak but not to vote.

ARTICLE V: THE COUNCIL

The Council shall be the governing body of the Society. It shall consist of the President, one Vice-President (Article VI), and seven elected members. The elected members shall each serve for three years. These members may be available for re-election but shall not serve for longer than six years in succession. If a current Council member is elected to the office of Incoming Vice-President (Article VI) the vacancy will be filled by the election of a further Council member for a term of three years. In this event if there are insufficient nominations to Council to cover this circumstance, then extra nominations will be called for immediately at the Annual General Meeting. Editors of any journals the Society may publish, if they are not already members of the Council, shall have the right to attend meetings and vote on matters pertaining to their journals. Council may co-opt further members for limited periods for specific purposes. In addition to the above members, one Council member will be a representative appointed by the New Zealand Association of Mathematics Teachers.

The Council shall determine the policies of the Society and shall supervise the affairs of the Society according to such by-laws as the Council may adopt. A by-law or amendment or repeal thereof shall come into effect thirty days after notification to the membership in a publication of the Society or otherwise in writing, unless during this thirty day period twenty members of the Society shall so petition and the by-law or amendment or repeal thereof shall then be submitted to a vote of the membership and shall not come into effect unless approved by a majority of those voting. However, this restriction shall not apply to those by-laws adopted by the time this constitution is first ratified.

The Council may enter into working arrangements and reciprocity agreements with other societies and organizations.

The Council shall meet at least once a year, and at other times if requested by the President or at least three members of Council. Members of Council shall be notified at least two weeks before any such Council meeting. In addition, a special meeting of the Council shall be held as soon as possible after the Annual General Meeting (Article VII) to appoint a Secretary and a Treasurer (Article VI) who shall be chosen from among the seven elected members of Council. Five members of the Council shall constitute a quorum, provided that at least one of the members present shall be the President or the Vice-President. The President, or in his absence the Vice-President, shall normally preside as Chairman at each meeting of the Council. All matters at Council meetings shall be decided by a majority vote of members of Council present and voting. In the case of a deadlock, the Chairman shall have a casting vote.

Any vacancy in the Council or Offices (Article VI) occurring other than by the normal expiration of a term of office, may be filled by an appointment of the Council. Officers and members thus appointed shall hold office until the next Annual General Meeting. When the vacancy is in the office of President of the Society (Article VI) the Vice-President shall be appointed President. In the event of the Incoming Vice-President resigning during his/her term of office, the next President shall be elected at the following Annual General Meeting.

ARTICLE VI: OFFICERS

The Officers of the Society shall be as follows:

(1) The President

(2) The Vice-President

(3) The Secretary

(4) The Treasurer

The term of office of the President shall be two years. The Vice-President shall normally be either the person who held the office of President immediately before the President in office (in which case he/she shall be known as the 'Immediate Past President') or the person elected towards the end of the first year of a President's term of office to succeed the President in the following year (in which case he/she shall be known as the 'Incoming Vice-President'). The term of office of the Vice-President shall be one year. The term of office of the Secretary and Treasurer shall be one year, but these officers shall be eligible for re-election."

The President shall be ex officio a member of all committees. He shall deliver the Annual Report of the Council at the Annual General Meeting (Article VII). The Secretary shall be responsible to Council for the records of meetings and correspondence of the Society. The Treasurer shall be responsible to the Council for the records of membership and the management of the financial affairs of the Society in accordance with the policies determined by the Council. He shall keep the Society's financial records and prepare the necessary financial statements.

ARTICLE VII: MEETINGS

There shall be an Annual General Meeting of the Society at such a time and in such a place as the Council may determine. The business of the Annual General Meeting shall be:

(1) To receive the Annual Report of the Council.

(2) To receive the duly audited Annual Statement of the income and expenditure and assets of the Society.

(3) To elect the Incoming Vice-President in alternate years (Article VI).

(4) To elect members of Council.

(5) To appoint an Auditor for the ensuing year.

(6) To transact any other business of which notice in writing has been given to the Secretary at least six weeks prior to the Meeting.

Special General Meetings may be convened at any time by the Secretary or the President under the direction of the Council or upon the requisition of a petition of not less than 20 members of the Society to discuss only those matters specified in the petition.

Four weeks' notice of any Annual General Meeting or Special General Meeting shall be given to members.

At every Annual General Meeting or Special General Meeting the Chair shall be taken by the President if present or in his absence by the Vice-President, failing him/her, a Chairman to be nominated from members of the Council by the persons present at the Meeting. The quorum for General Meetings of the Society shall be twenty members. All business shall be decided by a majority vote of those present and voting. In the case of a deadlock the Chairman shall have a casting vote.

For election of officers voting shall be done by secret ballot; other matters shall be voted by voice, or by a show of hands if called for by any members of the Society present at the meeting.

ARTICLE VIII: AMENDMENTS

An amendment to the Constitution may be proposed by five members of the Society.

An amendment shall be adopted by a majority of not less than three-fourths of the members who vote on the amendment by mail or at a General Meeting, provided the amendment has been duly proposed and the membership notified at least four weeks before the vote is taken.

ARTICLE IX: COMMON SEAL

There shall be a Common Seal of the Society which shall be that as appointed by the Council which shall be responsible for the safe custody and control thereof. Whenever the Common Seal of the Society is required to be affixed to any deed, document, writing or other instrument, the Seal shall be affixed pursuant to a resolution of the Council or of the Society by the President or Secretary and any two other members of the Council. The person so affixing the Seal shall at the same time sign the document to which the Seal is so affixed.

ARTICLE X: CONTROL AND INVESTMENT OF FUNDS

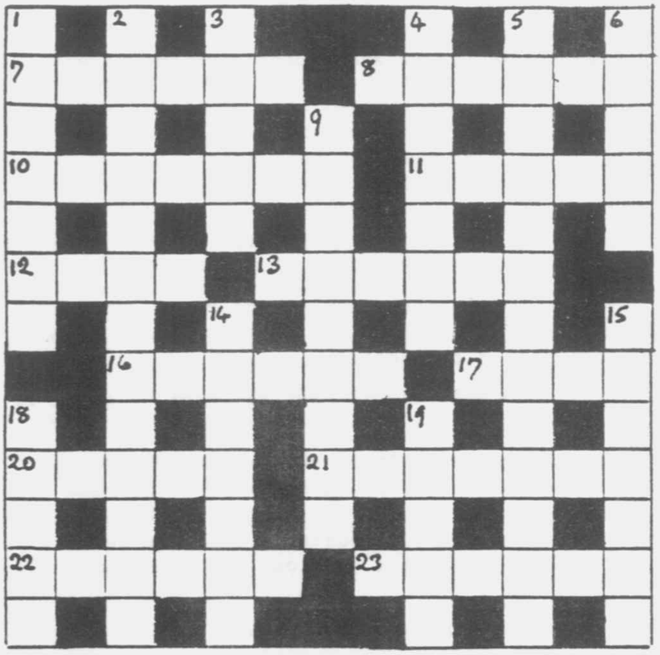
All monies received by or on behalf of the Society in an account with any bank or savings bank from time to time to be fixed by the Council and all cheques or withdrawal slips drawn on the account shall be signed by any two of the President, Secretary and Treasurer. The Society may from time to time invest and reinvest in such securities and upon such terms as it shall think fit, the whole or any part of its funds which shall not be required for the immediate business of the Society.

ARTICLE XI: DISSOLUTION

The Society may be wound up voluntarily if the members, at a Special General Meeting duly called for the purpose, pass a resolution requiring the Society to be so wound up and the resolution is confirmed at a subsequent General Meeting called together for that purpose and held not earlier than thirty days after the date on which the resolution so to be confirmed was passed. Any assets remaining after all debts have been paid shall be given to organizations whose objects are similar to those of the Society.

# Crossword

No. 15 **HOUND DOG NUMBER** by Matt Varnish



*Some of the clues evoke certain names that are to be used to fill in the lights.*

CROSSWORD No. 14 SOLUTION

Across:

1. Love and kisses (XOX),
8. Unsigned,
10. Vega,
11. En,
12. Student,
13. Gyrate,
15. Too,
16. Arose (is a rose ...),
18. Own,
19. Gotten,
20. Exerted,
21. Up,
23. Iona,
24. Tacnodes,
27. Straight lines.

Down:

1. Laurent series,
2. Visa,
3. Algebra,
4. Dee,
5. If,
6. Skeleton,
7. Starting posts,
9. Detest,
12. Stood,
13. Governor,
14. Argent,
17. External,
22. Aden,
25. Ash,
26. Pi.

Across:

7. (See 4 down)
8. +ve or −ve on ac? (6)
10. Best when al. (7)
11. 216, 343. (5)
12. Associated with $xy = yx$. (4)
13. A Laker state. (6)
16. $2^{2^n} + 1$. (6)
17. (See 19 down)
20. 17. (5)
21. Think past. (7)
22. rt adornment. (6)
23. Resolve land and pay up. (6)

Down:

1. $<$ or $>$ . (7)
2. 00. (1,5, 2,5)
3. (and 14 down) US53310761. (5,7)
4. (and 7 across) Ml. (3,4,6)
5. MI. (7,6)
6. 666. (5)
9. Throws font of predictable characters. (9)
14. (See 3 down)
15. In set sets insects. (7)
18. Signs of celts. (5)
19. (and 17 across) $1^\circ 1^\circ$ (d?). (5,4)

The Newsletter is the official organ of the New Zealand Mathematical Society. It is produced in the Mathematics Department of the University of Otago and printed at the University Printery. The official address of the Society is:

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C/- The Royal Society of New Zealand
Private Bag
WELLINGTON

However correspondence should normally be sent direct to the Secretary, Dr C.H.C. Little, Department of Mathematics and Statistics, Massey University, Palmerston North.