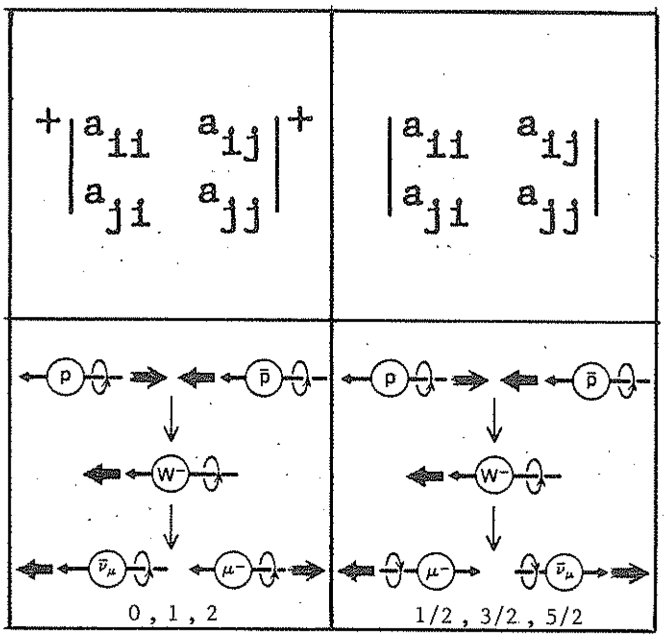
THE NEW ZEALAND MATHEMATICAL SOCIETY

NEWSLETTER



PERMANENTS, DETERMINANTS
BOSONS AND FERMIONS

CENTREFOLD
PROFESSOR GORDON PETERSEN

CONTENTS

Number 29 December 1983 ISSN 0110-0025

Editorial

This issue continues coverage of the Eighteenth New Zealand Mathematics Colloquium and includes the invited addresses by Professor David Vere-Jones and Dr Alistair Watson. Also featured is one of the talks delivered by Mr Trevor Boyle, the 1983 New Zealand Mathematical Society Visiting Lecturer, on his tour of the main centres. Dr Brent Wilson, a former editor, has again contributed to the Newsletter with the centrefold on Professor Gordon Petersen, whom we wish well in his coming retirement.

News items, notices, articles of general interest and suggestions for centrefolds are always welcome and may be sent to the editor or one of the honorary correspondents. Copy date for the next issue is 15 March, 1984.

John Curran
Editor

Notices

THE AUSTRALIAN MATHEMATICAL SOCIETY LECTURE SERIES

The Cambridge University Press is to publish a series of books under the above heading for the Australian Mathematical Society. The series is to contain books of 100-300 pages which are either university textbooks or more specialist monographs. The plan is that texts would be attractive not only to the Australasian market but also the United States and British markets. Monographs would clearly be addressed to the world community of mathematicians. It is expected that Cambridge University Press will keep the books in the series relatively inexpensive and to assist with this authors will be asked to prepare their typescripts in camera-ready form. Authors will of course receive royalties. It is expected that about six books will be published each year. The editorial board for the series is S.A. Morris (Pure Maths), A.J. van der Poorten (Pure Maths) , C.J. Thompson (Applied Maths), C.C. Heyde (Statistics).

The editors are keen to receive manuscripts in the near future. Further information about the series can be obtained by writing to Dr S.A. Morris, Editor in Chief, Australian Mathematical Society Lecture Series, Department of Pure Mathematics, La Trobe University, BUNDOORA, VIC. 3083, AUSTRALIA.

RESULTS OF TEACHERS PROJECT COMPETITION

The results of this competition, judged by Associate Professor Peter Lorimer, Dr David Robinson and Mr Bruce Sutton are as follows:

1st A.J. McNaughton of Mangere College, for 'Dynamic Transformations'.

2nd P. Tomlinson and others of Timaru Boys' High School, for their videotape introduction to the use of the electronic calculator.

3rd R.F. Browne of Cashmere High School for his 'Appple' disc of noughts-and-crosses type games for teaching coordinates.

The judges had no difficulty in selecting the winning entry, which provided a somewhat unusual treatment of matrix transformations, following the images of points under repeated application of the transformation and sketching curves. The great merit of this project was that it encouraged experimentation by the student. For example there were many interesting problems to follow up. The judges felt there were many areas of mathematics which could be treated in this way and this project might well encourage other teachers to construct explorations of their own.

There were a number of entries which the judges felt could worthily have filled the other placings but they were impressed by P. Tomlinson's technically well produced video-tape and R.F. Browne's computer games. The video not only provided an informative introduction to the use of a particular type of calculator, but also spent time considering the logical organisation of computation and discussing associated mathematical ideas. The computer games were a good attempt at using the computer to reinforce mathematical ideas, the main objective in this case being to teach the use of coordinates.

Local News

AUCKLAND UNIVERSITY

DEPARTMENT OF MATHEMATICS & STATISTICS

Professor Jan Jaworowski of Indiana University, arrived in late June to take up a visiting lectureship until the end of November 1983.

In July, Professor Peter Lorimer introduced a series of '*Group Theory*' seminars and later in the year, Professor David Gauld gave several seminars on '*Prime Ends*'.

In August, Mr Trevor Boyle visited the Department and gave a series of lectures in his capacity as the New Zealand Mathematical Society Visiting Lecturer for 1983. Trevor is the Principal of St Peter's College, Palmerston North. Over and above his long association with secondary in-service training for the Teachers' Refresher Course Committee and contributions for the School Certificate and University Entrance Syllabus, Trevor had an ANZAC Fellowship in 1973, a Woolf Fisher Award to Australia in 1978 and has served as a Teaching Fellow in the Mathematics Department of Victoria University.

Dr Chris Wild left for the University of London, U.K. at the beginning of the second term. Mr Chris King has gone on sabbatical leave to the Max-Planck-Institut, Göttingen, Germany. Mr Richard Cutler, Assistant Lecturer, has left for Berkley University, U.S.A., to study for a PhD. Richard was accompanied by his wife Adele.

Congratulations to Dr Jeffrey J. Hunter who has had Volumes 1 and 2 published of his textbook '*Mathematical Techniques of Applied Probability*'.

Professor Shigeru Tanaka of the Tsudo College, Tokyo, is visiting Auckland until March 1984. He has been accommodated in the Mathematics & Statistics Department for the duration of his stay.

Seminars:

Mr Richard Cutler (Auckland University - Valedictory Seminar), '*Hausdorff Notions for Fuzzy Topological Spaces*'.
Mr Trevor Boyle (St Peter's College), '*Out of School and into What? Mathematics from School to University*' and '*What have we done to School Mathematics?*'.
Mr Harry Romana (Auckland University), '*The Alexander Invariant via Seifert Surfaces*'.
Dr C.M. Triggs (Applied Maths Division, DSIR), '*A Sampling Scheme to Estimate the Kiwifruit Crop*'.
Dr Jocelyn R. Dale (Applied Maths Division, DSIR), '*Sparse Contingency Tables and Goodness of Fit*'.
Professor Richard D. Anderson (Louisiana State University), '*A Survey of Recent Developments in Infinite-Dimensional Topology*'.
Professor Jan Jaworowski (Indiana University) Lecture Series: '*Borsuk-Ulam Theorem*', '*Fibre Preserving Free Involutions*', '*Vector Space Bundles and their Characteristic Classes*', '*Antipodal Coincidence set for Fibre Preserving Maps*', and '*Free Fibre Preserving Involution*'.

I.L.R.

DEPARTMENT OF COMPUTER SCIENCE

Peter Henrici's 60th birthday was celebrated at Zürich by a conference on numerical analysis. At the conference banquet, he was presented with a calligraphic scroll, inscribed by the members of the Runga-Kutta Club at Auckland. Leslie Fox's retirement from the Directorship of the Oxford University Computing Laboratory was commemorated by a conference on numerical analysis. The Runga-Kutta Club sent a commendatory letter to him, at that conference.

Peter Gibbons is going on leave to the University of Toronto, until August 1984.

Ralph Jones, a numerical analyst from the South Australian Institute of Technology, is visiting the Department from August 1983 to January 1984. Dr Tjalling J. Ypma, a numerical analyst from the University of Witwatersrand, is visiting the Department from September 1983 to January 1984.

The Subject Conference on Computer Science, held at VUW in August, was attended by John Butcher, Kevin Burrage, Bob Doran and Bruce Hutton. Richard Lobb attended the Australian Computer Engineering Symposium at the University of Newcastle, in August.

The departmental seminars on ordinary differential equations have continued to be held weekly.

Seminars:

Professor G.A. Watson (University of Dundee), *'A finite algorithm for computing centres in the Jaccard matrix'*.
Professor Graham Birtwistle (University of Calgary), *'Silicon compilation'*.
Dr Gerald D. Chandler (Hong Kong Polytechnic), *'Sorting'*.
Dr Arndt Bode (Universität Erlangen-Nürnberg), *'The EGPA multiprocessor'*.

G.J.T.

DEPARTMENT OF THEORETICAL & APPLIED MECHANICS

Local News

Staff:

M.J. O'Sullivan	Claude McCarthy Trust Visiting Fellowship to Lawrence Berkeley Laboratory and Los Alamos National Laboratory, California, during June and July.
P.J. Hunter	Invited Lecturer at the 29th International Union of Physiological Sciences Conference, Sydney, August.
I.C. Medland	Invited Lecturer at a Conference in Manchester celebrating the retirement of Professor Michael Horne, FRS, University of Manchester, who was Ian's Ph.D. supervisor.
M.J. O'Sullivan and R. McKibbin	1982 N.Z. Geophysics Prize for paper entitled *'Heat transfer in a layered porous medium heated from below'*; J. Fluid Mech. III: 141-173, 1981.
Emeritus Prof. C.M. Segedin	is still alive but with only epsilon less kick!

Students:

Simon J. Tavener	Awarded Unilever Scholarship for Ph.D. study in Bioengineering, at Oxford from September.
Jeff D. Pugh	Awarded Fulbright Scholarship for Ph.D. study in Aeronautics at Caltech from September.

Seminars:

Dr G.A. Watson (University of Dundee), *'Numerical methods for Semi-Infinite Programming Problems'*.
Dr Fred Chipman (Acadia University), *'Problems in Software Design for the Numerical Solution of Initial Value Problems'*.
Dr P.J. Thomson (Victoria University), *'Speech Recognition: an application of time series methodology'*.
Professor D.A. Spence (Imperial College), *'Self-similar solutions for elastodynamic cavities'*.
Professor Gilbert Strang (Massachusetts Institute of Technology), *'Minimum Principles in Applied Mathematics and Design'*.

D.M.R.

WAIKATO UNIVERSITY

Professor Zulauf is spending 6 months study leave in Hamilton, till February.

Professor Hosking begins study leave shortly, with 6 months in the IAS at ANU (Canberra) and 6 months at the Department of Applied Mathematics and Theoretical Physics, Cambridge, and will also attend the Australasian Conference on Fluid Mechanics in Newcastle.

A.G. French drew the short straw, and has just begun to supervise a P.E.P. worker to extend the REMATH C.A.I. programme to basic calculus, trigonometry, and maybe vector mechanics.

A.D. Sneyd will shortly begin another P.E.P. scheme, to use computer aided graphics to create movie film of wave motion, diffusion and other physical processes, for classroom use.

The sudden discovery of money in some forgotten crevice has enabled the university to expand terminal facilities (but not as far as a funeral parlour) for next year, and arrange for a C.A.I. Laboratory Supervisor (at least for next year), to interact with students in our burgeoning REMATH sessions.

Seminars:

Dr E. Kalnins (Waikato), '*Separation of variables on n-dimensional Riemannian manifolds: the sphere and Euclidean space*'.
Professor K. Mariwalla (La Trobe), '*Black holes and fundamental matter constituents*'.
Dr A. Odlyzko (Bell Labs), '*Random shuffles and the representation theory of finite groups*'.
Professor C. Wulfman (University of the Pacific, Stockton, California), '*Symmetries of partial differential equations*'.
Dr P.J. Eccles (Manchester University), '*Multiple points of immersions*'.
Dr K.A. Broughan (Waikato), '*Vaxima for the working mathematical modeler*'.
T. Boyle (St Peter's, Palmerston North), '*Out of school and into what? Mathematics from school to university*', and '*What have we done to school Mathematics?*', a seminar given to the Waikato Mathematical Association.
Dr M. Schroder (Waikato), '*Why Lagrange multipliers work: geometry in the tangent space and matrix quasi-inversion*'.
Professor R. Hosking (Waikato), '*Codes for plasma stability investigations*'.
Dr A. Sneyd (Waikato), '*The aerodynamics of snails*'.
G. Reid (Waikato), '*Schrödinger, symmetry and separation*'.
J. Davys (Waikato), '*Waves on ice floes: caustics and super-caustics*'.
M. Rollo (Waikato), '*Thermal stability of coronal loops*'.

The Waikato Statistics Group, an informal group of people interested in Statistics, maintained its activity, and organised the following seminars:

Dr J. Hills (University of California, Davis), '*Misuse of mean separation techniques*'.
Dr J. Dale (A.M.D., Auckland), '*Spurse contingency tables and goodness of fit*'.

The Computer Science Department began a regular seminar series, often of mathematical interest:

Dr E. Lewis, '*Computer simulation of biochemical and medical processes*'.
Dr L. Groves, '*Automatic programming techniques based on formal logic*'.
Dr M. Livesey, '*Proofs by induction*'.

Other topics included - Petri nets, computer graphics, Pascal software, path Pascal and concurrent programming, Amdahl 580 and IBM 370 from the inside, databases, story comprehension, and interactive PROLOG in schools.

Recently, John Turner instigated an informal semi-regular series known as the P.B. Splurge (there is no prize for guessing why).

B. Beder, '*Branching processes - a problem on random generation of binary trees*'.
J.C. Turner, '*On random trees*'.
M. Livesey, '*Computational processes*'.

M.S.

MASSEY UNIVERSITY

Dean Halford returned in August from two months leave in Italy and the United Kingdom. His report on the 10th International Conference on General Relativity and Gravitation mentions, among other things, the fact that computers are now being used to handle the enormously complex algebraic manipulations encountered in this subject. Dean also attended a conference in Exeter on the teaching of mathematical modelling; it is clear that this is an area of growing importance in mathematics education.

Susan Byrne left at the end of August for six months leave, to be spent mainly at Imperial College, London.

Our congratulations go to Gordon Knight, who received his Ph.D. in Education for a study of the reasons why otherwise able adults have difficulty in understanding mathematics.

Seminars:

Trevor Boyle (NZMS Visiting Lecturer), *'Out of school and into what? Mathematics from school to university'*, *'What have we done to school mathematics?'*.
Gordon Knight, *' 'Careless' errors in algebraic manipulation'*.
L.C. Lee, *'A traffic simulation'*.

M.R.C.

VICTORIA UNIVERSITY

Dr Jim Ansell, left on 30 November 1983 for extended research leave (14 months) at Cambridge University, U.K. where he will join the V.U.W. group of geophysicists who are also currently at Cambridge (Dr John Harper, Professor Dick Walcott, newly appointed Professor of Geology at V.U.W.).

Professor Wilf Malcolm resumes full-time membership of the Mathematics Department in 1984 after a period of half-time duties in the Senior Administration as Pro-Vice-Chancellor (Academic). We welcome his "other-half" back into the continuing excitement of the mathematical world.

Professor Rob Goldblatt of this Department will be on exchange leave in 1984 at Auckland University. In return we are to be joined by Dr Marston Conder from that Department. This arrangement has successfully enabled Marston to be united with his (soon-to-be) wife at the Wellington Clinical School. Marston, like yours truly, is one of the few Brasenose Alumni (Oxford) resident here in N.Z.

Dr Fabio Musmeci, of the Comitato Nazionale per la Riceria e per lo Sviluppo dell' Energia Nucleare e delle Energie Alternative, Rome, Italy is currently visiting the Institute of Statistics and Operations Research, and the Institute of Geophysics until mid-1984. His interests are the statistical study of earthquakes.

Mr Roger Young, previously a Junior Lecturer in the Department has been reappointed as a Teaching Assistant for 1984 while he completes his Ph.D. studies in seismology under the direction of Dr J.H. Ansell.

G.C.W.

DSIR

APPLIED MATHEMATICS DIVISION, WELLINGTON

Mark McGuiness has joined the staff after graduating from Canterbury and spending 3 years at University College Dublin and 2 years at Cal. Tech.

Dick Sedcole, formerly of Grasslands Division, has seen the light and joined AMD's sub-station at Palmerston North.

Kelly Mara has returned to rejoin the staff after teaching at the University of Minnesota for 2 years and one year at the University of Georgia.

Jocelyn Dale has returned after 3 years at the University of London after completing her Ph.D. thesis *'Statistical Methods for ordered Categorical Responses and Sparse Contingency Tables'*.

There was a visit by Roald, Shirrell and Bulher from the PSTAT Project at Princeton to talk about PSTAT.

A meeting will be held in Auckland in late November between the maths-physics section and T.A.M. to meet each other and exchange notes.

Robin Wooding is organising a conference at Wairakei in May 1984 on Convective flows in Porous Media.

There will probably be no news next time from AMD with the writer being in Japan between January and May.

G.J.W.

MT. ALBERT RESEARCH CENTRE, AUCKLAND

Seminars:

Dr G.R. Edwards, '*Scheduling problems*'.
Mr J.H. Maindonald, '*Computer methods for statistical analysis; Linear models*'.
Dr C.M. Triggs, '*Experimental design; Linear models; Robust regression*'.
Dr J.R. Dale, '*Statistical methods for categorical data*'.

J.H.M.

CANTERBURY UNIVERSITY

In August, Gordon Petersen suffered a stroke which caused partial paralysis of his left side. Once again he showed great powers of recovery, and is now moving about without assistance. He is living at home, and making occasional visits to the Department. However, he decided to retire from the end of January, 1984, some three years earlier than he had originally planned.

As a consequence of Professor Petersen's retirement, the department had to carry out, for the first time, the democratic exercise of electing a new Head. The choice fell, unanimously, on Roy Kerr, for a term of four years. He is enthusiastically carrying out his duties with the aid of a faithful confidante, a desk-top Panasonic computer.

The Department has also set up an Executive Committee, to act in an advisory capacity to the Head on matters which do not need to be considered by the whole Department. In this way we hope to reduce the frequency and duration of Departmental meetings.

Yet another innovation is the publication, each Friday, of a small Departmental newsletter, containing reminders, notices and general items of interest, past, present and future.

Bill Taylor has departed for a year's study leave at Westfield College, University of London.

Peter Renaud leaves in December for a year at Macquarie University, N.S.W.

Brian Woods, on leave at the University of Oregon, is adapting well to his new role as a mathematical vulcanologist. We have received photographs of him walking confidently on the slopes of Mt St Helens.

In early October the Department was again host to the twenty finalists in the national BNZ Senior Mathematics Competition. The examination paper was devised by David Robinson, and at the prize-giving ceremony John Deely spoke on '*Sample Surveys - the bad news and the good news*'.

Seminars:

Dr Sum Chow (Australian National University), '*Finite element error estimates for blast furnace gas flow problems*'.
Professor Jan Jaworowski (University of Indiana), '*Antipodal coincidence theorems for continuous families of maps*.

R.S.L.

OTAGO UNIVERSITY

Professor Ivor Francis attended the 44th Session of the International Statistical Institute (ISI) held in Madrid, Spain, 12-22 September 1983, where he was an invited discussant in a session on the microcomputer in statistics. The meeting of the ISI and its section, the Bernoulli Society for Mathematical Statistics and Probability, the International Association for Statistical Computing, and the International Association of Survey Statisticians, attracted some 800 delegates plus accompanying persons who were welcomed at the opening session by King Juan Carlos. The ISI is a bridge between statisticians of different callings, such as academic, research, and government, and the scientific sessions reflect these broad interests. In recent years considerable attention has been paid to the problems of developing countries. The Sections of the ISI frequently sponsor regional meetings. The hospitality of the Spanish government was splendid. It included the presentation to each delegate of a facsimile of the proceedings of the International Statistical Congress from 1853 to 1869 prepared for the Congress of 1872 in St. Petersburg. The driving force behind these international meetings had been the Belgian mathematician Lambert Adolphe Jacques Quetelet. They led to the establishment of a Permanent Commission in 1872 and to the founding of the ISI in 1885. The centennary of the founding of the ISI will be celebrated at its next meeting in the Netherlands in 1985.

Dr John Harris will be on Sabbatical Leave in 1984. His plans include work on programming logic in the Computer Science Department at Indiana University in the U.S.A.

Dr Roselyne Joyeux of the Economics Department gave a seminar on "*Harmonizable Processes: Characterization Theorems and Applications*.

Mr Trevor Boyle, Principal of St. Peter's College, Palmerston North, and this year's NZMS Visiting Lecturer gave a talk on "*WHAT have we done to school MATHEMATICS*". This talk was jointly arranged by the Otago University Mathematics Department and The Otago Mathematics Association.

Professor W.B. Bonnor, head of the Mathematics Department, Queen Elizabeth College, University of London, will be a William Evans Visiting Professor at the University of Otago, 30 March - 30 June, 1984, attached to the Mathematics Department. Professor Bonnor is a relativist of distinction and has published a large number of papers dealing with important aspects of general relativity and its applications. These include solutions of the Einstein and Einstein-Maxwell equations, singularities of the Einstein equations, gravitational waves, gravitational instabilities in cosmology and the problem of the formation of galaxies, the Cauchy problem and the mechanics of general relativity. He has also published three books on relativity and cosmology. At this stage it seems very likely that Professor Bonnor will attend the 1984 Mathematics Colloquium and deliver a lecture. Professor Davidson will be hosting Professor Bonnor's visit.

G.O.

UNIVERSITY OF THE SOUTH PACIFIC

Donald Joyce has resigned to take up the chair at the University of Papua New Guinea in Port Moresby. He is currently on leave in England and will be returning via India, Singapore and Malaysia.

Richard Pollard is acting head, pending the appointment of a professor. Hasmukh Morarji is acting Director of the Computer Centre, in charge of a staff of five and an ICL 2903 computer system.

The maths department now has nine Commodore microcomputers and has introduced computing to nearly 3000 Pacific Island residents.

Subhan Ali, Kevin Donegan and Hasmukh Morarji will all be on study leave next year.

Clive Britton and Tim Dalby took up posts at the beginning of the year, Tim returning after a master's programme at Canterbury.

Sunil Kumar submitted his M.Sc. thesis (the first in mathematics at USP) and left to begin his Ph.D. programme at the University of New South Wales.

Parul Deoki was appointed for a further term as coordinator of preliminary/foundation maths.

D.J.

19TH NEW ZEALAND MATHEMATICS COLLOQUIUM

The nineteenth New Zealand Mathematics Colloquium will be held at Victoria University of Wellington from Monday, May 7th to Wednesday, May 9th, 1984.

Preliminary registrations are being solicited for the 1984 Colloquium. There will be sessions for papers to be presented by participants, as well as several lectures by invited speakers in various disciplines. Details of these will be announced in the next circular to be sent out early in the new year.

Presented papers will be expected to take of the order of 20-30 minutes, and papers which present survey material at a level likely to be accessible to a general mathematical audience are specifically solicited.

A program of papers, etc. concerned with Mathematical Education is being planned for Wednesday 9th May, and a complete day devoted to Time Series and multivariate analysis is planned for Thursday 10th May. This latter program will be held under auspices of the New Zealand Statistical Society.

For further details write to: Dr B.P. Dawkins, Maths Department, Victoria University, Private Bag, Wellington, New Zealand.

Book Review

STARTING FORTH: by Leo Brodie, FORTH, Inc, Prentice-Hall, Englewood Cliffs (1981).

INVITATION TO FORTH: by Harry Katzan, Jr, Petrocelli Books, Inc. New York (1981).

FORTH is a language of growing popularity among microcomputer users, for a number of reasons. Among these is the fact that a FORTH system can be implemented quite easily on a large variety of systems, while programs written in high-level FORTH enjoy a great deal of portability. One of its most important features, however, is its extensibility: the FORTH programmer has the ability to add not only new procedures to the language, but also new data structures, new control structures , and even new procedures for creating new control structures! The above two books both purport to be introductory texts for the beginning FORTH programmer.

There is a truly wonderful contrast between these two books, in terms of the elusive substance called 'quality' by Robert Pirsig in "Zen and the Art of Motorcycle Maintenance'. On the one hand is the book by Brodie, full of valuable and useful information, written in a lucid and good-humoured style and liberally sprinkled with illustrations which are not only entertaining but also relevant and entirely consistent. On the other hand, there is the book by Katzan.

The Brodie book has been praised frequently and well elsewhere; its undeniable high quality will ensure its continuing success. It is my purpose to warn the prospective buyer away from the other. It was my misfortune that when I accepted the 'Invitation', I was foolish enough to write my name on the fly-leaf before reading it - alas, the bookshop will no longer take it back.

Let's be specific: I'll deal with the Brodie book first, then the Katzan book. Starting FORTH contains twelve chapters, which lead the reader from a gentle but nevertheless brief introduction to Polish postfix notation through to the sophisticated and powerful aspects of FORTH - extending the compiler and building new control structures. The concluding chapter contains three non-trivial programming problems, and offers FORTH solutions. These are enormously valuable aids for those beginning to program in FORTH. In summary, the Brodie book is an excellent, authoritative and innovative text on an innovative language - I recommend it without hesitation.

The Katzan book has nine chapters - after three of these chapters (some 75 pages!) the author has still not adequately explained Polish postfix notation! After 130 pages (up to Chapter 6 inclusive), the text has managed to deal with elementary calculations, trivial stack manipulations and the creation of FORTH words. Still the fundamental control structures of FORTH have not been mentioned! Katzan then spends 35 pages on the elementary control structures (DO loops, IF statements and the like), and the final chapters deal with double precision and 'Information Management'. This final chapter gives an explanation of the way memory is organized by a FORTH system, and gives some information on input and output facilities.

The major omission which I find most disturbing in the Katzan book is that of the BUILDS ... DOES construct. It is this control structure which makes FORTH so powerful - the ability to control the compile-time and run-time behaviours of the computer. This is surely an essential topic of discussion in any text on FORTH.

This book was written with the aid of a word-processor, and is a good example of how badly it is possible to write with such an aid. The author has a number of stock phrases, possibly inserted using macros, which are used throughout the book. For example, at the end of each chapter, we are given a short vocabulary (not in itself a bad idea), and this is always introduced as follows:

A general familiarity with the following terms and FORTH words is necessary for learning the FORTH language:

There are several other such stock phrases which Katzan uses throughout. These give a curios and perversely interesting flavour to the book.

In conclusion, I can strongly recommend the Brodie book, and cannot adequately condemn the Katzan book. If the reader is interested in learning about FORTH, buy the former, not the latter!

Peter R. Mullins

Secretarial

MINUTES OF THE 14TH COUNCIL MEETING

Held at Science Centre, Wellington on 6th December 1983

PRESENT: W. Davidson (President, in the Chair), M.R. Carter, P.D. Hill, A. McNabb, J.L. Schiff, J.A. Shanks, D.J. Smith.
In Attendance: C.H.C. Little (by invitation).

1. APOLOGIES: J.H. Ansell, I.D. Coope.

   Moved from the Chair that the apologies be accepted. CARRIED

   It was also noted that Ian Coope had resigned from his position on the Council.

   Moved from the Chair that the resignation be accepted and that Ian be thanked for his contributions to the running of the Society. CARRIED

   A welcome was extended to Charles Little.

2. MINUTES:

   Moved from the Chair that the minutes of the 13th Council Meeting held on 22.5.83 and the Brief Council Meeting held on 24.5.83 be taken as read. CARRIED

   Moved (MRC/DJS) that the minutes be accepted as a true and correct record of those meetings. CARRIED

3. MATTERS ARISING FROM THE MINUTES:

   (a) Prince and Princess of Wales Award Scheme - JAS reported that $300 had been donated for 1983-84 (through RSNZ) and that the scheme was now under-way.

   (b) Project Competition for Teachers - it was noted that prizes had been presented to first, second and third place getters. JAS tabled copies of certifcates that had been sent to the prize-winners. JLS tabled a letter of thanks from the overall winner with copies of news items that had appeared in the local press.
   The organiser and judges were thanked for their work. There was some discussion about publication of winning entries in view of a letter from the editor of the N.Z. Maths Magazine.

   Moved (DJS/JLS) that the editor of the N.Z. Maths Magazine be approached and encouraged to contact the authors of winning entries with a view to publication, with adequate mention of the organisers of the competition. CARRIED

   The holding of another competition was discussed and PDH suggested that every 2 years would conveniently interleave this with the Pre-doctoral thesis competition. DJS commented that there may be some clash with the Ernest Duncan award, but this had not become evident as yet.

   Moved from the Chair that the editor of the N.Z. Maths Magazine also be informed of the Society's intention to continue to hold the competition, normally every 2 years with publication becoming a regular matter. CARRIED

   It was noted that the NZMS judge, P.J. Lorimer, had suggested that in future all judges be from one centre.

   (c) N.Z.M.S. Visiting Lecturers - a report was received from W.D. Halford. The 1984 lectureship had been offered but acceptance had not yet been received. The report raised the matter of a suitable honorarium for a distinguished visitor.

   Moved (MRC/DJS) that $400 be appropriate for the visitor under consideration, should he accept. CARRIED

   Comments had been received from several sources concerning the financing of the NZMS lecturer. It was decided to remind the Visiting Lecturer Selector that the NZMS holds all responsibility for meeting expenses, and that while the various centres should be asked for support, their help is in no way obligatory.

(d) Two year term for Presidents - JAS reported on the postal vote (34 for, 1 against the proposal) and the meeting noted that the change of constitution was therefore adopted. As the change will not take full effect until 1985, it was decided to announce the adoption at the next AGM and subsequently have the amendment registered and the updated constitution published in the Newsletter.

(e) Conditions of contract for authors of NZMS publications - a report was received from G.C. Wake and J.H. Ansell, and discussion took place on the recommendations contained in the report.

Moved from the Chair that
(i) existing contracts continue for the seventh form syllabus series,
(ii) the Society establish new contractual arrangements for future publications (including the Calculus text) where royalties would be calculated as a negotiable percentage of the recommended sale price, the percentage normally being 10% (between all authors); exceptional expenses incurred in the production of the text should be met by the Society separately,
(iii) the copyright of NZMS publications shall be vested in the Society. CARRIED

It was noted that joint publications would require other arrangements.

(f) University of the South Pacific fund - a report was received from D.J. Smith. The existence of the fund was announced in the USP Bulletin of 16.9.83 and copies of the announcement were circulated to those at USP likely to be interested. To date, one application has been received.

4. CORRESPONDENCE:

Moved (DJS/MRC) that the inwards correspondence be accepted and the outwards correspondence be approved. CARRIED

(a) New publication policy for RSNZ Journal - W.D. reported that he had suggested the names of 3 people who could possibly submit review articles to the Journal. The editor had replied with a note that "heavy" mathematics should be avoided. Professor G. Seber had already produced a paper which was being passed on to the editor of the Journal. As reported in the Newsletter, it was noted that any member can submit an article and it was not up to the Council to select writers.

(b) Nomination to RSNZ for Honours award - following a recommendation from the Royal Society the list of three names was reduced to one. A fully detailed submission would be compiled and presented in support.

(c) NZMS Lecturer at 1984 Maths Colloquium - the meeting endorsed the selection of Professor W.B. Bonnor who will be William Evans Visiting Professor at Otago, March to June 1984. A brief discussion of the history of selection of the Colloquium Lecturer followed.

Moved (DJS/MRC) that necessary decisions concerning the NZMS Colloquium Lecturer be made by Council at the mid-term meeting. CARRIED

(d) Collaboration in publications - deferred to 5(d).

(e) Gift from J.T. Campbell - JLS reported that Professor Campbell had contributed $500 to the publication fund. This generous donation was gratefully received and the Secretary would be writing to Professor Campbell to thank him for his gift.

(f) Delegates for People-to-People International - the choice of a leader was discussed for a delegation to the USA by a group of N.Z. mathematicians.

Moved (DJS/PDH) that a subcommittee (President, Secretary and Treasurer) be established to select up to 3 suitable candidates and with their permission to submit their names to the Citizen Ambassador Program, and that subsequent to a final decision the subcommittee would liaise with the leader in selecting delegates and in arranging other matters. CARRIED

It was suggested that the subcommittee explore the possibility of applying to the Prince and Princess of Wales Award Scheme for financial assistance for delegates.

(g) Support by NZMS for Mathematical Reviews - an appeal had been received from the President of the American Mathematical Society for funds to support the high cost of producing Mathematical Reviews.

Moved (JLS/MRC) that, for 1984, $200 be given to the AMS to support Mathematical Reviews, pointing out that this represented about 5% of our subscription, and asking whether the application of new technology might help to reduce costs. CARRIED

On a similar issue, it was decided not to take up a subscription to Siam News, now that the complimentary subscription had lapsed. Problems with distribution and the fact that the main libraries received copies meant that the receipt of Siam News by the Society would be inappropriate.

(h) Donation to 11th Combinatorial Mathematics meeting - JAS reported that $100 had been sent to CMSA to assist in the running of its 11th conference in Christchurch.

(i) Hamilton Award - WD brought the attention of the meeting to this award administered by RSNZ.

(j) Purchase of membership list by outside interests - no minuted resolution on this issue had been made in the past, although one publisher had been offered the list in 1982 for a certain sum. DJS thought that the Newsletter was intending to publish the list of members anyway so that it would become generally available. WD considered that the Council may not have the right to distribute the membership list. It was decided to defer the matter until the AGM when the feeling of the membership could be gauged.

(The meeting adjourned for lunch.)

5. PUBLICATIONS:

A report was received from I.L. Reilly.

(a) Applied Mathematics booklets - these continue to sell well and 2400 have been sold over recent months.

(b) Calculus book - 1100 are currently being produced to meet adoptions at Auckland and Waikato with a few spares for distribution to other centres. Discussion covered encouraging adoption elsewhere and the possibility of an Algebra text.

(c) Other publications - work has begun on a new 6th form Maths book for the new prescription for 1985. This is a joint venture with NZAMT with Dr L.C. Johnson convenor.

Moved from the Chair that in keeping with last year's resolution, copyright and profits should be shared between the two organisations and that NZAMT should be informed of our new contractual agreements. CARRIED

(d) Collaboration with overseas publishing companies - the meeting discussed an approach from Harcourt Brace Jovanovich Group and a letter concerning the Australian Mathematical Society Lecture Series (to be published by Cambridge U.P.). Reservations were expressed about co-operation with commercial publishers as the idea behind the NZMS publishing venture was to produce low cost texts relevant to New Zealand which could be easily modifed in response to local demand; the Society may also have to relinquish its copyright.

The secretary would write to the HBJ Group describing the present limited aims of the Society with regard to publications.

Moved from the Chair that the Publications Committee be asked to consider the general question of collaboration with commercial publishers as one means to extend the market for our tests, for example a second edition of Calculus. They should also be asked whether direct marketing in Australia was feasible. CARRIED

6. NEWSLETTER:

JLS presented a report from M.J. Curran (Newsletter editor). Printing costs for the last Newsletter were considerably down on previous issues, but postage was now a significant cost, especially for airmail overseas.

Moved (JLS/PDS) that all copies of the Newsletter be sent surface mail and that if members so wished it could be sent airmail at an extra charge of $1.50 each issue. CARRIED

7. FINANCIAL REPORT:

(a,b) An interim financial report was presented by JLS.

Moved from the Chair that the interim finanancial report be received. CARRIED

There was some discussion about the use of the general account for some publication expenses but JLS replied that it was so much more convenient and that anyway the end-of-year audited accounts would clearly separate the outgoings. JLS encouraged Council members to promote increased membership.

(c) C.J. Atkin's membership - there was some discussion of Dr Atkin's association with the Society during which his subscriptions had fallen into arrears.

Moved (JLS/AMcN) that Dr Atkin be recorded as a full member since 1981, subject to his paying arrears of $58, and that he be thanked for his services to the Society in preparing the "Postgraduate Topics" booklet. CARRIED

9. PRE-DOCTORAL THESIS COMPETITION:

A report was received from I.D. Coope, who was to be asked to continue in his position as organiser. Response is so far sparse; judges are being sought and previous sponsors are being approached. The Council was unclear about which theses were eligble, as there could be some final year Honours projects that would be worthy of consideration. MRC agreed to discuss a clarification of the rules with IDC.

10. OVERSEAS CONFERENCES:

The meeting noted
(a) Australian Maths Society 28th Annual Meeting, May 14-18, 1984, Monash.
(b) 1985 Australasian Maths Convention at University of New South Wales.

11. SUMMER RESEARCH INSTITUTE 1985:

A report was received from D.B. Gauld, and discussion centred on financial help that could be offered.

Moved (DJS/MRC) that $500 be denoted to the organising body of the Summer Research Institute to be held in Auckland. CARRIED

DJS agreed to liaise between the organisers and NZMS.

12. GRADUATE INFORMATION:

MRC presented a brief report, indicating that plans were in motion and that 3 years hence we would have at least 20 replies from graduates.

13. VISITOR INFORMATION:

A detailed report was received from W.D. Halford. The scheme was proving popular and hopefully useful. Over 50 enquiries from overseas have been answered with the help of a "kitset" of information.

As time was pressing, matters raised in the report were deferred until the May meeting.

14. INVOLVEMENT OF NZMS IN CURRENT ISSUES:

MRC raised the question of whether it was the place of the Society to become involved in current issues such as the UE controversy and the use of computers in mathematics teaching. A discussion was deferred until May. WD suggested that in the meantime, members may like to comment on these matters, in general or particular, through the "Letters to the Editor" columns of the Newsletter.

15. GENERAL BUSINESS:

(a) WD suggested a fund to assist students to attend conferences. Detailed discussion was deferred to the May meeting.

(b) Nomination for fellowship of RSNZ - WD reported that the material supporting the Society's nomination had been updated and submitted to the RSNZ.

(c) Other business.

Moved (DJS/MRC) that the Society donate $150 to the organisers of the 1984 Colloquium. CARRIED

Moved from the Chair that Charles Little be co-opted on to the Council with a view to his becoming Secretary for 1984/85 subject to his election at the next AGM. CARRIED

The meeting closed at 4.50 p.m.

J.A. Shanks
Secretary

Problems

Sub-edited by A. Zulauf, University of Waikato

PROPOSALS of problems should be sent to the sub-editor and should be accompanied by solutions and/or relevant references, comments, etc.

SOLUTIONS should be sent to the sub-editor within three months from the publication of each problem. If you discover that a problem has already been mentioned or solved in literature, please send full details to the sub-editor.

****

Comments on Problem 8 *(Erdös' $25 Question)*

The reward for the answer to this problem has not been claimed as yet: Is it true that for every n there are n distinct points in the plane, no three on a line, no four on a circle, so that they determine $n - 1$ distances so that the i-th distance (in some order) occurs i times? Let us say that an n-gon is remarkable iff its vertices have the property in question, and let $K(n)$ be the number of similarity classes of remarkable n-gons. Obviously, $K(3)$ and $K(4)$ are infinite. I know that $K(5) > 60$ and that $K(6) \geq 14$, but all remarkable hexagons that I have come across have central symmetry. The following problems are related to, but possibly easier than, Erdös' $25 question.

Problems 8a

1. Is $K(5)$ finite?
2. Is $K(n)$ finite for all $n \geq 5$?
3. Is there a remarkable hexagon which is not centrally symmetric?
4. Is there any remarkable n-gon with $n \geq 7$?

Sub-editor

Problem 10 *(Flight time along closed path.)*

The following is an extension of G. Wake's problem 9.

An aircraft travelling with constant air speed V describes a piecewise smooth closed space curve C, while the wind blows with constant speed mV in a fixed direction. Show that the total flight time $T(m)$ strictly increases with m $(0 \leq m < 1)$.

A.G. Mackie, Edinburgh

Problem 11 *(A diabolic square.)*

Find sixteen positive integers in arithmetic progression so as to complete the diabolic square shown on the right, where $x < y < z < t$. Show also that your solution is unique.

(Note that, in a diabolic square, the four row-sums, the four column-sums and the eight diagonal sums are all equal.) The heavy lines drawn in the top-centre of the diagram have no significance until the solution has been completed. The unknown w has no relevance here, but is referred to in a mini-problem below.

	y		
z	69	x	
w	t		
			24

Sub-editor

Mini-problems:

The following easy problems are offered simply for the sake of interest or amusement. Readers are NOT invited to submit their solutions.

A property of the parabola

Mr T. O' Helu of Nuku'alofa, Tonga, points out that a quadratic polynomial $f(x) = ax^2 + bx + c$ satisfies

(*) $$f'(x) = \frac{f(x+h) - f(x-h)}{2h} \quad \text{for all real } x \text{ and } h .$$

(In geometrical terms this means: If L is a straight line parallel to the axis of a parabola, meeting the parabola at P, then L bisects the system of chords that are parallel to the tangent at P.) Show conversely that all solutions of the functional equation (*) are given by $f(x) = ax^2 + bx + c$.

Problem 11 simplified

Readers unfamiliar with the theory of diabolic squares may find the solution to the above problem 11 tedious. Given the additional clue that $w < x$, the problem can be tackled quite easily from first principles.

Sub-editor

THE INTERNATIONAL CONFERENCE ON THE TEACHING OF MATHEMATICAL MODELLING

A report by H. Levy (University of Otago) and D. Halford (Massey University)

Over 100 mathematicians from universities, polytechnics, secondary schools and research establishments attended this conference held at the University of Exeter, 12-15 July 1983.

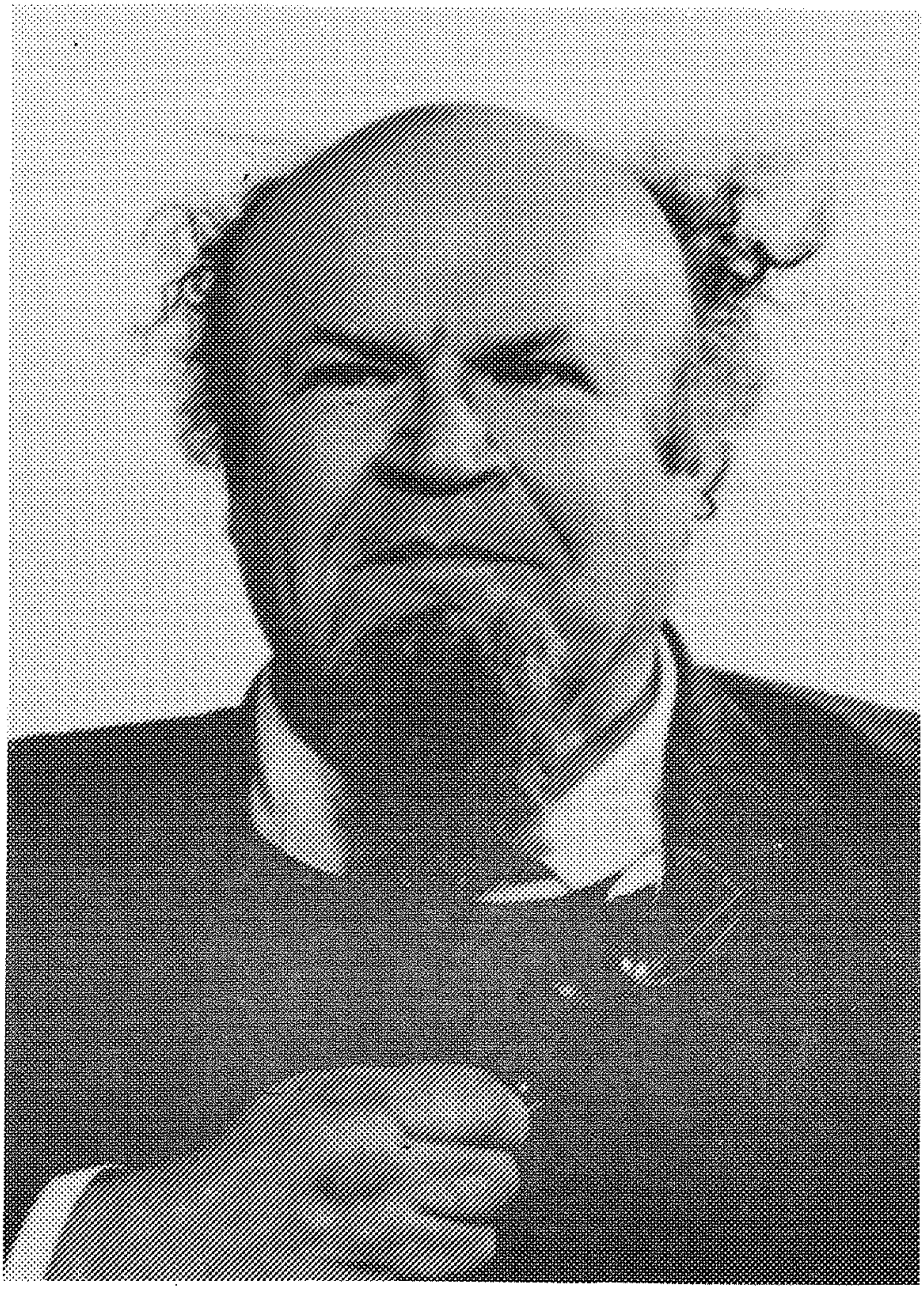
Mathematics graduates need, and are expected by industry, to be able to solve practical problems. In order to teach this skill more successfully it was felt, by those attending the conference, that students needed at least one course which would require them to be actively involved in the modelling process. To illustrate how this might be done we were shown some video films prepared by the Open University [1].

A number of speakers discussed the problems and difficulties associated with giving a modelling course. Assuming that it is possible (and this was never established beyond reasonable doubt!) the teaching of modelling requires the teacher to be flexible, confident in his subject and preferably to have had some industrial experience. The student too needs to be mature, and should only be required to exhibit mathematical skills acquired in earlier years, otherwise the exercise is unlikely to prove successful. It was generally agreed that students should undertake an investigation, perhaps working within a small group on the same project; the difficulties associated with assessment could, by and large, be overcome by double marking of the student's final report. It was not felt that collaboration between students presented any serious assessment problems, though in some cases an oral examination might be necessary. A number of speakers gave examples of projects which they had found to be successful, most of which can now be found in books [2,3]. We were also repeatedly reminded of the importance attached to validating models, and that much could be learnt from the exercise even when the model proved to be unrealistic.

Although most of the papers presented were concerned with tertiary level mathematics, there was also some lively discussion on what was wrong with the school curriculum, particularly in the United Kingdom. It was generally felt that the material presented tended to be too abstract, with too much emphasis given to structure and terminology, the syllabus perhaps being appropriate only for the minority intending to take mathematics at a university. In order to justify the emphasis on mathematics in the school curriculum, teachers must demonstrate that mathematics is both useful and relevant in the modern world, and this can perhaps best be done by teaching it through its applications. An interesting experiment along these lines is now taking place in the Netherlands [4].

In conclusion we could not help but be impressed by the amount of interest and work now going on in the field of mathematical modelling in the Open University and the polytechnics in the United Kingdom and Europe. We suggest that in New Zealand the Universities should keep abreast with these developments and embark on more experiments of our own.

[1] J. Jaworski, BBC/Open University, U.K.
[2] D.N. Burghes and M.S. Borrie, *Modelling with Differential Equations*, Ellis Horwood (1981).
[3] D.N. Burges and A.D. Wood, *Mathematical Models in the Social, Management and Life Sciences*, Ellis Horwood (1980).
[4] J. de Lange and M. Kindt, *The Hewet Project*, University of Utrecht, Netherlands.



Centrefold

GORDON PETERSEN

No New Zealand mathematician would fail to recognise one of their number described thus: "Larger-than-life-size, American with Danish and English origins, Canadian and Welsh overtones, Chinese undertones and New Zealand associations".

Gordon Petersen was born in San Francisco in 1921. He was an undergraduate at Stanford in 1941-43, taking lectures from Seger, Pólya, J.V. Uspensky and Blickfield. After a spell of school-teaching at Deep Springs he returned, in 1946, to do an M.Sc. degree under D.C. Spencer, who with A.C. Scheffer was studying univalent functions. The University of British Columbia was just then expanding from about 2,000 students to 12,000 with returned servicemen, (100 army barracks were brought in on rafts from Washington to act as lecture-rooms) and Gordon spent two years lecturing there. Then off to Toronto to do a Ph.D. in functional analysis under G.G. Lorentz. With G. De B. Robinson, Stanton and Coxeter also at Toronto it was an enjoyable place and Gordon's thesis gave rise to three papers in Fourier series. Off to Manitoba in 1951, which he found awful; thence to Arizona where he started upon his continuing interest in mathematics for top students, writing a problem book with R. Graesser; on to Oklahoma in 1954, which he dexcribes as god-awful.

Fed up with Americans, Gordon headed off to Swansea in 1955 where, except for an interlude in 1957 in Alberqueque, he stayed for a decade. This was a productive period, with a number of papers on divergent series, some jointly with F.O. Keogh.

In late 1965 Gordon took up the new chair of pure mathematics at Canterbury, joining Derek Lawden (then head of department), "Billy" Andress, Mary Harding and others (who are still on the "scene") making a total of 13 staff. The department was still at the old town site and computing within the university was under its umbrella. In 1967 Derek Lawden returned to Birmingham and Gordon took over as head, presiding over the rapid growth of staff numbers up to the present 26.

Of his association with mathematical life in New Zealand Gordon looks back on the establishment of the Colloquia as a great success. He recalls making the suggestion at a Steering Committee meeting in Professor Campbell's office in Wellington and having the balance tipped in its favour by Professor Jowett's enthusiasm. The First New Zealand Mathematics Colloquium took place in Wellington the following May (1966).

To the formation of a New Zealand Mathematical Society, Gordon initially expressed opposition - on the grounds of its financial insecurity, the confusion with the affairs of the Colloquium and the inability of the country to support another mathematics journal. (All of these considerations remain current!) However, when its birth became inevitable, he supported the Society, especially in its links with Australia, and served as President for two years.

His local university contributions have been towards fostering indigenous M.Sc. and Ph.D. programmes, getting rid of the language requirement for B.Sc.Hons. and not least, visiting Roy Kerr in America and persuading him to return.

Internationally, Gordon's name is synonymous with matrix summability theory - an area which, in a less complicated era, G.H. Hardy was content to refer to vaguely as divergent series. The last 30 years has witnessed a stream of impressive papers with the result that now summability theory is an important and stimulating mix of some of the best and noblest results in classical and functional analysis. His book "Regular Matrix Transformations", which appeared in 1966, was a welcome attempt to survey this field, to which he has contributed over seventy papers, and to plot directions for future growth. It is still essential reading for any researcher (raw or seasoned) who wishes to appreciate this beautiful branch of analysis.

G.M.P. retires in January, prematurely, because of ill-health which has also affected a proposed trip to his beloved China.

W. Brent Wilson

Feature Articles

PERMANENTS, DETERMINANTS, BOSONS AND FERMIONS

D. VERE-JONES

VICTORIA UNIVERSITY

An invited lecture at the 18th N.Z. Mathematics Colloquium, Massey, May, 1983.

As a mathematical fortune-seeker, one spends much time overturning mathematical stones under which there lies only mud, or at best a few specks of gold. Recently, however, I had the good fortune to overturn a mathematical stone under which there lay concealed a jewel; not quite new, but striking, unexpected, and certainly unfamiliar not only to myself but also to my immediate colleagues. This jewel is the following expansion for the reciprocal of the characteristic polynomial $|I - \lambda A|$ of a square $n \times n$ matrix $A = \{a_{ij}\}$.

$$(1) \qquad |I - \lambda A|^{-1} = 1 + \lambda \sum a_{ij} + \frac{\lambda^2}{2!} \sum {}^{+}\begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}^{+} + \ldots$$

The plus signs here signify the <u>permanent</u>, defined as for a determinant but without the alternating signs: thus for example

$$ {}^{+}\begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}^{+} = a_{ii}a_{jj} + a_{ij}a_{ji}$$

The summation in the 2nd order term in (1) is extended over all distinct permutations of two indices i, j out of n , <u>allowing repetitions</u>. There are then $\frac{n(n+1)}{2}$ distinct permanents in the sum, the $\frac{n(n-1)}{2}$ permanents with distinct indices each appearing twice, and those with a repeated index appearing once each. The higher order terms are constructed similarly. The series continues indefinitely, and converges for $|\lambda| < \frac{1}{\max \lambda_i}$, where the λ_i are eigenvalues of A . In general the k-th sum has $\binom{n+k-1}{k}$ distinct permanents, that with k_1 indices equal to 1 , k_2 to 2 , ... k_n to n $(k_1 + k_2 \ldots + k_n = k)$ being repeated $\binom{k}{k_1 \ldots k_n}$ times, its effective coefficient in the expansion being therefore $\prod_1^n \frac{1}{k_i!}$.

The identity (1) is of course dual to the well known expansion of the characteristic polynomial itself, namely

$$(2) \qquad |I - \lambda A| = 1 - \lambda \sum a_{ii} + \frac{\lambda^2}{2!} \sum \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} + \ldots + (-1)^n |A| .$$

Here the series terminates after n terms, and no repeated indices appear, since they lead to repeated rows and hence to zero determinants.

After stumbling across the identity (1) in the course of working up some examples for Chapter 5 of a book I am supposed to be writing, I have had a rather fascinating time trying to trace it to its source. In the rest of this talk I shall try to summarize my findings to date. I am sure there is more to be told, however, and would be glad to receive further references or suggestions.

There are three main angles that I have tried to follow up:-

(i) the combinatorial background (determinants, symmetric functions etc.);
(ii) the relation to moments of the multivariate normal and χ^2 distributions;
(iii) the quantum theory context.

I shall say a little about each of these.

2. The Combinatorial Background

The term "permanent" goes back to a memoir [1] by Cauchy, in which he refers to "fonctions symmétriques permanentes" to distinguish them from those with alternating signs which we now refer to as determinants. Since then they have figured sporadically on the mathematical scene. Aitken, in his little monograph [2] , says somewhat disparagingly "... its properties are neither so simple nor so rich in application as those of determinants, but it has an importance in the theory of symmetric functions and in abstract algebra". He points out that permanents share with determinants many of the well-known expansion theorems, such as the expansion by co-factors and Laplace's expansion, but depart from them in not reflecting the multiplication of matrices, so that in general $^+|AB|^+ \neq {}^+|A|^+ \, ^+|B|^+$. In the last few decades they have had something of a resurgence in connection with the Ising problem in mechanics, and "Van der Waerden's Conjecture" (just recently solved, as I am informed):- the permanent attains its minimum value for doubly stochastic matrices (non-negative, row and column sums all equal to unity) when all elements are equal to $\frac{1}{n}$.

I have, however, found no statement of the identity (1) in the literature relating to permanents. It is not to be found, I am fairly confident, among the works of Cayley, Muir, MacMahon, Turnbull or Aitken. It is not referred to in the monograph on permanents by Minc [3] .

There is, however, an essentially equivalent and quite celebrated result which goes by the name of "MacMahon's Master Theorem".

It was proven by MacMahon in 1894 [4] and so-called by him in his book (1915) on combinatorics [5] . MacMahon (1854-1929) himself is worthy of some comment, as a mathematician whose true stature is only recently emerging. George Andrews, whom many of you will remember as the NZ Maths Society Lecturer a few years back, recently published an edition of his collected papers [6] , in the Preface to which G.C. Rota writes, "With his moustache, his "British Empah" demeanour, and worst of all his military background, MacMahon was hardly the type to be chosen by Central Casting for the role of the Great Mathematician. Ramanujan, with his Eastern aura, his frail physique and his swarthy good looks, qualified all the way. It should have been fascinating to have been present at one of those battles of arithmetical wit at Trinity College, when MacMahon would regularly trounce Ramanujan by the display of fast mental computation (as reported by D.S. Spenser, who heard it from G.H. Hardy)".

What the Master Theorem actually states is as follows:-

<u>Theorem</u> Let $\underset{\sim}{X} = A\underset{\sim}{x}$. Then the coefficient of $x_1^{k_1} x_2^{k_2} \ldots x_n^{k_n}$ in the expansion of $X_1^{k_1} X_2^{k_2} \ldots X_n^{k_n}$ is the same as the coefficient of the same term in the expansion of $|I - DA|^{-1}$ where $D = \mathrm{diag}(x_1 \ldots x_n)$.

To see that the Master Theorem is equivalent to the identity (1), or rather its polarized form (4), one has to check that the coefficient of $x_1^{k_1} \ldots x_n^{k_n}$ in $X_1^{k_1} \ldots X_n^{k_n}$ is equal to $\Pi\left(\frac{1}{k_i!}\right)$ times the permanent obtained from A by repeating the index 1 i_1 times, the index 2 i_2 times, and so on. Thus for $i_1 = 2$, $i_2 = 1$, the quantity

$$\frac{1}{2!} \times \frac{1}{1!} \; {}^+\begin{vmatrix} a_{11} & a_{11} & a_{12} \\ a_{11} & a_{11} & a_{12} \\ a_{21} & a_{21} & a_{22} \end{vmatrix}^+$$

should appear as the coefficient of $x_1^2 x_2$ in the expansion of $(a_{11}x_1 + a_{12}x_2)^2 (a_{21}x_1 + a_{22}x_2)$. It is not particularly difficult to verify that this is the case, and a similar identification is possible, in general.

Many proofs of the Master Theorem have been put forward, none of which are particularly easy. MacMahon's own proof used a generating function argument; others have used more directly combinatorial arguments, or methods using contour integrals. Andrews [6] gives one variant and lists numbers of further references.

In applications, the Master Theorem is used typically as a means of evaluating certain combinatorial expressions; one well-known example, used as such by MacMahon himself, concerns the sum of the cubes of the binomial coefficients:-

$$\sum_{r=0}^{n} \binom{n}{r}^3 (-1)^r = 0 \qquad (n \text{ odd})$$

$$= (-1)^{3m} \frac{(3m)!}{(m!)^3} \qquad (n = 2m) .$$

You may like to try and establish this identity from the Master Theorem, taking

$$A = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} .$$

Whether or not the identification of the coefficient which figures in the Master Theorem with a permanent will materially assist progress in this field is debatable. Perhaps one may hope that the association will suggest some new insights or applications.

3. Moments of the Complex Multivariate Normal and χ^2 distributions

Moment generating functions of the form

$$\psi(s_1 \ldots s_n) = |I - 2DA|^{-1} \quad (D = \text{diag } s_1 \ldots s_n)$$

are well-known in statistical literature. They arise in the following way. Consider two independent random vectors $\underset{\sim}{U}$, $\underset{\sim}{V}$ each $1 \times n$ and with multivariate normal distribution having zero mean vector and covariance matrix A. Then consider the joint distribution of the sums of squares

$$\begin{aligned} X_1 &= U_1^2 + V_1^2 \\ X_2 &= U_2^2 + V_2^2 \\ &\vdots \\ X_n &= U_n^2 + V_n^2 \end{aligned} .$$

It is not difficult to show that $\psi(s_1 \ldots s_n)$ is just the multivariate moment generating function $E[e^{\Sigma s_i X_i}]$. More generally, we can take $\underset{\sim}{U}$, $\underset{\sim}{V}$ to be the real and imaginary components in a complex normal vector $\underset{\sim}{Z} = \underset{\sim}{U} + i\underset{\sim}{V}$ with complex (Hermitian non-negative definite) matrix $A = E[\underset{\sim}{Z}\bar{\underset{\sim}{Z}}']$. Then the X_i are just the squared moduli of the components of $\underset{\sim}{Z}$.

Evidently the polarized form of (1), namely

$$(4) \qquad |I - 2DA|^{-1} = 1 + 2\sum s_i a_{ii} + \frac{2^2}{2!}\sum s_i s_j \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}^+ + \ldots,$$

is just the Taylor series expansion of ψ about the origin, and so the successive terms are the corresponding moments of the X's. Thus for example we find in the complex case

$$E[|Z_1^4||Z_2^2|] = E[U_1^2 U_2] = 8\begin{vmatrix} a_{11} & a_{11} & a_{12} \\ a_{11} & a_{11} & a_{12} \\ a_{12} & a_{12} & a_{22} \end{vmatrix}^+$$

$$= 32\, a_{11}|a_{12}|^2 + 16\, a_{11}^2 a_{22} .$$

The fact that these moments can be represented as permanents is apparently known in the statistical literature; at least they are quoted by Brillinger [7] who cites as his reference Dubman and Goodman [8]. The latter, however, provides neither reference nor proof, and at this stage in my quest I have drawn a blank in trying to track this aspect any further. Of course any proof that the moments could be represented in permanent form would imply a proof of the identity (1), at least for A positive definite.

4. The Quantum Theory Context

Quantum theory, to the statistician, is rather frightening territory. Not only is it inherently difficult, but represents a lurking menace, for it claims its own version of probability theory which transcends and even contradicts that familiar version on which the statisticians' creed is founded. It is, therefore, with some trepidation that I make even the most modest attempt to set out any of its doctrines. What has, nonetheless, become clear to me in the course of this investigation, and especially after insight provided by Dr Crispin Gardener at Waikato University, is that quantum mechanics not only provides a most natural and striking setting for the identities (1) and (2), but also has tools available which provide alternative and very effective methods of proving these identities. Both identities are to be found in an article by Schwinger [9], embedded in a most rebarbative discussion of field equations, annihilation and creation operators, and other entities whose interpretation in plain mathematics remains (I hate to admit it) a mystery to me at the present time.

It was, in passing, from this context that my original interest in this problem arose, in connection with certain point process models for particle beams developed by Bénard and Macchi (see [10] for a review of their work). These authors do not refer to (1) or (2) explicitly, but in effect they use the Fredholm versions of these identities to develop the properties of their models.

Since an adequate account of the quantum mechanical context is beyond me, I shall content myself by giving a simplified and somewhat personal interpretation of the identities in this context, and an indication of the approach to their proofs that the quantum mechanical formalism suggests to me.

Under suitable conditions, and when properly normalized, both expansions (1) and (2) can be viewed as probability generating functions for the numbers of particles in various energy states. Thus for example (1), or more generally the polarized form (4), describes a convergent series of positive terms in which those of order k determine (after normalisation) the probabilities of finding just k particles distributed in various ways among the n energy levels. A sufficient condition for the validity of this interpretation is that A be positive definite, for under this condition $^+|A|^+ \geq |A| \geq 0$. The same condition also implies that the expansion of $|I + \lambda A|$ can be normalized to provide a probability generating function. If the particles are of boson type (e.g. photons) there are no restrictions on the number of particles occupying a given state, and the permanent expansion (1) is relevant. If the particles are of fermion type (e.g. electrons) no more than one particle per state is permitted, and the expansion (2) is relevant.

Let us take, by way of example, the very simple situation that A is a 3×3 diagonal matrix with all diagonal entities equal to a say, and consider the possible ways of distributing 2 particles between the three states. In the boson case these are $\binom{4}{2} = 6$ possible configurations, 3 corresponding to two particles in a single state and three to 1 particle in each of 2 states. Each of these 6 ways has a probability proportional to one of the $\binom{4}{2}$ distinct 2×2 permanents that can be made up from a 3×3 matrix A. Taking into account the factorial coefficients multiplying each of these permanents, it turns out that all six configurations have equal probabilities. In general, with n states and k particles and the same simple type of matrix, there will be $\binom{k+n-1}{k}$ distinct configurations of k particles in n states, each with the same conditional probability given the number of particles. The p.g.f. of the total number of particles has the general form $|I - A|/|I - zA|$, which in our special case reduces to $(1 - a)^3/(1 - za)^3$, representing a negative binomial distribution. It can be regarded as deriving from the sum of three independent geometrically distributed random variables, each representing the number of particles in one of the 3 possible states.

The corresponding fermion model has just $\binom{3}{2}$ configurations possible, corresponding to the different ways of allocating the two particles to a pair of states. Again the probabilities of these 3 configurations are equal. In this case the total number of particles has the ordinary binomial p.g.f. $(1 + za)^3/(1 + a)^3$, which can be regarded as derived from the sum of the three $0-1$ variables representing the number of particles allowed in each of the 3 states.

In this way we can see that the probability models derived from the expansions (1) and (2) provide extensions of the elementary Bose-Einstein and Fermi-Dirac statistics described (for example) in Chapter 2 of Feller [11].

In fact permanents arise as inevitably in dealing with particles with symmetric wave functions as determinants do in dealing with those which have anti-symmetric wave functions. The algebraic background to this comment is the basic fact, attributed to Marcus and Newman [12], although it seems to me that it must be implicit in earlier quantum mechanical discussions, that the permanent arises as an inner product in a symmetrized tensor product space. The argument goes as follows.

Suppose that we have a finite dimensional vector space spanned by basis vectors $e_1 \ldots e_n$. From this we form a k-fold tensor product space with basis elements of the form

$$e_{i_1} \otimes e_{i_2} \otimes \cdots \otimes e_{i_k}$$

(where again repetitions are allowed). If the original space admitted an inner product $\langle e_i, e_j \rangle$, then the tensor space inherits an inner product defined by

$$\langle e_{i_1} \otimes e_{i_2} \cdots \otimes e_{i_k} | e_{j_1} \otimes e_{j_2} \cdots \otimes e_{j_k} \rangle = \prod_{r=1}^{k} \langle e_{i_r}, e_{j_r} \rangle$$

We now consider the subspace of all symmetric tensors, spanned by elements of the form

(5) $$e_{i_1} \cdot e_{i_2} \cdots \cdot e_{i_k} = \frac{1}{k!} \sum_\sigma e_{\sigma_1} \otimes e_{\sigma_2} \otimes \cdots \otimes e_{\sigma_k}$$

where the summation is taken over all $k!$ permutations $\sigma = (\sigma_1 \ldots \sigma_k)$ of the k indices $(i_1 \ldots i_k)$. It is now easy to verify that

$$\langle e_{i_1} \cdot e_{i_2} \cdots \cdot e_{i_k} | e_{j_1} \cdot e_{j_2} \cdots \cdot e_{j_k} \rangle = {}^{+}\left| A \begin{pmatrix} i_1 & \cdots & i_k \\ j_1 & \cdots & j_k \end{pmatrix} \right|^{+}$$

where $a_{ij} = \langle e_i, e_j \rangle$, and the notation indicates the permanent formed by letting i run through the indices $i_1 \ldots i_k$ and j run through the indices $j_1 \ldots j_k$. Similarly determinants arise as the form of the inner product on the subspace of antisymmetrized products

(6) $$e_1 \wedge e_2 \wedge \cdots \wedge e_k = \frac{1}{k!} \sum_\sigma (-1)^{\in(\sigma)} e_{\sigma_1} \otimes e_{\sigma_2} \otimes \cdots \otimes e_{\sigma_k}$$

where $\in(\sigma)$ is the index of the permutation.

Here
$$\langle e_{i_1} \wedge e_{i_2} \cdots \wedge e_{i_k} | e_{j_1} \wedge e_{j_2} \cdots \wedge e_{j_k} \rangle = \left| A \begin{pmatrix} i_1 & \cdots & i_k \\ j_1 & \cdots & j_k \end{pmatrix} \right|$$

where the determinant on the right is non-zero only if the i- and j- indices are distinct.

The hint provided by this interpretation is that the k-particle configurations should be associated with the compound matrix of order $\binom{n+k-1}{k} \times \binom{n+k-1}{k}$ formed from all $k \times k$ permanents obtained by selecting in all possible ways k row and column indices from the n available (allowing repetitions). The probabilities are taken from the diagonal elements of this compound matrix. In particular the sums of permanents appearing as the coefficients of the successive powers of λ in (1) are just the traces of these compound matrices. The whole of the RHS of (1) may therefore be regarded as the trace of one gigantic infinite dimensional matrix of block diagonal form, each block being one of these permanental compounds.

Similarly in the fermion case we may regard the RHS of (2) as the trace of a large but finite-dimensional matrix of block diagonal form, in which each block is a compound matrix whose elements are determinants formed from the original matrix A.

In the case that A is symmetric and positive definite, as would be implied by its representation in terms of an inner product, one can do more, for the unitary (or orthogonal) transformation that reduces A to diagonal form induces a unitary transformation on the k-fold tensor product space which turns out to reduce both the symmetric (permanental) and anti-symmetric (determinantal) k-th order compounds to diagonal form also. When all the compound matrices are so reduced it is easy to see that in the permanental case, the trace of the k-th order compound is the sum of all distinct k-fold products of the eigenvalues of A, allowing repetitions. Thus, if A is 3×3, and we take $k = 2$, the trace is the sum of the six terms $\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$. Summing over all orders we obtain the product of geometric series corresponding to the expansion of the reciprocal $|I - \lambda A|^{-1}$ on the LHS of (1). Similarly in the determinantal case, the trace of the 2nd order compound is $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$ and summing over all orders we obtain the product of binomial series corresponding to the expansion of the determinant $|I + \lambda A|$ in LHS of (2) (with $-\lambda$ replaced by $+\lambda$). All of these elegant results are implicit in the quantum mechanical formalism, but how far they have been made explicit, and how well-known they are outside this context, are matters of some uncertainty.

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DATA FITTING WHEN ALL VARIABLES CONTAIN ERRORS

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An invited lecture at the 18th N.Z. Mathematics Colloquium, Massey, May 1983.

1. Introduction

A common problem in data analysis is that of fitting a linear model, say a linear combination of given basis functions $\phi_j(x)$, $j = 1, 2, \ldots, n$, to observations y_i taken at points x_i, $i = 1, 2, \ldots, m$ (which need not be scalars). Generally m is (much) larger than n, so that the system of linear equations

$$y_i = \sum_{j=1}^{n} a_j \phi_j(x_i), \quad i = 1, 2, \ldots, m \tag{1.1}$$

for the unknown coefficients is overdetermined. A standard procedure is then to introduce perturbations r_i, $i = 1, 2, \ldots, m$, so that

$$r_i + y_i = \sum_{j=1}^{n} a_j \phi_j(x_i), \quad i = 1, 2, \ldots, m, \tag{1.2}$$

and to determine a_j, $j = 1, 2, \ldots, n$ so that some norm of the vector $\underset{\sim}{r} \in R^m$ (whose i^{th} component is r_i) is minimised. If the components of $\underset{\sim}{r}$ represent errors which are independent and normally distributed, then the maximum likelihood estimate is obtained by taking the norm to be the least squares norm; on the other hand if there are isolated gross errors (corresponding to wild points in the data) then the ℓ_1 norm $\|\underset{\sim}{r}\|_1 = \sum_{i=1}^{n} |r_i|$ is more appropriate. Indeed, any of the ℓ_p norms

$$\|\underset{\sim}{r}\|_p = \left(\sum_{i=1}^{m} |r_i|^p\right)^{1/p}$$

where $1 \leq p \leq 2$, may be useful in the data fitting context.

An underlying assumption in (1.2) is that the errors in the data are confined to the y_i values. However this can often be an oversimplification, as the x_i values may also be incorrect. In this case if perturbations e_i are introduced into the x_i values, then (1.2) is modified to become

$$r_i + y_i = \sum_{j=1}^{n} a_j \phi_j(x_i + e_i) , \quad i = 1, 2, \ldots, m \tag{1.3}$$

$$= \sum_{j=1}^{n} a_j [\phi_j(x_i) + \psi_j(x_i, e_i)] , \text{ say,} \quad i = 1, 2, \ldots, m$$

or

$$\underset{\sim}{r} + \underset{\sim}{y} = (A + E)\underset{\sim}{a} ,$$

where $\underset{\sim}{y} \in R^m$ has i^{th} component y_i , and the (i,j) component of the $(m \times n)$ matrices A and E is, respectively, $\phi_j(x_i)$ and $\psi_j(x_i, e_i)$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. The 'errors in variables' problem has long been of interest to statisticians. However, even when some information about the error distribution is available, the maximum likelihood estimation of the functional relationship does not necessarily have an immediate interpretation. In this paper, some ways of dealing with the errors in variables situation are considered, and some appropriate numerical methods outlined. The emphasis of the paper inevitably reflects the fact that it has been written by a numerical analyst and not a statistician.

2. Total ℓ_p approximation

One approach which has been suggested is to treat the matrix E as a perturbation matrix of the elements of A . Then if the errors represented by the elements of both $\underset{\sim}{r}$ and E are independent and normally distributed, it is appropriate to minimize the sum of squares of those elements, or equivalently the Frobenius norm of the $m \times (n + 1)$ matrix $[E \vdots \underset{\sim}{r}]$. The problem is then referred to as the 'total least squares problem'. Here the more general total ℓ_p problem is considered of minimizing $\| [E \vdots \underset{\sim}{r}] \|_p$ subject to (1.4), where the norm is defined (for a general matrix M) by

$$\|M\|_p = (\sum_{i,j} |m_{ij}|^p)^{1/p} ,$$

with m_{ij} the (i,j) component of M , and $1 \leq p \leq 2$. The relevance of values of p other than $p = 2$ may be argued as in the simpler situation when (1.2) is appropriate. The problem under consideration may thus be stated

$$\text{minimize} \quad \| [E \vdots \underset{\sim}{r}] \|_p$$

$$\text{subject to} \quad (A + E)\underset{\sim}{a} = \underset{\sim}{y} + \underset{\sim}{r} \tag{2.1}$$

where $1 \leq p \leq 2$.

If $p > 1$, and $\| [E \vdots \underset{\sim}{r}] \|_p > 0$, then (2.1) is a differentiable optimization problem, and attention will be restricted for the moment to this case. To develop multiplier relations for the problem, it is then necessary that the Jacobian matrix of the constraints have full rank, and it is readily verified that this is so. Then the appropriate Lagrangian function is

$$L = \| [E \vdots \underset{\sim}{r}] \|_p + \underset{\sim}{\lambda}^T((A + E)\underset{\sim}{a} - \underset{\sim}{y} - \underset{\sim}{r}) ,$$

and necessary conditions for a solution are that there exists $\underset{\sim}{\lambda} \in R^m$ such that

$$(A + E)^T \underset{\sim}{\lambda} = \underset{\sim}{0} \tag{2.2}$$

$$(A + E)\underset{\sim}{a} = \underset{\sim}{y} + \underset{\sim}{r} \tag{2.3}$$

$$G + \underset{\sim}{\lambda} \begin{pmatrix} \underset{\sim}{a} \\ -1 \end{pmatrix}^T = 0 , \tag{2.4}$$

where G is the $m \times (n+1)$ matrix of partial derivatives of $\| [E \vdots r] \|_p$ with respect to its components. It is possible to eliminate E, r and λ in terms of a from (2.3) and (2.4) and it may be verified that

$$[E \vdots r] = (y - Aa) w^T(a) \,/\, \left\| \begin{smallmatrix} a \\ -1 \end{smallmatrix} \right\|_q , \tag{2.5}$$

$$\lambda = u(a) \,/\, \left\| \begin{smallmatrix} a \\ -1 \end{smallmatrix} \right\|_q ,$$

where $w(a) = \nabla_a \left\| \begin{smallmatrix} a \\ -1 \end{smallmatrix} \right\|_q$, $u(a) = \nabla_a \| y - Aa \|_p$, and $\frac{1}{p} + \frac{1}{q} = 1$. It follows from (2.5) that the function to be minimized in (2.1) can be written

$$f(a) = \| y - Aa \|_p \,/\, \left\| \begin{smallmatrix} a \\ -1 \end{smallmatrix} \right\|_p \tag{2.6}$$

where the minimization is to be taken over all $a \in R^n$. Direct calculation gives

$$\nabla_a f(a) = (A + E)^T \lambda$$

so that (2.2) will automatically be satisfied at a minimum of $f(a)$. Letting

$$v = \begin{bmatrix} a \\ -1 \end{bmatrix} , \quad (\tau \neq 0) \tag{2.7}$$

$$Z = [A \vdots y] . \tag{2.8}$$

(2.6) may be written as $\| Zv \|_p \,/\, \| v \|_q$, so that the minimization problem can be expressed in the form

$$\text{minimize} \quad \| Zv \|_p \quad \text{subject to} \quad \| v \|_q = 1 . \tag{2.9}$$

This turns out to be a very convenient form in which to attack the problem (2.1). It is possible to drop the differentiability requirement (and thus in particular to include the case $p = 1$) by using the results of Hiriart-Urruty [2]. The essential difference in the argument is that gradients are replaced by subgradients; the resulting form (2.9) remains unchanged.

In summary, then, the conditions (2.2) - (2.4) may be satisfied by finding a solution v to (2.9), and determining a to satisfy (2.7). Notice that it is necessary for $v_{n+1} \neq 0$ for this to be possible, and this is an existence condition on solutions to the problem (2.1). Notice also that the problems (2.1) and (2.9) are non-convex, so that it may only be possible to calculate a local solution to (2.9), and thus to (2.1).

Finally, in this section, attention is drawn to the fact that it is straightforward to take account of some columns of Z being <u>exact</u>. It is merely necessary to exclude components of v corresponding to exact columns from the normalization condition of (2.9). Details of this are given in [8].

3. <u>The calculation of total ℓ_p approximations</u>

Consider first the case $p = 2$ (and so $q = 2$) in (2.9). Then the problem becomes

$$\text{minimize} \quad v^T Z^T Z v \quad \text{subject to} \quad v^T v = 1 , \tag{3.1}$$

which is just the problem of finding the smallest eigenvalue of $Z^T Z$ and a corresponding eigenvector, or equivalently the smallest singular value of Z and a corresponding right singular vector. For an analysis of this problem, and a method based on the singular value decomposition of Z, see [1].

When $p > 1$, (2.9) can be written in the form

$$\text{minimize} \quad v^T Z^T D(v) Z v \quad \text{subject to} \quad v^T C(v) v = 1 \tag{3.2}$$

where

$$D(v) = \text{diag}\{ |(Zv)_i|^{p-2} , \; i = 1, 2, \ldots, m \}$$

$$C(v) = \text{diag}\{ |v_i|^{q-2} , \; i = 1, 2, \ldots, n \} .$$

It will be assumed that $D(v)$ and $C(v)^{-1}$ exist for all v of interest. Then first order necessary conditions for a solution to (3.2) are that there exists a scalar μ such that

$$Z^T D(\underset{\sim}{v}) Z \underset{\sim}{v} = \mu C(\underset{\sim}{v}) \underset{\sim}{v} , \tag{3.3}$$

and

$$\underset{\sim}{v}^T C(\underset{\sim}{v}) \underset{\sim}{v} = 1 .$$

Consider the simple iteration process

$$\left.\begin{aligned} Z^T D(\underset{\sim}{v}_i) Z \underset{\sim}{d} &= C(\underset{\sim}{v}_i) \underset{\sim}{v}_i \\ \underset{\sim}{v}_{i+1} &= \underset{\sim}{d} / \|\underset{\sim}{d}\|_q \end{aligned}\right\} \quad i = 1, 2, \ldots \tag{3.4}$$

which may be regarded as a generalization of the inverse iteration calculation applicable when $p = q = 2$. It is possible to show that this iteration is locally convergent for all p, $1 < p \leq 2$ [9]. In practice, convergence appears to occur from any feasible starting point, although theoretically this question is open. It is easy to guarantee convergence, however, by exploiting the fact that the direction of progress implicit in the iteration (3.4) is a feasible descent direction for (2.9). This direction $\underset{\sim}{s}$ is given by

$$\underset{\sim}{s} = \underset{\sim}{d} / \underset{\sim}{d}^T C(\underset{\sim}{v}_i) \underset{\sim}{v}_i - \underset{\sim}{v}_i$$

where $\underset{\sim}{d}$ satisfies (3.4). The incorporation of a line search is all that is now required; the step length must eventually be unity. (For details, see [9].)

When $p = 1$, different techniques must be used to solve (2.9) because of the non-differentiability of the objective and constraint functions. However, it is possible to exploit the piecewise linear nature of the objective function (and indeed of the constraint function). An algorithm for this problem based on the use of a reduced gradient algorithm to generate descent directions is given in [4]. Under a suitable non-degeneracy assumption, the method will converge to a local minimum in a finite number of steps. An interesting feature of the problem in this case is that, in the absence of degeneracy, the solution is such that only one column of the matrix $[E \vdots \underset{\sim}{r}]$ is non-zero.

4. Orthogonal ℓ_p approximation

Consider now the problem of fitting the straight line $y = a_1 + a_2 x$ to given data, as in the illustration in Figure 1. When both x_i and y_i values are in error, one possible approach is to minimize the sum of squares of the perpendicular distances from the points to the line (see, for example [3], p. 408). It is readily seen that if the vertical distance from (x_i, y_i) to $y = a_1 + a_2 x$ is $|r_i|$, then the perpendicular distance is $|r_i| / \sqrt{1 + a_2^2}$, where in this case

$$r_i = a_1 + a_2 x_i - y_i , \quad i = 1, 2, \ldots, m .$$

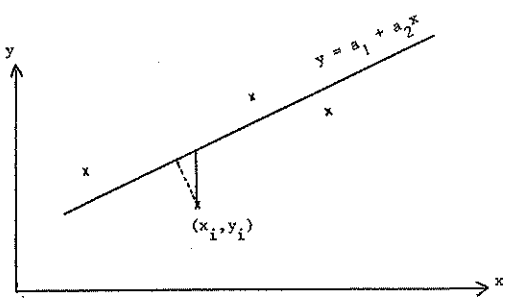

Figure 1

The sum of the p^{th} powers of these distances is thus

$$\frac{\|\underset{\sim}{r}\|_p^p}{(1 + a_2^2)^{p/2}} = \frac{\|Z\underset{\sim}{v}\|_p^p}{(v_2^2 + v_3^2)^{p/2}} \tag{4.1}$$

where (2.7) and (2.8) have been used, with Z the $m \times 3$ matrix with i^{th} row $[1 \; x_i \; y_i]$. The function (4.1) may thus be minimized by solving the problem

$$\text{minimize} \quad \|Z\underset{\sim}{v}\|_p \quad \text{subject to} \quad v_2^2 + v_3^2 = 1 \,. \tag{4.2}$$

Comparison with (2.9) shows that the structure of the problem is similar, but the norm in the constraint is now the 2-norm, and the first component of $\underset{\sim}{v}$ is missing in the normalization condition (reflecting the fact that the first column of Z is exact). When $p = 2$, the best ℓ_2 line may be shown to pass through $(\bar{x},\bar{y})$, where $\bar{x}$ and $\bar{y}$ are means. If the origin is shifted to this point the resulting problem (4.2) becomes a 2×2 eigenvalue problem which may be solved explicitly. This technique is well-known to statisticians, being referred to as 'principal component analysis' or 'latent root regression analysis'.

There are two obvious ways in which these ideas can be extended to more general linear models than straight lines. Späth [6] gives the name 'orthogonal ℓ_p approximation' to approximations obtained through the solution of the problem

$$\text{minimize} \quad \|Z\underset{\sim}{v}\|_p \quad \text{subject to} \quad \|\underset{\sim}{v}\|_2 = 1 \tag{4.3}$$

where Z is now a general $m \times (n + 1)$ matrix defined by (2.8). Methods similar to those mentioned in the last section may be used to solve (4.3) (see, for example, [7]).

However, (4.3) no longer necessarily has the same geometrical interpretation as before. Consider the general model $f(x,\underset{\sim}{a})$, and let

$$r_i + y_i = f(x_i + e_i, \underset{\sim}{a}) \,, \quad i = 1, 2, \ldots, m \,. \tag{4.4}$$

Then the distance from the point (x_i,y_i) to the point on the curve $y = f(x,\underset{\sim}{a})$ with x-coordinate $x_i + e_i$ is easily seen to be $(e_i^2 + r_i^2)^{\frac{1}{2}}$. Thus the minimum sum of squares of 'orthogonal distances' is in fact given by the minimization of

$$\sum_{i=1}^{m} (r_i^2 + e_i^2) \,.$$

Substituting for r_i from (4.4), the result is an unconstrained minimization problem for the unknown vectors $\underset{\sim}{a}$ and $\underset{\sim}{e}$. In particular if f is linear in $\underset{\sim}{a}$, then standard methods for separable problems may be applied: see, for example the review paper by Ruhe and Wedin [5]. It is possible that similar ideas may be helpful in the minimization of the sum of p^{th} powers, but this has not been investigated.

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WHAT HAVE WE DONE TO SCHOOL MATHEMATICS ?

A summary of an address by Mr Trevor Boyle, the 1983 NZMS Visiting Lecturer.

About 20 to 25 years ago, mathematics began in the primary school with one arithmetic book, written in such a way as to provide a volume a year. It's largely arithmetic work was followed in secondary schools by three years preparation for four hours of School Certificate examining divided equally between Algebra and Arithmetic in one paper and Geometry and Trigonometry in another. Of the 50 percent who (then) passed that hurdle, some presented for University Entrance and then, in very small numbers, returned to the Upper Sixth.

Those were the days when girls did not do mathematics if they could avoid it and when core mathematics was quite acceptable to most people. The system certainly made it clear then that there has to be two sorts of mathematics offered and one was less demanding than the other. The effects of that structure are with us now, of course, in the 40 year olds (mostly women) who were treated like this and accepted it.

In the late 1950's there came a change from both the USA and the UK - Yale, SMP, Scottish Maths Group - these were the often heard names and we adopted their material and later adapted it. The early publications were, as always in this kind of change, extreme. New material was being introduced and justified on the grounds that it could be taught so it ought to be taught. The introduction of matrices would probably be the worst example of this kind. In particular, a set of what were called SPOTTY BOOKS was produced by a Dr Matthews at St. Dunstan's School and so inveigling was his matrix material that it got taught at unreasonable length. It appeared at form four level and was examined (then) in the optional or alternative part of the S.C. syllabus. It made its way into the 6th form where matrices were applied to a study of transformations of the plane. Now, if you recall the latest UE draft, matrices have gone again. The point to be made here is that because it can be taught does not mean it ought to be taught and apparently this lesson is now being practiced by some of our writers of syllabi.

As these overseas changes took hold and their influence was felt more clearly here, copy-cat groups sprang up all over the place. In New Zealand CMG was one which tended to lead the way but others were well established. The effects of this activity were two-fold. Firstly, it stimulated mathematics teachers no end. Out went the old core mathematics, from even the most conservative schools; the refresher course boom began and there was, fortunately, an expansion in school rolls, new schools opening with the total effect being one of growth and enlightenment. The mood of those times certainly contributed to the expansion of mathematics.

The second effect was slower in becoming observable, however. We began to see that it was possible to think independently about courses and their content. Not much happened at first - everyone was off after the Modern mathematics (or new maths even!). But the seeds were sown for later growth. While they were germinating, we got rid of core mathematics, put everyone onto full mathematics (and new at that!) and even dallied a while with recreational and creative materials. What a hey day!

But things were not all simple and back we went to a new idea - local mathematics certificates. Not core, you understand, but "an alternative". Not easier, you understand, but "different". Not refusing to acknowledge inability to pass S.C. you understand, but "catering for individual differences".

Another change began to shape up when we asked the question, why have only one assessment scheme? Why should there not be alternatives to the usual School Certificate? The Canterbury-Westland scheme, the Northlands scheme and the Nelson-Marlborough scheme were born.

Through all these changes, however, the annual S.C. routine went on. Whatever else one may say about that examination, at least it is a plodder! As regular as clock work, always in November, a new set of tide marks laps up on the shores of mathematics land. Just as sedimentary deposits can tell a lot about what went on in days gone by, so do questions from past S.C. papers

(Some of the questions then discussed included

(a) divide $x^4 - 5x^2y^2 + 4y^4$ by $x^2 + xy - 2y^2$ using factors.

(b) if $x^3 + mx + n$ is divisible by $x + 3$ and $x - 3$ without remainder, find m and n .

[c] a man can cycle 8 mph faster than he can walk. If he cycles a mile, then walks a mile, his time for the journey is 20 minutes. Find his walking speed.

[d] if $\tan x = 3$ prove that $\frac{\sin x - \cos x}{\sec x - \operatorname{cosec} x} = 0.3$.]

There are two matters that arise:

Where do these questions belong now, in our current teaching programmes?
Where have the fifth formers gone, who used to be able to do this?

The real question is, what have we done to school mathematics?

I think we have

[a] made it available to more people
[b] lowered our standards to do so
[c] redesigned the entire system to cover up the most able.

I look back over twenty years or so and on the basis of this observation, want to suggest a way ahead. What should we do to school mathematics?

There are three suggestions I have to offer.

Firstly, where mathematicians have before led the school field the time has come for them to lead it again. I believe the time is appropriate for the introduction of an honours level paper at S.C. level. It could be an additional two hour paper or an alternative but of a higher standard and on a wider syllabus than the present prescription. I would estimate that 25% to 30% of present candidates should be prepared for it and they should sit. Not all candidates, however, should sit all papers.

My second suggestion is that the mathematicians press to introduce an accrediting procedure in their subject at form five level as a pilot scheme for more general adoption in the next few years. The advantages this has are

[a] decreased cost
[b] more professional acceptance by schools
[c] national moderation and a national qualification.

Thirdly I consider it a good time for the introduction of a re-sit opportunity at S.C. level in May for those who miss out in November of the preceding year. Once again, mathematics is an ideal subject to provide a trial programme in this. Probably one or two other subjects may seek to join in but that would only be to the good of the idea overall. There are implications which complicate this issue of course, not the least of which is how to cope with a pupil who sits an examination in May and passes, thus rendering himself eligible for form six.

I have referred to what I call a lowering of standards in school mathematics. This is not exactly the way to consider the situation because it is related closely to the size and nature of the population facing the examination. I think I have shown you that the level of performance of candidates is different now from what it was and that the degree of skill and of mathematical ability required to pass S.C. now is less than it once was. This is what I call a lowering of standards.

There is another factor at work, too, which I would describe as an inflation in the value of school qualifications. Where School Certificate would once buy entry to a particular future, a higher level of attainment is essential now. The value of the qualification has been eroded - that is inflation.

The two pronged erosive effects of "lowered" standards and qualification inflation are serious and my three suggestions are possible one way to reverse the move.

ACCOMMODATION IN ABERDEEN

Academic staff who are planning to visit the Department of Mathematics at the University of Aberdeen are advised that the Department has family accommodation available at a very reasonable rental. The fully furnished two-bedroom house with central heating located on campus just three minutes walk from the Department of Mathematics would suit a family of up to four persons. Enquiries should be directed to

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University of Aberdeen
Dunbar Street
Aberdeen AB9 2TY
SCOTLAND

Conferences

Compiled by Dr M.R. Carter, Massey University.

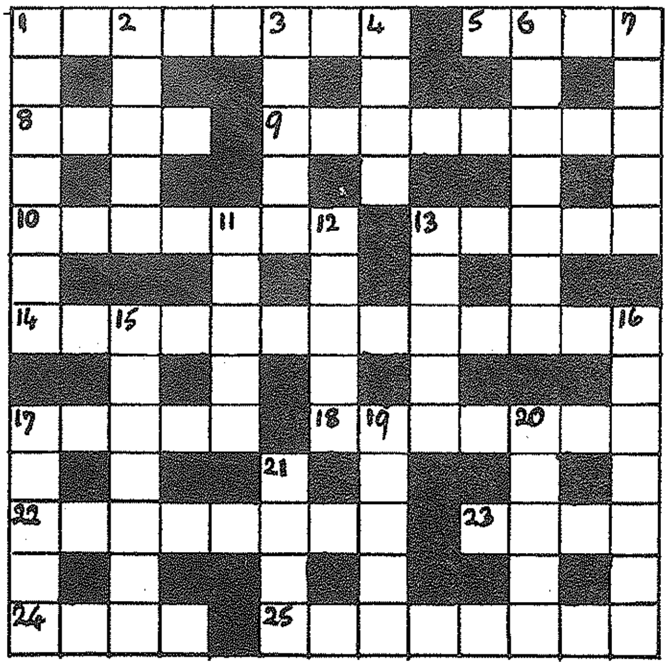
1984

January 6-10 (New York)	*NSF-CBMS Regional Conference on some Global Problems concerning Curvature of Riemannian Manifolds* Details from L.M. Sibner, Polytechnic Institute of New York, 333 Jay Street, Brooklyn, New York 11201, U.S.A.
January 9-13 (Coral Gables, Florida)	*NSF-CBMS Regional Conference on Minimax Methods in Critical Point Theory and Applications to Differential Equations* Details from Shair Ahmad, Department of Mathematics and Computer Science, University of Miami, Coral Gables, Florida 33124, U.S.A.
February 5-9 (Merimbula, New South Wales)	*Australian Applied Mathematics Conference* Details from Ashley Plank, AMC Conference Secretary, School of Information Sciences, Canberra College of Advanced Education, Box 1, Belconnen, ACT 2616, Australia.
February 6-9 (Orlando, Florida)	*Second International Modal Analysis Conference* Details from Union College, Graduate and Continuing Studies, Wells House, 1 Union Avenue, Schenectady, New York 12308, U.S.A.
March 12-16 (Old Tampa Bay, Florida)	*Seventeenth Annual Simulation Symposium* Details from Alexander Kran, IBM Corporation, B/300-40E, Hopewell Junction, New York 12533, U.S.A.
March 19-21 (Munich)	*International Conference on Numerical Analysis* Details from C. Zenger, Technische Universität München, Institut für Informatik, Arcisstr. 21, D-8000 München 2, Federal Republic of Germany.
April 9-13 (Bristol)	*36th British Mathematical Colloquium* Details from H.E. Rose, School of Mathematics, University of Bristol, University Walk, Bristol BS8 1TW, U.K.
April 11-13 (Paris)	*Symposium on Theoretical Aspects of Computer Science* Details from Symposium Office, STACS/AFCET, 156 Boulevard Pereire, 75017 Paris, France.
April 19-20 (Pittsburgh, Pennsylvania)	*15th Annual Pittsburgh Conference on Modeling and Simulation* Details from William G. Vogt, Modeling and Simulation Conference, 348 Benedum Engineering Hall, University of Pittsburgh, Pittsburgh, Pennsylvania 15261, U.S.A.
April 23-27 (Princeton, New Jersey)	*Conference Celebrating the Sixtieth Birthday of Professor Harish-Chandra* Details from V.S. Varadarajan, Department of Mathematics, University of California, Los Angeles, California 90024, U.S.A.
May 1-4 (Uxbridge, U.K.)	*Conference on the Mathematics of Finite Elements and Applications* Details from The Secretary, The Institute of Computational Mathematics, Brunel University, Uxbridge, Middlesex UB8 3PH, U.K.
May 2-4 (Montreal)	*Optimisation Days 1984* Details from C.L. Sandblom, Department of Quantitative Methods, Concordia University, 7141 Sherbrooke Street West, Montreal, Quebec H4B 1R6, Canada.
May 4-6 (West Lafayette, Indiana)	*Midwest Algebraic Geometry Conference* Details from Joseph Lipman, Department of Mathematics, Purdue University, West Lafayette, Indiana 47906, U.S.A.
May 7-9 (Wellington)	*19th New Zealand Mathematics Colloquium* Details from Dr B. Dawkins, Mathematics Department, Victoria University, Private Bag, Wellington, New Zealand.
May 13-17 (Anaheim, California)	*Computer Graphics '84* Details from National Computer Graphics Association, 8401 Arlington Boulevard, Fairfax, Virginia 22031, U.S.A.
May 17-18 (Liège, Belgium)	*IMACS International Symposium on Modelling and Simulation of Electrical Machines and Converters* Details from H. Buysse, Unité Courant Fort et Electrotechnique, Université Catholique de Louvain, Batiment Maxwell, Place du Levant 3, B-1348 Louvain-la-Neuve, Belgium

May 28-June 1 (Becici, Yugoslavia)	*16th Yugoslav Congress of Theoretical and Applied Mechanics* Details from J. Jaric, Yugoslav Society of Mechanics (16th Congress 1984), Kneza Milosa 9/i, 11000 Belgrade, Yugoslavia.
June 4-8 (Kalamazoo, Michigan)	*Fifth International Conference on the Theory and Applications of Graphs, with Special Emphasis on Computer Science Applications* Details from Directors, Graph Theory Conference, Department of Mathematics, Western Michigan University, Kalamazoo, Michigan 49008, U.S.A.
June 6-8 (San Diego, California)	*1984 American Control Conference* Details from AACC Secretariat, 1051 Camino Velasquez, Green Valley, Arizona 85614, U.S.A.
June 10-14 (Berlin)	*Conference on Global Differential Geometry-Global Analysis* Details from Dirk Ferus, Fachbereich 3-Mathematik, Teknische Universität Berlin, Strasse des 17 Juni 135, 1000 Berlin 12, Federal Republic of Germany.
June 12-14 (West Lafayette, Indiana)	*Tenth International Symposium on Machine Processing of Remotely Sensed Data* Details from Paul E. Anuta, Purdue University/LARS, 1291 Cumberland Avenue, West Lafayette, Indiana 47906-1399, U.S.A.
June 18-20 (Copenhagen)	*26th International Meeting of the Institute of Management Sciences* Details from Julie Eldridge, TIMS, 146 Westminster Street Providence, Rhode Island 02903, U.S.A.
June 19-21 (Bethlehem, Pennsylvania)	*Fifth IMACS International Symposium on Computer Methods for Partial Differential Equations* Details from William E. Schiesser, Department of Chemical Engineering, Whitaker Laboratory #5, Lehigh University, Bethlehem, Pennsylvania 18015,U.S.A.
June 20-22 (Dublin)	*Third International Conference on Boundary and Interior Layers* Details from BAIL III Conference, 39 Trinity College, University of Dublin, Dublin 2, Ireland.
June 25-29 (Linz, Austria)	*International Workshop on Applied Optimisation Techniques in Energy Problems* Details from Hj. Wacker, Math. Institut, Johannes-Kepler-Universität Linz, Altenbergerstrasse, A-4040 Linz, Austria.
July 2-11 (Vancouver)	*Canadian Mathematical Society Summer Seminar on Algebraic Geometry* Details from J.B. Carrell, Department of Mathematics, #121-1984, Mathematics Road, Vancouver, Canada V6T 1Y4.
July 11-14 (Charleston, S. Carolina)	*Conference on Universal Algebra and Lattice Theory* Details from S.D. Comer, Department of Mathematics and Computer Science, The Citadel, Charleston, South Carolina 29409, U.S.A.
July 23-27 (Campinas, Brazil)	*Conference on Complex Analysis and Approximation Theory* Details from Jorge Mujica, Instituto de Matematica, Universidade Estadual de Campinas, Caixo Postal 6155, 13100 Campinas SP, Brazil.
July 23-August 8 (Bad Windsheim, W. Germany)	*NATO-ASI Conference on Computational Mathematical Programming* Details from K. Schittkowski, Institut für Informatik, Azenbergstrasse 12, D-7000 Stuttgart 1, Federal Republic of Germany.
July 24-27 (Leuven, Belgium)	*International Congress on Computational and Applied Mathematics* Details from F. Broeckx, University of Antwerp (RUCA), Faculteit Toegepaste Economische Wetenschappen, Middelheimlaan 1, B-2020 Antwerpen, Belgium.
July 25-Augst 4 (St. Andrews, Scotland)	*Edinburgh Mathematical Colloquium* Details from Dorothy M.E. Foster, Colloquium Secretary, Department of Pure Mathematics, University of St. Andrews, The Mathematical Institute, The North Haugh, St. Andrews, Scotland KY16 9SS
August 19-25 (Lyngby, Denmark)	*16th International Congress of Theoretical and Applied Mechanics* Details from ICTAM, Technical University of Denmark, Building 404, DK-2800 Lyngby, Denmark.
August 24-30 (Adelaide)	*Fifth International Congress on Mathematical Education* Details from ICME 5, GPO Box 1729, Adelaide 5001, Australia.

Crossword

N° 12 NOTHING SPECIAL by Matt Varnish



CROSSWORD N° 11 SOLUTION

Across:

3. Deep sighs, 8. Foot, 9. Sectioned, 10. Eleven, 11. India, 14. Meets, 15. Clue, 16. Eases, 18. Tart, 20. Pie in, 21. Shaft, 24. Pearls, 25. Crossword, 26. Mice, 27. Polygraph.

Down:

1. Aftermath, 2. Forebears, 4. Even, 5. Piton, 6. Ironic, 7. Heed, 9. Sense, 11. Inset, 12. Algebraic, 13. Tennessee, 17. Speed, 19. The sky, 22. Fewer, 23. Trio, 24. Prop.

Across:

1. Takes around, keeps the beat and passes the charge. (8)
5. National who if free is without penalty. (4)
8. Paths of points from sprung coil. (4)
9. Cribbage impossibility. (8)
10. Hypothetical surface man. (7)
13. Body always found in a dearth of planets. (5)
14. Geometer's dictum used in traffic flow. (7-6)
17. Connected account. (5)
18. Devious deviser uses a thousand cheers. (7)
22. Outdoes with nearly conical phenomena. (8)
23. The middle light seen either way. (4)
24. $x^2 + y^2 \leqslant 1$. (4)
25. Gain the lost object of three in the official. (8)

Down:

1. Heat content of the car coil. (7)
2. Recess of the chine. (5)
3. Cleric in musical imitation. (5)
4. Latin without the ratio. (4)
6. $\bigvee$. (7)
7. The unfortunate in decimation? (5)
11. A right beam for set pieces. (5)
12. 00. (5)
13. A lapse of time. (5)
15. Used by the wise who are soon led. (7)
16. Ye rings! give the needle. (7)
17. Velocity without direction. (5)
19. Equivalence class of hard gas? (5)
20. Sir Thomas about nothing for Henry or E.M. (5)
21. Leader leading tars, star, arts and rats. (4)

The Newsletter is the official organ of the New Zealand Mathematical Society. It is produced in the Mathematics Department of the University of Otago and printed at the University Printery. The official address of the Society is:

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C/- The Royal Society of New Zealand
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However correspondence should normally be sent direct to the Secretary, Dr J.A. Shanks, Department of Mathematics, University of Otago, Dunedin.