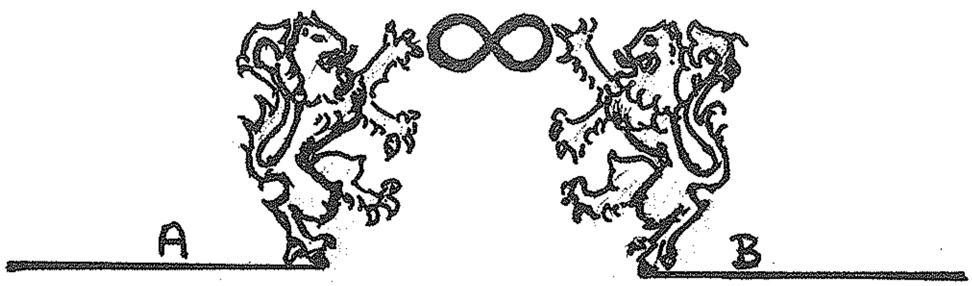
THE NEW ZEALAND MATHEMATICAL SOCIETY

NEWSLETTER



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Number 25 October, 1982 ISSN 0110-0025

Editorial

Coverage of the Seventeenth New Zealand Mathematics Colloquium continues in this issue with three of the survey papers presented at Dunedin. Meanwhile the Eighteenth Colloquium draws ever nearer and readers are directed to the notice on page 9.

Readers will also be pleased to see that Brent Wilson, the previous editor, (now overseas on study leave) has not lost touch with the Newsletter. His centrefold article on Derek Lawden should be well received.

News items, notices and any articles for publication in the Newsletter should be sent to: The Editor, NZMS Newsletter, Department of Mathematics, University of Canterbury. However, institutions with Honorary Correspondents are encouraged to submit items through their correspondent.

COPY DATE FOR THE NEXT ISSUE IS DECEMBER 20.

HONORARY CORRESPONDENTS OF NEWSLETTER

Dr. M.R. Carter	Mathematics Department, Massey University, Palmerston North.
Dr. L. Fradkin	D.S.I.R., Physics & Engineering Labs, Gracefield, Wellington.
Dr. J.F. Harper	Mathematics Dept., Victoria University of Wellington, P.B. Wellington 1.
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Dr. M.A. Jorgensen	Biometrics Section, Ministry of Agric. & Fish., P.O. Box 1500, Wellington.
Dr. D.C. Joyce	Mathematics Dept., USP., Box 1168, Suva, Fiji.
Mr. R.S. Long	Department of Mathematics, University of Canterbury, Christchurch.
Mr. J.H. Maindonald	D.S.I.R.-A.M.D., Mt. Albert Research Centre, Private Bag, Auckland.
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Dr. G. Olive	Mathematics Department, University of Otago, P.O. Box 56, Dunedin.
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Dr. M. Schroder	Mathematics Department, University of Waikato, P.B. Hamilton.
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Dr. G.J. Weir	Applied Mathematics Division, D.S.I.R., P.B. Wellington.
Mr. K. Perrin	Dept. of Maths. Education, Teachers College, Secondary Division, P.O. Box 31065, Christchurch 4.

OFFICERS OF THE SOCIETY, JUNE 1982 - MAY 1983

President:	Dr. J.H. Ansell,	Victoria University of Wellington
Incoming Vice-President:	Prof. W. Davidson,	University of Otago
Immediate Past President:	Prof. D.B. Gauld,	University of Auckland
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	Dr. I.D. Coope,	University of Canterbury (to 1985)
	Dr. P.D. Hill,	University of Waikato (to 1985)
	Dr. A. McNabb,	DSIR, Wellington (to 1984)
	Dr. D.J. Smith,	University of Auckland (to 1984)
Editor:	Dr. I.D. Coope,	University of Canterbury (to 1983)
NZAMT Alternates:	Mr. N.J. Gale,	Papanui High School
	Mr. B.R. Stokes,	Hamilton Teachers' College

THE ROYAL SOCIETY OF NEW ZEALAND
NOMINATIONS FOR FELLOWSHIP OR HONORARY MEMBERSHIP

As a member body of the Royal Society of New Zealand, the N.Z.M.S. may nominate people for Fellowship or Honorary Membership of that Society. (You will recall that Professor David Vere-Jones was elected a Fellow in May, 1982 on the nomination of the N.Z.M.S.). A nominee must be resident in New Zealand and must have been a New Zealand resident for at least three years during his or her scientific career.

The Council of the N.Z.M.S. is always pleased to receive suggestions of people who should be considered for nomination. Please send the names of suggested nominees, together with brief biographical details, to the Secretary.

BNZ SENIOR MATHEMATICS COMPETITION

This is an annual competition, organised by the Canterbury Mathematical Association, with the Bank of New Zealand as a major sponsor. This year's competition drew 2300 competitors from sixth and seventh forms in all parts of the country, though mostly in Canterbury and Auckland.

The 20 finalists included nine from Auckland, and Aucklanders won all three major placings

First: Adam Grove (Auckland Grammar School)
Second: Martin Kealey (Mount Albert Grammar School)
Third: Graham Coop (Auckland Grammar School)

The Association and Bank also award about 100 certificates to the 'Top Hundred' candidates. Here Canterbury did best, with 32, followed by Auckland with 29 and South Auckland-Waikato with 12.

D.F.R.

THE WORLD DIRECTORY OF MATHEMATICIANS

About every four years a new edition of the World Directory of Mathematicians, published under the auspices of the International Mathematical Union is issued. It is an austere work; the entries consist only of the names and addresses of mathematicians, without references to their works, highlights of their careers, or their hobbies. The qualification for inclusion is equally simple. To appear in the pages of the Directory, a mathematician must have had two articles reviewed either in Mathematical Reviews, or in the Zentralblats, or in the Referativnyi Zhurnal.

The collection of names in each country is delegated to the National Committee, and on the basis of our experience during 1981 in assembling the list for the latest edition, it has seemed to me that this work would be made easier, and the result more reliable, if the National Committee maintained a file of those who qualify, to be added to and updated continuously rather than (as has been the case up to now) at four-yearly intervals. Accordingly I invite individuals who believe that they may qualify, but have not yet been approached in the matter, to write to me, citing the appropriate references to their works in the reviews listed above.

B.A. Woods,
Convenor, National Committee for Mathematics,
Mathematics Department,
University of Canterbury.

GUGGENHEIM FELLOWSHIP

Professor George E. Andrews, Evan Pugh Professor of Mathematics at Pennsylvania State University, has been awarded a Guggenheim Fellowship for research into statistical mechanics and partition theory. He was a N.Z. Mathematics Society visiting lecturer in 1979 and the Society published his book *Partitions: Yesterday and Today.* Professor Andrews is also an expert on the work of Ramanujan and has found unpublished notes by this great Indian mathematician. These discoveries he has described in an entertaining and memorable lecture entitled "The Lost Notebook of Ramanujan".

MATHEMATICAL VISITORS IN NEW ZEALAND

Information is arranged as follows: Name of visitor; home institution; whether accompanied; principal field of interest; dates of visit; principal host institution; principal contact; comments.

Professor Thomas Berger; University of Minnesota; ?; group theory; July-November 1982; University of Auckland; Professor P.J. Lorimer.

Professor F.H. Chipman; Acadia University, Canada; wife and two sons; numerical methods for ordinary differential equations; 28 July 1982 - 20 July 1983; University of Auckland; Professor J.C. Butcher.

Dr. Michael Fawcett; University of Cambridge; ?; mathematical physics; August 1982 - December 1983; University of Otago; Professor W. Davidson; Beverley Fellow.

Professor Max Jammer; Bar-Ilan University, Israel; ?; history of physics; 1 March-30 September 1983; University of Otago; Professor J.N. Dodd.

Dr. M. Ludvigsen; -; wife and two children; mathematical physics; all 1982; University of Canterbury; Professor R.P. Kerr; Research Fellow.

Dr. G.A. Read; Open University; wife and three children; numerical analysis, distance teaching technology; all 1982; Massey University; Professor B.I. Hayman.

Professor D.A. Spence; Imperial College, London; ?; continum mechanics, applicable analysis, perturbation methods; March-May 1983; University of Canterbury; Professor B.A. Woods; Erskine Fellow.

Dr. G.A. Watson; University of Dundee; ?; numerical analysis; 1 March-10 May 1983; University of Canterbury; Dr. I.D. Coope; Erskine Fellow.

Professor Carl E. Wulfman; University of the Pacific, USA; wife; Lie groups and application to physical problems; January 1983-June 1983; University of Canterbury; Professor B.G. Wybourne.

Dr. M. Avriel; Technion-Israel Institute of Technology; wife; operational research; energy modelling; February 1983 for one month; Applied Mathematics Division, DSIR; Dr. H. Barr; to be confirmed.

Professor Brian F. Gray; Macquarie University; wife and two children; mathematical chemistry especially thermal ignition and reaction kinetics; 26 January 1983-26 March 1983; Victoria University of Wellington; Dr. G.C. Wake; interested in visiting elsewhere in New Zealand; to be confirmed.

THE NEW ZEALAND MATHEMATICAL SOCIETY

MATHEMATICS SYLLABUS SERIES:

SEVENTH FORM APPLIED MATHEMATICS

PROBABILITY and STATISTICS - J.C. Turner and R.M. Cornwell, Waikato;
COMPUTING and NUMERICAL MATHEMATICS - R.L. Broughton and A. Ramsey, Canterbury;
MECHANICS - J.F. Harper, Wellington.

Each booklet supplies definitions, formulae, worked examples, problems and historical notes, together with tables and selected questions (with answers) from past examination papers.

These booklets were first printed in 1981 and proved to be very popular. They are being reprinted in substantially the same form for the 1983 school year. Provided advance orders are received by mid November, we will ensure these are delivered by the beginning of February, 1983. The price for 1983 has been set at $6.00 per booklet. A separate teachers booklet is available to accompany the Probability and Statistics booklet (price $1.25).

Orders and enquiries to: Dr Graeme Wake, Mathematics Department, Victoria University, Private Bag, Wellington.

Local News

AUCKLAND UNIVERSITY

DEPARTMENT OF MATHEMATICS & STATISTICS

Professor Tom Berger, University of Minnesota, arrived in July to take up a visiting appointment. Tom will be with the department until November.

Dr. T. O'Hagan returned to the University of Warwick in August. He described his visit to Auckland University as 'extremely stimulating'.

Two of our assistant lecturers left the department with their sights set on undertaking Ph.D's in the U.S.A. Paul Goodyear has gone to Harvard University and Peter Danaher to the University of Indiana.

In August, our Post Doctoral Fellow, Ms. Mila Mrsevic, returned to Belgrade, having spent 12 months with the department. While with the department, Mila completed four papers on Topology.

At the end of the second term, Professor Kalman went on leave. He will commence his travels in the U.S.A. and finish up in the U.K.

On 6th September, Dr. G.A. Read of the Open University visited the department and gave a series of lectures in his capacity as the New Zealand Mathematical Society Visiting Lecturer. Dr. Read's interests lie in the uses of television, audio-cassettes and micro-computers in the teaching of mathematics, and in the development of educational communications systems.

On 20th September, Mr. and Mrs. Chris King were blessed with a son.

Seminars:

Dr. Alan J. Lee (Auckland University), *'Regression Diagnostics in SAS'*.
Professor P. Lorimer (Auckland University), Stars on Saturday symposium.
Topic: *'The Theory of Groups'*. Contributions from Professor T. Berger, Dr. M. Lennon and Dr. P. McInerney.
Dr. J.J. Hunter (Auckland University), *'Queues with Feedback'*.
Professor M.L. Slater (Texas Christian University), *'Linear Differential Inequalities'*.
Professor Don James (Pennsylvania State University), *'Generalizations of the Fundamental Theorem of Projective Geometry and some applications'*.
Professor James Taylor (University of Liverpool), *'The use of small sets in Analysis and Probability'*.
Dr. Danny Summers (University of Sydney), *'An Applied Mathematical Problem in Astrophysics'*.
Dr. J.L. Schiff (Auckland University), *'Fractals'*.
Mr. Tony Aldridge (Applied Maths Division, D.S.I.R.), *'N.Z. Lamp Shells'*.
Mr. John Maindonald (Applied Maths Division, D.S.I.R.), *'Statistical Packages : Performance and Promise'*.
Dr. Simon Fitzpatrick (Auckland University), *'Best Approximation in Banach Spaces'*.
Dr. Michael Stenzel (Hamburg), *'Topologizing the Rational Number Field'*.

E.D.

DEPARTMENT OF COMPUTER SCIENCE

Mini-Symposium in Numerical Analysis

Professor Germund Dahlquist of the Royal Institute of Technology in Stockholm will be visiting Auckland for a week from 6th December and we are taking the opportunity to organise a mini-symposium while he is here. It will take place in the Department on the 7th, 8th and 9th and will deal with numerical methods for ordinary differential equations and other areas of numerical analysis. Also taking part will be Professor Fred Chipman from Acadia University in Canada and a number of participants from various parts of New Zealand. Anyone interested in attending should contact Dr. Kevin Burrage or Professor John Butcher.

Visitor to the Department

Professor Fred Chipman from Acadia University Nova Scotia began his year long stay in Auckland in July. While he is here he will collaborate again with Kevin Burrage and John Butcher in the development of software for solving ordinary differential equations.

P.S.

CANTERBURY UNIVERSITY

COURSE CHANGES

In recent years the Stage 2 Vector Methods course has attracted few students. The department has replaced it by a 'Special Topic' paper which will be 'Graph Theory' in 1983. A parallel change sees the introduction of Special Topics at Stage 3: two are proposed for 1983, a Topology course, formerly confined to Honours students, and an Applied Algebra course.

VISITORS

The department has been host to an unusually large number of visitors this term.

Professor C.C. Lindner (Auburn University) was with us for three weeks in September as an Erskine Fellow. He gave twelve lively lectures on Steiner Triple Systems and Latin Squares.

Dr. Graham Read (Open University) was here for two days in his capacity as NZMS Visiting Lecturer. He gave two talks on "University Mathematics" and "Innovations in Distance Education". One feels impelled to make two quotations from his first talk: "The primary purpose of an examination paper is to separate those who know a little about the subject from those who know nothing", and "it is impossible to make an examination paper too easy". He also spoke to the Canterbury Mathematical Association.

Professor S.J. Taylor (Liverpool University) was with us for several weeks. He gave a series of lectures on the history of analysis, as part of an Honours Part 3 course, a short series of lectures on notions of size in small sets, and two seminars, 'A probabilistic approach to the Prime Number Theorem' and 'Telling the time on a Brownian Path'. He also addressed the Canterbury Mathematical Association, and in private discussion and through their Newsletter told us of work to stimulate an interest in Mathematics in the schools on Merseyside, including an interschool competition broadcast on local radio. His report of the rarity of mathematically-trained teachers on Merseyside makes New Zealand schools look very well staffed by comparison.

Seminars:

Dr. D. Summers (Memorial University of Newfoundland), *'An Applied - Mathematical Problem in Astrophysics'.*

Professor M.L. Slater (Christian University of Texas), *'An Integral Equation'*, and two seminars on *'Linear Differential Inequalities'.*

Professor R.W.H. Sargent (Imperial College), *'The Solution of Sparse Systems of Nonlinear Equations - Decomposition Versus Simultaneous Solution'*, and *'New Recursive Quadratic Programming Algorithm with Global Superlinear Convergence'.*

R.S.L., D.F.R.

MASSEY UNIVERSITY

Followed by the envious glances of his colleagues, Howard Edwards took off for Hawaii for a week in August, to attend an international workshop on inference procedures associated with statistical ranking and selection (at least, that's his story). Howard gave an invited paper on a computer package (RANKSEL) of ranking and selection procedures that he has been developing on Massey's PRIME computer, which attracted considerable interest. Keeping in the news, Howard and Joy did their bit towards balancing the sex ratio by adding a daughter to their family in September; this in contrast to Greg and Barbara Arnold, who increased the number of their sons to three in May.

Two of our senior students have just headed overseas to pursue their studies. Stephen Joe is working on the numerical solution of integral equations, at the University of New South Wales, and Russell Blyth is following a Ph.D. program in algebra at the University of Illinois.

Seminars:

Dr. Paul Gandar (Plant Physiology Division, DSIR), *'Siege works at the Tower of Babel: using mathematics to describe plant growth'.*

Dr. Graham Read (The Open University), *'Innovations in distance education'.*

Stuart Birks (Economics Department, Massey), *'An application of modified Herfindahl indices'.*

Dr. Richard Cowan (Division of Mathematics and Statistics, CSIRO, Sydney), *'Models of road traffic congestion processes'.*

Russell Blyth, *'Galois theory and the classical construction problems'.*

Dr. Charles Little, *'A new characterization of planar graphs'.*

Professor R.W.H. Sargent (Imperial College, London), *'The solution of sparse systems of non-linear equations - decomposition vs simultaneous solution'.*

M.R.C.

OTAGO UNIVERSITY

Professor Ivor Francis has been appointed to the Chair of Statistics and will take up his appointment in February, 1983. He is a New Zealander - and now comes to us from the Department of Economics & Social Statistics at Cornell University (where he has been Professor of Statistics).

Dr. Michael Fawcett (a graduate of Otago University) has just taken up his Postdoctoral Research Fellowship in Mathematics - and has the title of "Beverly Research Fellow". Michael has recently completed a Ph.D. in Theoretical Physics (Quantum Gravity) in the Department of Applied Mathematics & Theoretical Physics at Cambridge University under Professor Stephen Hawking. The title of his thesis was "Quantum Field Theory Near a Black Hole".

The Department has received 2 teaching grants:

(a) John Harraway's grant of $3000 is for extending the use of microcomputers by students in large first year service courses (especially in Statistics). This grant provides for the development of statistical packages for multiple regression and analyses of variance on the CASIO FX-900P.

(b) Petronella de Roos' grant is for making a film (with the aid of "student role-players") to help train Tutors & Demonstrators.

This year's NZMS Visiting Lecturer, Dr. Graham Read of the Open University, gave a talk on "Innovations in Distance Education" (on Sept. 15) and had a variety of informal chats on teaching methods and assessment (on Sept. 15 & 16).

Dr. Peter Fenton will be on leave during the 1st term of 1983 - and will be visiting Imperial College, London.

G.O.

VICTORIA UNIVERSITY

Doug Harvie has now succeeded Terence Nonweiler as Chairman of the Mathematics Department.

Andrew Lacey, who was recently a Postdoctoral Fellow here, is now a Lecturer in Mathematics at Heriot-Watt University, Edinburgh.

Two overseas visitors early next year will be Professor B.F. Gray, Macquarie University, and Dr. D. Gubbins, Cambridge University, doing thermal ignition and seismology respectively.

Emeritus Professor J.T. Campbell will be going to Edinburgh for that University's 400th birthday celebrations next year.

J.F.H.

D.S.I.R.

APPLIED MATHEMATICS DIVISION

Hamish Thompson has been promoted to the position of Chief Director at Head Office, D.S.I.R., and will officially leave AMD on October 19th. Hamish joined AMD in 1947 (it was then the Biometrics Section) and was appointed Director in 1963. The new director will be Robert Davies, who was the head of the Statistics section at AMD.

Vicky Mabin (O.R. Section) has been seconded to the Department of Trade and Industry until the end of this year.

G.J.W.

MINISTRY OF AGRICULTURE AND FISHERIES

BIOMETRICS

Chris Dyson is to transfer from the Ruakura Group to the Lincoln Group in early December. Dr. Ray Littler (ex Waikato) and Ms. Barbara Dow (ex Teaching) have been appointed Biometricians at Ruakura. Jim Courtley has been reappointed to the HO Group in Wellington following a stint at the University of Malawi. Dr. Geof Jowett is leaving the Invermay Group this month to enter a well-earned retirement. John Waller has returned to Ruakura from doctoral studies at the University of Adelaide and Martin Upsdell has departed for doctoral studies at Nottingham. Peter Johnston is currently away from Invermay on a fellowship to the University of Reading.

Two technicians have recently been appointed: Anthony Byett (B.Sc. Waikato) to the Palmerston North Group and Rosemary Whewell (B.Sc. VUW) to the HO Group. Both have extensive experience with statistical packages.

It has been interesting watching the progress of a statistics vacancy around the country - the departure of Graeme Winn left a hole at Ruakura which moved about in the following pattern:

Ruakura → Lincoln Col. → Lincoln → MAF → Ruakura → Waikato

Filling in the arrows is left as an exercise. *M.A.J.*

Conferences

*** 1982 ***

November 2-4 (Boston, Massachusetts)	*SIAM Conference on Numerical Simulation of VLSI Devices* Details from Hugh B. Hair, SIAM Services Manager, 1405 Architects Building, 117 South 17th Street, Philadelphia, Pennsylvania 19103, U.S.A.
November 3-5 (Chicago, Illinois)	*Twenty-third Annual IEEE Symposium on Foundations of Computer Science* Details from David W. Bray, Department of Electrical and Computer Engineering, Clarkson College, Potsdam, New York 13676, U.S.A.
November 6-16 (Khartoum)	*Relational Representations of Biological and Environmental Systems* Details from M.E.A. El Tom, School of Mathematical Sciences, University of Khartoum, Khartoum, Sudan.
November 16 - December 10 (Trieste, Italy)	*Autumn Course on Mathematical Ecology* Details from ICTP, P.O. Box 586, I-34100 Trieste, Italy.
November 17-19 (Lille, France)	*Congress on Machine Architecture and Information Systems* Details from AFCET Informatique 1982, 156 boul. Périere, 75107 Paris, France.
November 21-30 (Khartoum)	*Deterministic Models in Population Biology* Details from M.E.A. El Tom, School of Mathematical Sciences, University of Khartoum, Khartoum, Sudan.
December 28 - January 4 (Honolulu)	*Conference on Abelian Group Theory* Details from Adolf Mader, Department of Mathematics, University of Hawaii, 2565 The Mall, Honolulu, Hawaii 96822, U.S.A.

*** 1983 ***

January 10-14 (College Station, Texas)	*Fourth International Symposium on Approximation Theory* Details from C.K. Chin, Department of Mathematics, Texas A & M University College Station, Texas 77843.
March 21-26 (Birmingham, Alabama)	*UAB International Conference on Differential Equations* Details from Ian W. Knowles, Department of Mathematics, University of Alabama in Birmingham, Alabama 35274.
May 23-25 (Palmerston North)	*Eighteenth New Zealand Mathematics Colloquium* Details from Colloquium Secretary, 18th N.Z. Mathematics Colloquium, Department of Mathematics and Statistics, Massey University, Palmerston North, New Zealand.
June 20-24 (Israel)	*International Symposium on the Mathematical Theory of Networks and Systems* Details from P.A. Fuhrmann, Department of Mathematics, Ben Gurion University of the Negev, Beer Sheva 84120, Israel.
June 28 - July 1 (Dundee, Scotland)	*Dundee Biennial Conference on Numerical Analysis* Details from Conference Secretary, Department of Mathematics, University of Dundee, DD14HH, Scotland.
July 12-15 (Exeter, England)	*International Conference on the Teaching of Mathematical Modelling* Details from Mrs S. Williams, Conference Secretary, University of Exeter, St Lukes, Exeter EX1 22U, England.
July 25-29 (Pittsburgh, Pennsylvania)	*Sixth International Symposium on Multivariate Analysis* Details from P.R. Krishnaiah, Centre for Multivariate Analysis, Ninth Floor, Schenley Hall, University of Pittsburgh, Pennsylvania 15260.

*** 1984 ***

August (Adelaide)	*Fifth International Congress on Mathematics Education* Details from John Mack, Department of Pure Mathematics, Sydney University, New South Wales 2006, Australia.

EIGHTEENTH NEW ZEALAND MATHEMATICS COLLOQUIUM

The Eighteenth New Zealand Mathematics Colloquium will be held at Massey University, Palmerston North, from Monday, May 23 to Wednesday, May 25, 1983.

The organising committee would like to continue the emphasis on increasing the proportion of papers accessible to a general mathematical audience. In particular, survey papers and papers detailing applications to other disciplines are encouraged, as are papers from outside the university scene. It is also hoped to encourage Masterate and Ph.D. students to talk about their work; shorter 20 minute sessions will be timetabled for these if required. Provision will be made for participants who may be unable to attend the full conference; it is hoped that papers of interest to mathematics teachers (secondary and tertiary) can be concentrated on one day. The annual meeting of the New Zealand Mathematical Society will be held during the Colloquium.

Invited speakers will include Professor R.P. Kerr (University of Canterbury), who will speak on Equations of Motion in General Relativity, and Professor G. Szekenes (University of New South Wales), who will speak on a topic featuring the interplay between combinatorics and analysis. For further details and registration form write to:

Colloquium Secretary, 18th N.Z. Mathematics Colloquium, Department of Mathematics and Statistics, Massey University, Palmerston North, New Zealand.

EIGHTH AUSTRALASIAN FLUID MECHANICS CONFERENCE

UNIVERSITY OF NEWCASTLE, 28 NOVEMBER - 2 DECEMBER 1983

CALL FOR PAPERS AND PRELIMINARY REGISTRATION

Topics covering all aspects of theoretical and experimental fluid mechanics are within the scope of the Conference.

Titles of proposed papers together with a brief synopsis (a maximum of 300 words) outlining the aims, content and conclusions of the papers should be received by 15 January 1983 and should be accompanied by a statement of the authors' intention to attend the Conference. Papers not presented by an author will not appear in the Proceedings.

Authors will be notified of the provisional acceptance of their papers by 28 February 1983. They will then be asked to submit the full text of papers by 30 June 1983 for final consideration. Accepted papers will be pre-printed by the photo-offset process direct from the authors' manuscript. The Proceedings will be available to registrants on arrival at the Conference.

Intending authors should note the following deadline dates:

Receipt of synopses	15 January 1983
Notification of provisional acceptance of papers	28 February 1983
Receipt of full text for final review	30 June 1983

All correspondence relating to the Conference and proposals for papers should be addressed to:
Professor R.A. Antonia, Department of Mechanical Engineering, University of Newcastle, NEW SOUTH WALES 2308, AUSTRALIA. Telex: TUNRA AA 28784

COMBINATORIAL MATHEMATICS CONFERENCE

The 10th Australian Conference on Combinatorial Mathematics was held this year in Adelaide from 23rd to 27th August and was attended by about 60 delegates from Australia and overseas who between them gave 41 papers. Invited speakers were C.C. Chen (Melbourne), J. Hirschfield (Sussex), D.A. Holton (Melbourne), A.D. Keedwell (Surrey), C.C. Lindner (Auburn, USA), N.J. Pullman (Queen's, Canada) D. Stinson (Manitoba) and J.A. Thas (Ghent, Belgium). As usual the Proceedings will be published by Springer-Verlag.

A feature of this series of conferences is the conviviality of the annual dinners. This year's included a specially written set of words to the tune of "My Bonnie Lies over the Ocean" of which this is a sample verse:

"The lonesome and lovelorn lion A,
He longs so to meet lion B;
But, ah, they are parallel lions,
They'll meet in infinity.
Going, going, here we all go to infinity,
Going, going, going to infinity."

At the annual general meeting of the Combinatorial Mathematics Society of Australasia it was decided that 11th conference in the series is to be held at the University of Canterbury, Christchurch, New Zealand, from Monday, 29th August to Friday, 2nd September 1983.
Dr. D.R. Breach was elected Director (and organiser), there being only one dissenting vote, his own, which was deemed to be ineffective.

D.R.B.

LIFE WITHOUT HAUSDORFF

IVAN L. REILLY
UNIVERSITY OF AUCKLAND

Although 1982 is the Fortieth anniversary of the death of Felix Hausdorff, this talk will not be a belated obituary for the great German topologist.

Recall that a topological space X is a Hausdorff space (or a T_2 space) if for every pair of distinct points $x,y \in X$ there are open sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. In 1955 Kelley's famous book "General Topology" caused some consternation by the omission of T_2 from several of its definitions. Bourbaki, on the other hand, went so far as to make T_2 part of the definition of compactness.

At first sight life without T_2 seems to be unpleasant: compact subsets need not be closed, sequences may have more than one limit point, compact spaces may fail to be normal, paracompactness may not imply the existence of a partition of unity subordinate to an open cover. This is the first major disadvantage of non-Hausdorff spaces - some of our favourite theorems are no longer true. We shall discuss some procedures for extending standard theorems to various classes of non-T_2 spaces.

The second major objection to the greater generality of non-Hausdorff spaces is that it is not compensated for by a significant increase in the number of examples covered. This view may have had some credence twenty years ago, but can be quickly laid to rest today with a considerable list of significant non-Hausdorff spaces.

Despite the fact that this talk has been advertised as not being an obituary for Felix Hausdorff, I feel constrained to give some biographical details by way of introduction. Since, for obvious reasons, I have no personal anecdotes about Hausdorff, let me quote some reminiscences of someone who has, namely P.S. Alexandrov [2, pages 316, 328, 330].

"In July 1924, Hausdorff welcomed Urysohn and I with exceptional kindness and warmth. In Bonn we went every morning to bathe in the Rhine and swam across it and back. These swims lasted not less than three hours altogether. The Rhine is wide at Bonn and its current is very swift. Every time we swam across it we were carried several kilometers downstream by the current, and then had to make up for these kilometers by walking along the bank. As Hausdorff (unsuccessfully) tried to convince us, this swim across the Rhine was not without its dangers, as there were steamers and barges along the Rhine (true, not in such numbers as nowadays). We spent the whole of the second half of the day, from lunchtime until late in the evening, with Hausdorff, mainly in mathematical discussions, which were very lively and interesting indeed.

"I went to the south of Switzerland, where I spent a whole month in the small town of Ascona on the banks of Lago Maggiore. Hausdorff and his wife were on holiday in Locarno at that time and we saw each other every day (it is a short walk from Locarno to Ascona.) The Hausdorffs walked it more than once, and I took them in the rowing boat that was at my constant disposal (it was amazingly light.) During these boat rides and especially when swimming I had only to take care that I did not suddenly find myself in Italy (the border between Switzerland and Italy is somewhere in the middle of Lago Maggiore), which in my case was entirely "non-trivial". The Hausdorffs and I had an exceptionally good time. Our parting was all the sadder, though we did not know then that it was to be forever.

"As I have already said, I met Hausdorff for the last time by the Lago Maggiore in the autumn of 1932. During the winter of 1932-1933 we carried on a lively correspondence at first. Then Hausdorff began to write to me more and more rarely and after some time it became clear to me that it was not safe for him to receive letters from me. Our correspondence ceased. At the beginning of 1942 Hausdorff learned that he was liable to be sent to a concentration camp, and in February of that year Hausdorff and his wife committed suicide in their home in Bonn."

The place of Felix Hausdorff in the historical development of point set topology is clearly indicated by J. H. Manheim in his book [7].

> "The appearance in 1914 of Hausdorff's *Grundzüge der Mengenlehre* marks the emergence of point set topology as a separate discipline. A *space* became merely a set of points and a set of relations involving these points, and a geometry was simply the theorems concerning the space. Hausdorff began his development of topology with a small non-categorical set of neighbourhood axioms. After deriving many of the properties of the general space thus defined he progressively introduced new axioms, ultimately deducing metric spaces and finally, particular Euclidean spaces.
>
> "The studies of Frechet, Riesz, and Weyl all advanced the subject of abstract spaces. It was, however, Hausdorff who saw how to give to these spaces both the generality and precision necessary to establish point set topology as a separate discipline. This achievement was possible because Hausdorff united a recognition of the utility of choosing the neighbourhood concept as fundamental with a set of axioms that was both general enough to handle abstract spaces and sufficiently restrictive to serve as a basis for developing a fruitful set of theorems."

By a topological space Hausdorff meant a set X, composed of elements x, together with certain subsets U_x, associated with x; the subsets U_x are called neighbourhoods of x and are subject to the following four conditions:

(A) To each point x corresponds at least one neighbourhood U_x. Each neighbourhood U_x contains the point x.

(B) The intersection of two neighbourhoods of x contains a neighbourhood of x.

(C) If y is a point in U_x, there exists a U_y such that $U_y \subseteq U_x$.

(D) If $x \neq y$, there exist U_x and U_y such that $U_x \cap U_y = \phi$.

Present day topologists would say that properties (A), (B) and (C) are a neighbourhood axiom approach to the definition of a topological space, and that the separation property (D) defines a special class of topological spaces which are known as Hausdorff (or T_2) spaces. Property (D) can be replaced by weaker properties (D') or (D").

(D') For $x \neq y$ there exists a U_x not containing y and there exists a U_y not containing x.

(D") For $x \neq y$ there is a neighbourhood of x not containing y or there is a neighbourhood of y not containing x.

These give us the classes of T_1 and T_0 topological spaces, respectively. It seems that Vaidyanathaswamy [13] was the first to make this observation.

Until about 1950 it seemed that, with one or two exceptions, topologists had a *theorem* which said *"all spaces are Hausdorff"*. Typical was the attitude of Bourbaki who had made T_2 part of the definition of compactness. J. L. Kelley's book [6] set the cat among the pigeons in 1955, by daring to omit Hausdorff from many of its definitions. Since then there has been a continuing and growing interest in the study of non-Hausdorff spaces. For example, I noticed recently the plea of the editors of [3] "for non-Hausdorff spaces as a legitimate object of study in general topology". The theory of continuous lattices is pervaded by sober spaces and the Scott topology, see [5], and neither of these are necessarily Hausdorff. Other examples of useful non-Hausdorff topologies are the Zariski topology of algebraic geometry, the topology generated by a non-separating family of semi-norms on a vector space, the Alexandrov one-point compactification, and the topology generated by a non-symmetric distance function or quasi-metric. I have been fascinated by quasi-metric spaces for some time, see [10], [8], and with my colleague Vamanamurthy [11], [12] have been making some progress on one or two of the outstanding problems. Let me mention some applications of quasi-metrics. Examples are the shortest-time distance and the minimum-energy distance, and they have relevance when consideration is taken of such things as topography, prevailing winds, river or ocean currents, or man-made barriers to travel such as one-way street systems. Waterman, Smith and Beyer [14] have considered some quasi-metrics of biological origin, and Domiaty [4] has discussed the relevance of quasi-metrics to the structure of space-time.

How then do we overcome the difficulties that life in non-Hausdorff spaces presents? Much of the remainder of this talk is drawn from the paper of Wilansky [15], who gives procedures for extending standard theorems, or modified forms of theorems which usually deal with Hausdorff spaces. Some of these procedures are as follows.

1. Use retracts instead of closed subspaces. *A retraction* $r: X \to S$ is a continuous map from X to a subspace S satisfying $r(s) = s$ for all $s \in S$. If there exists such a retraction onto S, then S is called a *retract* of X.

   Example. The classical theorem: *"If X is a Hausdorff space, the diagonal in $X \times X$ is closed"* fails for more general spaces. It may be replaced by the following generalization: The graph of continuous $f: X \to Y$ is a retract of $X \times Y$. (Proof: Consider $(x,y) \to (x, fx)$.) Thus we obtain: If X and Y are T_2 spaces and $f: X \to Y$ is continuous, then its graph is closed. The first mentioned result is the application of this to $i: X \to X$.

2. Use closed graph instead of continuous. We call $f: X \to Y$ a CG function if it has closed graph, i.e., the graph of f is a closed subset of $X \times Y$. Since a continuous map between T_2 spaces is CG, any theorem in which CG is a hypothesis generalizes the corresponding theorem for continuous maps between T_2 spaces.

   Example. If X is compact, Y is T_2, and $f: X \to Y$ is continuous, then f is closed (i.e., f preserves closed sets.) This fails if Y is not T_2, but we have the generalization: If X is compact and $f: X \to Y$ is CG, then f is closed.

   There are ways in which CG functions are better behaved than continuous ones. The following two remarks fail for continuous functions: If f is a CG bijection, f^{-1} is also CG. (Proof: It has the same graph as f.) If $f: X \to Y$ is CG, then f remains CG when the topologies of X and Y are enlarged. (Proof: A closed set remains closed when the topology is enlarged.) The latter remark is the Closed Graph Lemma which has numerous applications in functional analysis. Moreover, one of the most famous operators in all mathematics is CG and not continuous. This is $D: C^{(1)} \to C$, the differentiation operator. The relevant facts are that if

   (a) $f_n \to f$ uniformly and (b) $f'_n \to g$ uniformly, then $g = f'$, i.e.,

   (c) $f'_n \to f'$ uniformly; and that (a) alone does not imply (c).

3. Assume regular instead of T_2. Kelley's book [6] emphasizes this strategy. For regular spaces, compact implies normal and completely regular. All uniform spaces (in particular topological groups and vector spaces) are regular.

   Example. A regular second countable space is pseudometrizable. (A pseudometric is like a metric except that $d(x,y) = 0$ for $x \neq y$ is allowed.) Also (Nagata-Smirnov Theorem) a regular space is pseudometrizable if and only if it has a σ-locally finite base.

4. Assume *KC* instead of T_2. A *KC* space is one in which all compact sets are closed.

   Example. Let T be a compact T_2 topology for a set X. Then T is maximal among compact topologies for X. This result (with precisely the same proof) holds if T_2 is replaced by *KC*. But now we have the advantage that the converse holds, i.e. a compact topology is maximal among compact topologies if and only if it is *KC*.

5. Assume *US* instead of T_2. A *US* space is one in which convergent sequences have unique limits. In [9], I obtained a generalized Banach contraction theorem for *US* spaces.

6. Modify conclusions of theorems.

   Example. The result that a finite dimensional Hausdorff linear space has a unique topology is replaced by: Every finite dimensional linear topological space has its topology uniquely determined by the closure of $\{0\}$.

7. Modify definitions.

   Example 1. Neighbourhoods of ∞ in the Alexandrov one point compactification X^+ must be defined as complements of compact closed sets (rather than just compact sets.) Then we can show that X^+ is Hausdorff if and only if X is T_2 and locally compact.

Example 2. Paracompactness does not need T_2 in its definition. Then the famous theorem of A. H. Stone says that a pseudometric space is paracompact.

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GAṆITA BHĀRATĪ

The fourth volume of the Bulletin of the Indian Society for History of Mathematics will be published this year. Manuscripts of articles, books for review, news items etc. are welcomed. The editor's address is: Professor R.C. Gupta, Birla Institute of Technology, P.O. Mesra, Ranchin 835-215, INDIA.

[They are reviewing "Partitions: Yesterday & Today" by G. Andrews].

Centrefold

DEREK LAWDEN

Professor of Mathematics, University of Aston, Birmingham

Many readers of this magazine will be interested to hear news of Derek Lawden, sometime Head of the Mathematics Department, University of Canterbury, contentious talker over (New Zealand) radio, and space futurist. Well, judging from the correspondence columns of "The Times", the appearance of a new edition of his "General Relativity and Cosmology" and his editorship of the "Journal of Psychophysical System", it can be reported that his profile in England is high.

It is now 15 years since he returned to his native Birmingham so a biographical sketch is in order. Derek went to Cambridge in 1937 and volunteered for the Army at the outbreak of war. He was quickly commissioned into the Royal Artillery and assigned to radar, as instructor in techniques and later as officer controlling the radar stations in Gibraltar. Finally he became a lecturer at the Military College of Science.

Released from the Army in 1946, he returned to the Cambridge Tripos and qualified in 1947 as a Wrangler. Then back to the Military College, Shrivenham, as a mathematics lecturer, where he commenced research into the theory of control systems. His contributions to sampling systems gained an award from the Institute of Electrical Engineers in 1952.

On to the College of Advanced Technology, Birmingham (now the University of Aston) as senior lecturer in mathematics and the beginning of research into optimization of rocket trajectories. (He had evinced early interest in the possibility of space travel, and had been a "member" of the British Interplanetary Society since 1937.) Having established the foundations of the theory (and solved some major problems in the calculus of variations), the results were collected together in a book "Optimal Trajectories for Space Navigation", published in 1963.

Derek had come to New Zealand in 1957 and had presided over the move of the Mathematics Department to the Ilam campus and the beginning of the growth of its establishment. With the successful launching of the first sputnik in 1958, the tempo of research in rocket dynamics accelerated and he was in demand as a consultant to such companies as Boeing and Lockheed. This led to a number of visits to the U.S.A., thus greatly stimulating his research in this new field. Unfortunately, his opposition to the U.S. involvement in Vietnam ultimately resulted in the withdrawal of his visa and so put a stop to this useful traffic in ideas. Nevertheless, the research developed and led to the award of the Sc.D. degree by Cambridge University in 1962 (now taking space travel seriously!), election to a Fellowship of the Royal Society of New Zealand (1962), award of the Society's Hector Medal (1964) and the Mechanics and Control of Flight Award of the American Institute of Aeronautics and Astronautics (received by proxy, 1967).

In 1967 he returned to Birmingham to a Chair of Mathematics at Aston. His work for a book "Mathematical Principles of Quantum Mechanics" aroused an interest in the conceptual foundations of the subject, particularly the rôle of the conscious observer. There followed a close study of the scientific status of the mind and the results accummulated by parapsychologists and psychical researchers (even spoon-bending!). In 1980 he became editor of a journal whose object is to encourage research into the interaction between mind and matter on both the theoretical and experimental levels.

Professor Lawden is within a few years of retirement and is currently concerned with the cut-back of university funds. His wife, Mary, is heavily involved with the amateur stage. Their family thrives: the twins as practising mathematicians (Gregor at Sussex University, Michael as an astronomical computer man at Reading) and Mark as a researcher into physiology at Cambridge, soon to go to Oxford to finish his medical degree.

W.B. Wilson

THE CONSTRUCTION OF MATHEMATICS TELEVISION PROGRAMES AT THE OPEN UNIVERSITY

G.A. READ

OPEN UNIVERSITY

The Open University, in collaboration with the BBC, has been involved in educational television since 1969 and has produced a considerable number of programmes in the last thirteen years. The majority of these programmes deal with topics in Science, Technology and Mathematics, and I thought that it might be of interest to discuss the process by which they are produced.

Initially I should explain, for those readers not familiar with the organisation, that the Open University caters for about 60,000 extramural students and has no undergraduate students on campus. The students are provided with a variety of distance education materials including: specially prepared texts, home experiment kits, audio cassettes, broadcast television and radio; in addition they may attend short residential courses and local tuition centres. The bulk of the teaching load is carried by the texts which are comparable in academic content to a conventional university course, while designed to cater for the needs of distance learners. It would be quite mistaken to conclude that the Open University students obtain their degrees simply by watching television, or even that television occupies a high proportion of the students' study time. In fact at most 5% of a student's time would be devoted to television, and the average student would spend a far smaller proportion. Nonetheless television broadcasts make a very worthwhile contribution to our courses as do audio cassettes. In passing, I should like to mention the considerable success which we have had with audio cassettes designed to accompany specially prepared printed material. They are relatively cheap and easy to produce and have considerable potential for conventional 'on-campus' universities.

The audio cassettes, radio and television programmes and occasionally special films, are jointly produced by the Open University and the BBC. The BBC has a separate division, 'BBC Open University Productions', which is comparable with its other internal departments such as 'Outside Broadcasts' and 'Light Entertainment'. From 1970 until 1981 the television programmes were recorded at Alexandra Palace, in North London, but recently the BBC staff have moved to a new group of buildings, including a new studio complex, on the Open University campus at Walton Hall. This close proximity of the two groups on the same site will, it is hoped, contribute to an even closer collaboration, and will certainly save considerably in travel time. The close liaison which is required between staff with distinct affiliations is potentially fraught with difficulty; but the relationship works surprisingly well in practice. This must be due mainly to a far sighted decision by the BBC to appoint young academics to their staff, and to train them to become producer/directors, rather than to use production staff with no expert knowledge of the subject matter. The discussion of the programmes is thus centred around a common academic goal, and the BBC staff are regarded, quite rightly, as academic equals.

A typical course would consist of 32 written texts, plus a number of television programmes and cassettes, and would be produced by a Course Team of, perhaps, eight or nine people (sometimes more, or less, depending on the course). Generally it is the courses which can expect more than, say, 1000 students per year which justify a full complement of TV broadcasts, although special exceptions are made for certain subjects for which television is regarded as essential. This restriction is largely due to the shortage of broadcasting time, rather than recording facilities, but with the continued proliferation of video cassette recorders and, perhaps, satelite broadcasts, this situation could change very quickly.

The Course Team consists of academics, often from different faculties, several television producers and usually an educational technologist. The University has its own Institute of Educational Technology, and it is necessary to explain its function, since visitors to the campus almost invariably misunderstand its involvement in the construction of our courses. The educational technologists play a useful role organising the testing of materials with groups of pilot students and by making constructive comments on the materials after they are produced, but in my experience they make little contribution to the initial design of the course and the texts, and play no part in the design of the television programmes.

Generally the Team will delegate one of its members to write a particular text, and to collaborate with a television producer to produce the associated TV programme. The starting point for the discussions of the television component is thus a defined body of academic material; in practice however the text may only exist as a draft outline when these discussions begin. This is the first, and perhaps the most important, point to appreciate about the system of production. No text, or programme, is produced in isolation. The production of each of the texts has to fit into a complicated publishing timetable, and the TV programmes are recorded in a studio which is managed and staffed to a high professional standard. The academic staff are required to be equally professional in the preparation of materials, which means for example that they need to abide strictly to a production timetable, which may sometimes lead to a situation where the author is working simultaneously on a text and its TV programme. Actually this situation can be beneficial, since ideas which arise during the discussion of TV are very often incorporated in the printed material. In fact we make a positive effort to closely integrate the text and the broadcast component, and a piece of work started in one medium may well be continued in another, if the author feels that it is appropriate. For example, the data may be provided in the text for an experiment discussed on TV, or the student may be required to work through various exercises before the broadcast (as in the programme given in the reference).

The initial discussions might begin, for example, by attempting to list the skills which the student would be expected to master after working through the unit (text, TV programme, cassette etc). As an illustration, let us suppose that we are attempting to construct a programme to accompany a text on the Taylor Series within an elementary Calculus course. We might then begin with the following list of educational objectives:

1. To be able to write down a general expression for the Taylor (Maclaurin) Series for an arbitrary function, and to calculate the coefficients for various simple functions.

2. To be able to obtain a formula for the general term of a Taylor Series in various simple cases.

3. To appreciate that the series may converge for a range of values, and diverge otherwise.

4. To be able to calculate the first few terms of a series by manipulating series.

5. To be able to use the first few terms of a series to calculate limits and to find approximations to function values and integrals.

6. To appreciate that a proper discussion of the error requires an estimate for the remainder term.

The early part of the discussion would be designed to produce as many ideas as possible for the programme and, in the nature of things, such sessions are 'open ended' and far from logical. We might begin by suggesting a case study, or some practical situation, which leads to a mathematical model involving Taylor Series, and if we cannot think of a suitable physical problem, we would either coopt a member of the Technology or Science Faculties, or hire a consultant from a suitable industry. Quite often such an approach leads to a lively and interesting programme. I should explain at this point that one of our objectives is to produce a programme which is entertaining (or perhaps 'not boring' would be a better description), since a programme which does not keep the student's attention most certainly cannot teach him anything.

As an alternative, someone might suggest that the programme should aim to show the students why Taylor Series are important; which leads inevitably to a long discussion of why Taylor Series are in the course in the first place. We might design the programme around one or two 'mathematical' problems, for example:

How can we calculate the value of sin(0.5)?

or

How do we evaluate the integral $\int_{0.5}^{1} \frac{\sin x}{x}\, dx$?

One could conceive of an entirely satisfactory programme which, for example, contrasted the approximation of such an integral by, say, the Trapezium Rule and by the Taylor Series.

As a contrast, one could take the view that the programme should be regarded as an introduction to Mathematical Analysis, and we should discuss the convergence of the series, both pointwise and on an intervel (aimed towards uniform convergence). We might then discuss the possibility of producing computer animations of the Taylor Approximations to a given function.

Generally this first meeting will come to no firm conclusions, other than to note that if we intend to commission computer animations, models or get in touch with a consultant, then certain deadlines will have to be met. The meeting usually concludes with someone agreeing to circulate brief notes, and each of us delegated to do some research and to come up with some firm ideas for the next meeting. There may be two or three such sessions, until finally we produce a tentative 'Running Order' for the programme. Such a document is a skeleton of the script which will be produced later, and the BBC producer's concern at this stage will be not only the content of the programme, but what pictures will actually appear on the screen.

Proposed Running Order

What are Taylor Polynomials?
Calculate Taylor Polynomials for a general function f.
Calculate the coefficients for $\sin x$.
Show how we can calculate $\sin(0.5)$

and $$\int_{0.5}^{1} \frac{\sin x}{x}\,dx.$$

Talk about the error in approximating functions by polynomials.
Introduce idea of regions of high accuracy.
Show some examples: $\sin x$ and $\sin x + \cos x$, (with computer animations?)
Show an example where the series converges on an interval, $\frac{1}{1+x}$.

Having produced the Running Order we may then talk it through in detail and 'play-act' each part, with perhaps a sketch on the blackboard of the pictures which the camera will see. Very often the research will have produced material which gives a natural order to the programme, as for example in a recent programme on the Wave Equation. In this particular instance we received invaluable help from the Royal Society, and they were able to lend us various pieces of apparatus which contributed considerably to the structure of the programme. This illustrates the essence of this production process. The limits to the programme are governed only by your own imagination, and you are able to call on many different resources in order to fill the screen with the appropriate pictures. Certainly we have a budgetary limit, but this is by no means as serious as the limitations of your ability to construct imaginative ideas.

An attempt to translate the above running order into a script soon reveals some serious deficiencies. The early part of the programme looks particularly uninteresting, with perhaps a choice of a presenter seated at a desk (or at a blackboard), or a piece of animated film.

Television (in the context of Mathematics) is a very poor vehicle for communicating complex algebraic manipulation, and is much more successful at transmitting ideas. For example, the idea which lies behind an iterative process, repetitively passing the image under a function through the function, is likely to work quite well, whereas the actual calculation of those image values would be rather boring.

The following running order is in fact an outline of a programme actually produced for one of our courses, and was a result of the sort of discussions outlined above.

Note: Introduce the Taylor Polynomials in the text and get them to calculate $\sin 0.5$ and $\int_{0.5}^{1} \frac{\sin x}{x}\,dx$ before the programme.

Running Order: Mention the pre-programme work.
Calculate Taylor polynomials for $\sin x$ (presenter at desk?)
Show the graphs for increasing values of n (computer graphics?)
Talk about the error, regions of high accuracy and the power of the polynomials.
Short Summary
Generalize to an arbitrary function F.
Play around with the formula to get them to remember it.
Discuss $\sin x + \cos x$ and $\frac{1}{1+x}$.

It is not the only programme which we could have produced, and, in fact, a few years earlier we had produced a very different programme on the same topic. One cannot even be sure that it is a very good, or a very bad, programme, since the quality of a television programme is generally a subjective assessment (although some attempts have been made at objective testing).

Once the running order is agreed, there will be one or two further meetings to write the words for a script. The BBC producer then writes his camera script, containing the instructions for camera crew, vision mixers, special effects etc; and writes a specification for the model makers, designers and so on. (In fact, more likely, the pressures of time have required him to specify some of the items in detail before the script is finalized). From this point on, it is the producer's responsibility to assemble all the material required for the programme by the required date, although the academic will be involved in checking the specifications of such items as graphics and animated film, or may be involved in location filming and visiting consultants if required.

Two or three days before the programme the producer may assemble as much of the apparatus as possible, and you may attempt a 'walk-through' of the programme (often referred to as a 'stagger-through'). This is then followed by another day of rehearsal, and finally a recording day in which the programme is finally recorded onto tape. Everyone then breathes a sigh of relief; everyone that is except the producer, who has still to edit the programme and ensure that it will fit into a transmission slot of exactly 24 minutes and 30 seconds.

After editing, the programme has to be approved by the course team before it is passed for transmission. The academic then continues to process the written work, including any material which links the broadcast to the text.

I hope that the reader will gather from this brief description that the production of video material to professional standards can be a very time consuming, but nonetheless, extremely interesting task. There is no doubt that television has played an important part in the success of the Open University, both as a unique means of instruction, as a 'shop-window' by which potential students are made aware of our existence, and as a means of giving our widely spread population of distant learners of a sense of identity.

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BIFURCATION THEORY

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A review lecture given at the 17th N.Z. Mathematics Colloquium, Dunedin, May 1982.

§1. INTRODUCTION. A considerable number of phenomena studied in the physical sciences (and elsewhere) lead to a mathematical model of the general form

$$\frac{du}{dt} = F(\lambda,u) . \qquad (1)$$

Here t denotes time, λ represents a real parameter ("eigenvalue") and F is a (usually) nonlinear mapping from $\mathbb{R}\times B$ into C where B and C are Banach spaces - often infinite dimensional function spaces.

The transient behaviour of solutions to (1) is largely determined by the existence, multiplicity, and stability of steady-states to (1), that is

$$F(\lambda,u) = 0. \qquad (2)$$

The purpose of this paper (talk) is to study these questions using a body of knowledge described as "Bifurcation Theory" which copes easily with the cases when B and C are of finite dimension or, if they are not finite dimensional spaces, functions F which are compact perturbations of the identity. The aim will be to characterise the set $F^{-1}(0)$ in the (λ,u)-space and to describe this set in the $\mathbb{R}^2$ plane, $\lambda \in \mathbb{R}$, $\|u\| \in \mathbb{R}$.

§2. DEFINITIONS AND REMARKS.

(i) We suppose that equation (2) possesses a simple curve of solutions in $\mathbb{R} \times B$ given by

$$\Gamma = \{(\lambda, U(\lambda)) \ : \ a < \lambda < b\} \subseteq \mathbb{R} \times B, \ -\infty \leqq a,b \leqq \infty .$$

(ii) Further $(\lambda, U(\lambda)) \in \Gamma$ is said to be a BIFURCATION ($\equiv$ branching) POINT (BP) of F if F possesses solutions of equation (2) not on Γ in every neighbourhood of $(\lambda, U(\lambda))$ in $\mathbb{R} \times B$.

(iii) Two special and important families of such equations have the form

$$\left.\begin{aligned} G(\lambda,u) &= u \ , \quad G(\lambda,0) = 0 \ ; \\ \text{or} \quad \lambda G(u) &= u \ , \quad G(0) = 0 \ . \end{aligned}\right\} \qquad (3)$$

Here G is assumed to be compact and continuous (in u), which makes $F = I - G$ ($F=I-\lambda G$) a compact perturbation of the identity. In the case that G is Fréchet differentiable at $u \in B$ we have

$$G(\lambda,u) = L_\lambda(u) + o(\|u\|) \ ,$$

$$\text{or} \quad G(u) = L(u) + o(\|u\|) \ .$$

That is,

$$\left.\begin{aligned} L_\lambda(\phi) \\ L(\phi) \end{aligned}\right\} = (DG)(u)(\phi) \ ,$$

with the appropriate interpretation of the dependence on λ, is a linearization of G near u.

For the purposes of this paper (lecture) we shall take the simpler of these cases (equation (3)), that is λ appearing "linearly" in the equation. The term "non-linear eigenvalue problem" for equations (2), (3) applies here to the nonlinear behaviour of F or G with u.

(iv) We shall normally plot the solution set of (3) (or (2)) in the $(\lambda, \|u\|)$-plane. A vast range of possibilities can occur. In the main we shall meet cases in which the branches are connected, although simple cases can be constructed in which discontinuities arise.

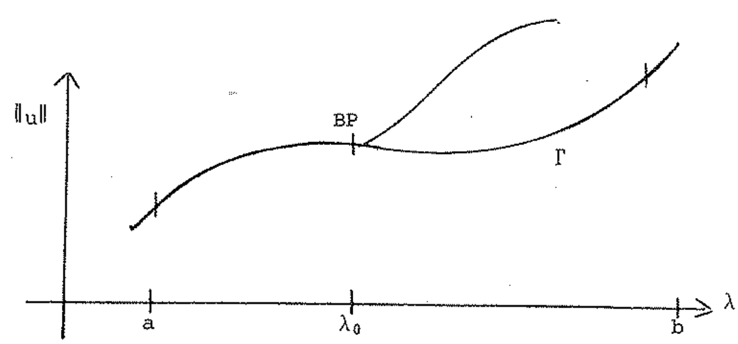


Figure 1: λ_0 is a bifurcation point of Γ. (BP).

Of course, if $G(0) = 0$, we could take Γ as the interval

$$\Gamma = \{(\lambda,0) \ : \quad a < \lambda < b\} .$$

(v) When we are solving a problem like (3), often the parameter λ is not present explicitly, that is, we might just have the equation

$$G(u) = u.$$

In order to characterise the solution set of this equation we embed the problem in that given by (3).

(vi) Clearly the basic question mentioned earlier - covering the multiplicity of solutions for a given λ - is related to the occurrence of bifurcation points. Unlike some other authors, here we shall agree that when a solution branch turns back on itself (for example $d\|u\|/d\lambda = \infty$) a bifurcation point occurs, since this situation is covered by the same theory as the more common type of branching shown in Figure 1.

§3. APPLICATIONS (for more applications see Keller and Antman [1])

(1) Steady-state heat generation (elementary). Consider a volume V of material with surface area S which is at a uniform temperature T ("well-stirred reactor") producing heat at a rate which varies only with temperature as $f(T)$, and which radiates heat to its surroundings at temperature T_0 in accordance with Newtonian cooling. In the steady state (heat produced = heat lost), we have,

$$Vf(T) = kS(T-T_0),$$

or

$$f(T) = \frac{kS}{V}(T-T_0).$$

Letting $u(\in \mathbb{R}) = \frac{T-T_0}{T_0}$, $\lambda = \frac{V}{kST_0}$, $f(T_0+T_0u) = G(u)$,

we obtain a nonlinear equation precisely in the form of (3). In the usual case where the Arrhenius law applies,

$f(T) = \exp(-{}^{K}/_{T})$, where K is a positive constant, we get the transcendental equation

$$\lambda \exp(-K/T_0(1+u)) = u \,. \tag{4}$$

Differentiation with respect to u shows that λ as a function of u has turning points where (4) is solved with

$$\frac{\lambda K}{T_0}\exp(-K/T_0(1+u)) = (1+u)^2.$$

Hence, if T_0/K is small enough ($< \frac{1}{4}$) we have two turning points which satisfy

$$(1+u)^2 = \frac{K}{T_0}u \quad \text{or} \quad u^2 + (2-\frac{K}{T_0})u + 1 = 0. \tag{5}$$

Accordingly (for $u > 0$) the bifurcation diagram looks like that in Figure 2.

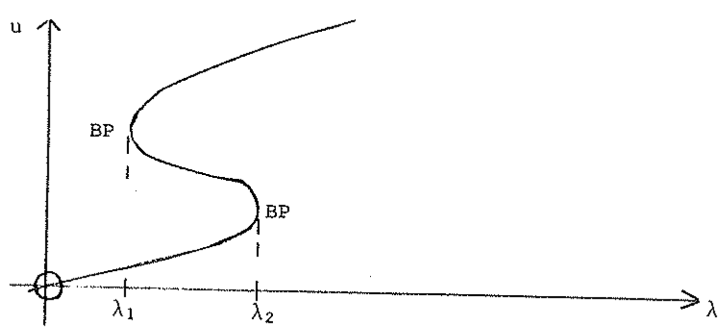


Figure 2: Bifurcation diagram for problem (1) when $T_0/K < 0.25$.

Here λ_1, λ_2 are the corresponding values of λ for the roots u_1, u_2 of the quadratic in (5). Hence, for

$\lambda < \lambda_1$, $\lambda > \lambda_2$ there is just one positive solution of (4)

$\lambda = \lambda_1, \lambda_2$ there are two positive solutions of (4)

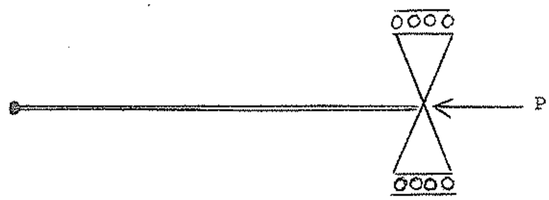
$\lambda_1 < \lambda < \lambda_2$ there are three positive solutions of (4).

Of course if $T_0/K \geq \frac{1}{4}$ we have a unique positive solution for all $\lambda > 0$. More general discussion of this problem appears in Boddington, Gray and Wake [2].

This example is finite dimensional ($B = C = \mathbb{R}$) and is essentially trivial. Yet it does illustrate the powerful results given in the next section. Before we examine these we will consider a second more complicated example where the Banach spaces are infinite dimensional spaces ($L_2(0,\ell)$).

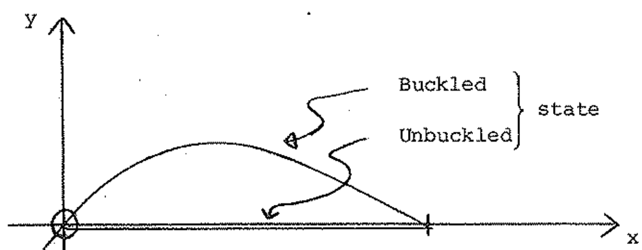
(2) Buckling of a Strut (see Stakgold [3]).

This problem is probably the most well-known in the field and illustrates very well the main general results in the following section. Consider a thin rod under compression with one end pinned and the other confined to $y = 0$. (shown below).



There are, of course, two possible states: unbuckled and buckled states. The important practical question is: what is the critical value of P which will result in the rod buckling? Using natural (intrinsic) coordinates it is possible to show under some physically reasonable simplifying assumptions, (see Stakgold [3]) that $u = \tan^{-1}\left(\frac{dy}{dx}\right)$ as a function of arc-length s satisfied the equation

$$\frac{d^2u}{ds^2} + \lambda \sin u = 0, \qquad 0 < s < \ell, \tag{6}$$
$$u'(0) = u(\ell) = 0.$$



Here $\lambda = \frac{P}{EI}$, P is the compressive load, E is Young's modulus and I is the moment of inertia of the X-section. This problem as stated can be easily analysed in terms of elliptic functions and integrals. In fact, for the buckled state, $u(0) \neq 0$ and $u(s) = 2\sin^{-1}(k\,\mathrm{sn}(\sqrt{\lambda s} + K))$, where $k = \sin\frac{u(0)}{2}$, $K = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} \geq \pi/2$, (7)

satisfies the differential equation and $u'(0) = 0$.

The condition $u(\ell) = 0$ gives

$$\mathrm{sn}(\sqrt{\lambda}\ell + K) = 0,$$

which is essentially the equations of the solution curves in the bifurcation diagram ($\lambda, u(0)$-plane). As the function sn is periodic with period $4K$, $\mathrm{sn}\,K = 1$,

so $\sqrt{\lambda}\ell + K = 2nK$, $\quad n \in N$

or $\lambda = \frac{(2n-1)^2K^2}{\ell^2}$.

As $K \geq \pi/2$, this means for $\lambda = \frac{P}{EI} \leq \frac{\pi^2}{4\ell^2}$ there is no buckled state and that there are a countable infinity buckled states emanating from the unbuckled state $u = 0$ at the points $((2n-1)^2\frac{\pi^2}{4}, 0)$, $n \in N$, in the bifurcation diagram; see Figure 3.

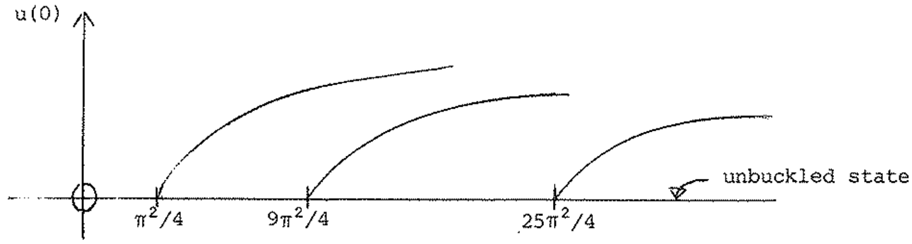


Figure 3: Bifurcation diagram for the buckling problem.

Thus we can analyse this problem completely essentially outside of the framework provided by bifurcation theory.

In order to analyse it in terms of this theory and to illustrate the results in Section 4, we need to write the problem in (6) in the form suggested in equation (3)

$$u(s) = \lambda \int_0^{\ell} g(s,t) \sin u(t)dt \equiv \lambda G(u)(s) \tag{8}$$

where g is the Green's function for the differential operator $\frac{-d^2}{ds^2}$ with the boundary conditions $u'(0) = 0$, $u(\ell) = 0$.

Actually

$$g(s,t) = \begin{cases} \ell - t, & 0 < s < t. \\ \ell - s, & t < s < \ell \end{cases}$$

Thus G is a nonlinear integral operator and our problem is now restated as a nonlinear Fredholm integral equation. On the domain $L_2(0,\ell)$, the operator G is compact. In order to further illustrate the following results we observe that

$$g(u) = \int_0^{\ell} g(s,t)u(t)dt + \int_0^{\ell} g(s,t)(\sin u(t) - u(t))dt$$

$$\equiv L(u) + H(u)$$

and $\|H(u)\| = 0(\|u\|^2)$, $\|u\|^2 = \int_0^{\ell} |u(s)|^2 ds.$

We will now give some general results which these examples illustrate nicely.

§4. GENERAL RESULTS.

There has been a large number of quite general results which apply to equations of the form of (3) and which predict certain features of the bifurcation diagram for (3). We shall look briefly at the more important of these, remembering that we shall take the basic branch Γ as $u = 0$.

Result 1: $\lambda = \lambda_0$ is a bifurcation point of (3) at $(\lambda,0) \in \mathbb{R} \times B$ implies that λ_0^{-1} is an eigenvalue of the linear operator L. (This result gives a *necessary* condition for bifurcation and is proved using general forms of the implicit function theorem, see Krasnosel'skii [4]).

Result 2: λ_0^{-1} is an eigenvalue of L of *odd algebraic* multiplicity then $(\lambda_0,0)$ is a bifurcation point of (3) from $\Gamma = \{(\lambda,0)\}$. (This result gives a *sufficient* condition for bifurcation and is proved using the ideas of analytic (Leray-Schauder) degree theory, see Temme [5, Vols. I & II]).

Result 3: If λ_0^{-1} is an eigenvalue of L and is of *odd algebraic multiplicity*, then the solution set S of (3) contains a maximal subcontinuum (that is a subset which is closed and connected in S) C_λ such that

(i) $(\lambda,0) \in C_\lambda$

(ii) C_λ either

(a) meets infinity in $\mathbb{R} \times B$, or

(b) meets $(\hat{\lambda},0)$ where $\hat{\lambda}^{-1}$ is also an eigenvalue of L and $\hat{\lambda} \neq \lambda_0$.

Note: By a "maximal" subcontinuum C_λ we mean C_λ is not a proper subcontinuum of any C having the same properties. The possibilities are shown in the diagram below (Figure 4).

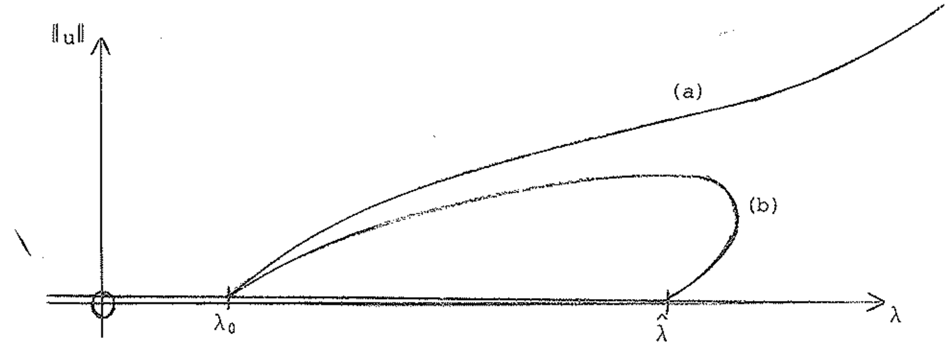


Figure 4: A possible bifurcation diagram illustrating the possibilities (a), (b).

We note that Result 1 implies that $\hat{\lambda}^{-1}$ is an eigenvalue of L but not necessarily of odd algebraic multiplicity. The proof of Result 3 is long and will not be given here (see Rabinowitz [6]). It relies also on the ideas of Leray-Schauder degree theory. It also gives rise to a nice corollary which covers the case when the operator G is not defined globally on B.

Result 3a: If Ω is a bounded open set in $\mathbb{R} \times B$ containing $(\lambda_0, 0)$ and $G(\lambda, u)$ is continuous and compact on $\bar{\Omega}$, λ_0^{-1} is an eigenvalue of L with odd algebraic multiplicity then the solution set of (3) possesses a maximal subcontinuum $C_\lambda \subset \bar{\Omega}$ such that

(i) $(\lambda_0, 0) \in C_\lambda$

(ii) either (a) C_λ meets $\partial\Omega$

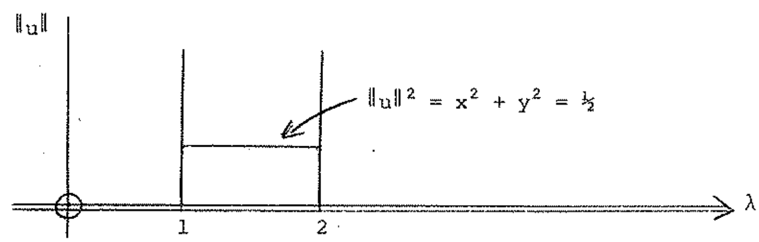
or (b) C_λ meets $(\hat{\lambda}, 0)$ where $\hat{\lambda}^{-1}$ is an eigenvalue of L, $(\hat{\lambda}, 0) \in \Omega$.

An example (somewhat contrived) in which the possibility (b) (which is less usual) of Result 3 occurs is the following:

Example $u = (x,y)^T \in \mathbb{R}^2$,

$$G(u) = (x - xy^2,\ \tfrac{1}{2}y + x^2 y)^T$$

(G is an odd operator). The equation $\lambda G(x,y) = (x,y)^T$ has the solution set shown below:



It is also useful to notice that L is the Jacobian matrix of G evaluated at $u = (0,0)^T$, that is

$$L = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

The last result we shall mention is also from the collection given by Rabinowitz [6].

Result 4: If λ_0^{-1} is a simple eigenvalue of L (with algebraic multiplicity = 1), then there are two subcontinuum of C_λ which near $(\lambda_0, 0)$ have only that point as a common point.

This result is one of the few results which gives an indication of the number of branches at a bifurcation point. Often these two branches have the same norm and are reflections of each other.

§5. CONCLUDING REMARKS

The last few years has seen a lot more results of the character of Results 1-4, but the results in Section 4 should suffice to indicate the sort of results which are possible. Together with the theory of singularities, bifurcation theory has provided a framework under which a large number of diverse phenomena can be described.

It should be seen that the two applications in Section 3 serve to illustrate the behaviour indicated by the Results in Section 4. The first one is not so striking since the bifurcation points were the turning points of λ as a function of the real variable u. However, clearly Example (2) illustrates these results nicely. The values obtained as branching points for $\{(\lambda,0) : \lambda \in \mathbb{R}\}$

$\lambda_n = (2n-1)\frac{2\pi^2}{4}$ are the reciprocal of the eigenvalues of the linearised integral equation

$$u = \lambda\int_0^{\ell} G(s,t)u(t)\,dt$$

and are therefore the eigenvalues of the linear eigenvalue problem

$$\frac{d^2u}{ds^2} + \lambda u = 0,\ 0 < u < \ell;\ u'(0) = 0,\ u(\ell) = 0.$$

For each value of n the algebraic multiplicity is odd, (equal to unity) and so they serve to illustrate the Results 3,4 as well (using u and $-u$ as different solutions.

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3. Stakgold I. Siam Review, Vol 13, 1971, p.289.
4. Krasnosel'skii M. *"Topological Methods in the Theory of Nonlinear Integral Equations"*, Pergammon, Oxford, 1963.
5. Temme N.M. (ed) *"Nonlinear Analysis, Vol 1,2"*, Mathematisch Centrum, Amsterdam, 1976.
6. Rabinowitz P.H. Journal Functional Analysis, Vol 7, 1971, p.487.

Problems

Readers are invited to send problems for this section. Some indication should be given of how a problem has arisen and whether a complete solution is known and attribution of sources should be provided for problems that are not original. Attempts at solutions should be sent to the setter or to the Editor.

Problem 8 Revisited.

A line was omitted from the definition of problem 8, in Newsletter 24. The corrected problem reads:

> Is it true that for every n there are n distinct points in the plane, no three on a line, no four on a circle, so that they determine n-1 distances so that the ith distance (in some order) occurs i times?

Readers are reminded that Professor Erdös has offered $25 for a proof or disproof.

Readers are also reminded that problem 6 *polygon regions*, in Newsletter 21, has so far evoked a null response.

Book Reviews

MATHEMATICS AND MODELS IN ENGINEERING SCIENCE; CONTRIBUTIONS PRESENTED TO C.M. SEGEDIN

Edited by A. McNabb, R.A. Wooding and M. Rosser.
Published by Department of Scientific and Industrial Research.

This book contains research papers presented at a symposium in August 1980 at the Applied Mathematics Division of the DSIR, in honour of Professor C.M. Segedin. It marked his retirement as head of the Department of Theoretical and Applied Mechanics at the University of Auckland.

The book opens with a brief biography of Professor Segedin's professional career followed by his thoughts on the nature of applied mathematics. He sums this up with a comment which could be said to link the papers:

> *...it is the interplay between the physical and the formal mathematical description that not only provides the challenge and the fascination of applied mathematics but is, indeed, the very essence of the discipline.*

The sixteen papers by New Zealand authors that follow illustrate this with a diverse range of topics from the general areas of mechanics, elasticity, fluid mechanics and heat transfer.

Most of the papers are of a specialised nature, although a few are of more general interest. An example of the latter is an interesting article on the application of mechanics to gymnastics, which indicates that gymnasts may have something to learn from bang-bang control theory!

In another paper, on hypersonic flow, it is suggested that although a mathematical model may introduce features not shared by the relevant physical problem, nevertheless all the necessary conclusions must be followed through. A paper on models in water quality management points out that two types of model should be used, one for research purposes and the other for management.

This is a well-produced volume which applied mathematicians interested in mechanics and fluid mechanics may find worthwhile.

ISBN 0-477-066992. Available from:- The Publications Officer, Science Information Division, DSIR, P.O. Box 9741, Wellington, New Zealand. Price NZ$14.50 plus postage and packing. (NZ 60c., overseas $1.50).

D.J.N. Wall

INTRODUCTION TO OPTIMAL CONTROL THEORY, by J. Macki and A. Strauss.
Springer-Verlag, 1982. XIII, 165p, 68 figs.
Cloth DM58 (N.Z.$48 app.)
ISBN 0-387-90624-X

To quote from the preface: "This monograph is an introduction to optimal control theory for systems governed by vector ordinary differential equations. It is not intended as a state-of-the-art handbook for researchers. We have tried to keep two types of reader in mind: (1) mathematicians, graduate students, and advanced undergraduates in mathematics who want a concise introduction to a field which contains nontrivial interesting applications of mathematics (for example, weak convergence, convexity, and the theory of ordinary differential equations); (2) economists, applied scientists, and engineers who want to understand some of the mathematical foundations of optimal control theory."

Formerly treated as part of the calculus of variations, the type of problem here considered is now usually handled by the maximum principle of Pontryagin. While simple in concept, the proof of this principle may fairly be described as difficult. There have been many attempts to simplify the presentation of the theory, and this book is a very worthwhile addition to that number.

Apart from the maximum principle itself, the book discusses the related questions of controllability, the attainable set of states, the existence of optimal controls and their uniqueness, for various special, but practically important, classes of systems. This being an introduction, the range of material considered is by no means complete. For example, the maximum principle is proved only for time-independent systems, with a single specified state as the target, rather than for a target set of states. This last omission is regrettable, but both the time-dependent case, and the extended target set, are discussed by means of theorem statements and examples, and references are given to other texts.

Within its chosen limitations of topics, the book is well constructed. It is mathematically rigorous, but with a clear motivation based on realistic examples. For example, it is made clear that while measurable controls form the natural mathematical assumption, yet piecewise continuous controls involve little loss of generality, and meet the practical requirements. To quote again from the preface: "We feel that a solid understanding of basic facts is best attained by at first avoiding excessive generality." That is a principle with which the reviewer heartily agrees. One simple example, the rocket car, is used throughout the book, with many variations appropriate to the particular theoretical ideas being discussed.

For most of the book, the mathematics required is not advanced, being based directly on the fundamental properties of differential equations, rather than on a more abstract treatment via Banach spaces. The necessary abstract concepts and results are well explained in a mathematical appendix. Chapter 4 is exceptional in having a much higher level of abstraction than the rest of the book. Since it is concerned with existence theorems, it can well be skimmed over on a first reading, as such theorems are usually of small practical utility.

In summary, this book provides an account of the important ideas of optimal control, which is concise, lucid, interesting, mathematically rigorous but not too abstract. It could serve as a basis for a course at the final year honours level, though the exact use to be made of it would depend on the preferences of the lecturer and the class concerned.

R.S. Long

GOPI JAIN APPEAL

Readers will be aware that Gopi Jain died on June 14. An obituary appeared in the August issue of the Newsletter. To preserve his memory mathematicians at Otago have started an appeal to establish a Gopi Jain Memorial Prize in Statistics at the University of Otago. Many people who knew Gopi personally and through his talks at colloquia and through his published work will wish to contribute to the appeal. The following is an extract from the appeal letter.

> On Monday, June 14, Gopi Jain collapsed and died while out jogging in the Botanic Gardens in Dunedin. His sudden and tragic death, so totally unexpected, has been a great blow to all who knew him.
>
> Gopi was always kind and considerate; he would always go out of his way to help his friends and neighbours and would himself never refuse an appeal for help. To his colleagues Gopi was above all else a dedicated and innovative statistician, and it seems only fitting to us that it should be through his work in this area that we should preserve his memory. It is for this reason that we, his friends and colleagues, with the expressed approval of his widow Usha, wish to establish a prize in statistics to be awarded annually to a student of merit. This prize, to be known as the Gopi Jain Memorial Prize in Statistics, will ensure that Gopi will be remembered here for many years to come.
>
> Just as Gopi never said no, we feel sure that all who knew him will now rally round and guarantee that this appeal will be not just successful, for of its success we have no doubt, but that it will be a fitting tribute to his memory.
>
> As regards practicalities, the Inland Revenue has agreed that all donations in excess of $5 will qualify for the annual 50% tax rebate, and we ask that all cheques should be made out to the University of Otago and sent to:
>
> The Secretary, G.C. Jain Appeal, Department of Mathematics,
> University of Otago, P.O. Box 56, Dunedin.
>
> All contributions will be individually acknowledged and a receipt sent.

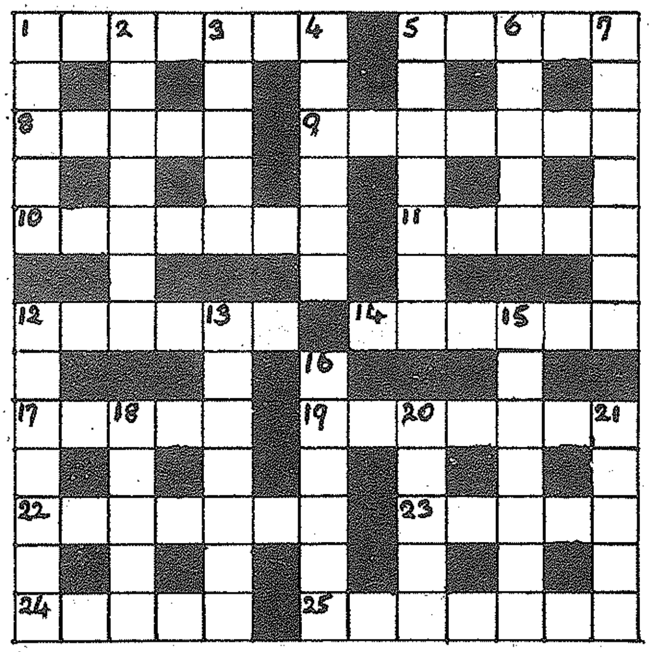
* * * * *

"There are two kinds of foolishness: one is to say, 'this is old, therefore it is good'; the other is to say, 'this is new, therefore it is better'."

Dean W.R. Inge (1860-1954)

Crossword

N$^{\underline{o}}$ 8 ....US, GLEEK, OG.... by Matt Varnish



CROSSWORD N$^{\underline{o}}$ 7 SOLUTION

The letters a,e,o,u do not appear in the clues or in the answers.

Across:

1. Light switch, 8. Irk, 9. Skink, 11. Lying, 12. Icy, 14. Rhys, 16. Stir, 17. Rimini, 18. Bilk, 20. Twit, 22. Illish, 23. List, 25. Kiln, 27. Shy, 29. Grill, 30. Idris (airport of Tripoli), 32. Gin, 33. Skirmishing.

Down:

2. Ink, 3. Tiki, 4. Wily, 5. Trim, 6. Hight, 7. Sky writings, 9. Scribblings, 10. Idyll, 13. Chilli, 14. Kitsch, 16. Sit, 19. Kit, 21. Whirr, 24. Signs, 26. Mimi ('Your tiny hand is frozen'), 27. Slim, 28. Yips, 31. Inn.

Across:

1. X. (7)
5. One of the 48? (5)
8. c. (5)
9. ST. (7)
10. XI XI. (7)
11. Of a wood. (5)
12. Union is disorder if conned. (6)
14. G^T shocked. (6)
17. Rover 1000 BC. Yes? (5)
19. Unaligned. (7)
22. Am crass? Tart speech. (7)
23. $\mathbb{P}$? (5)
24. 2, 4, 6, ... (5)
25. $\leqslant 0$. (7)

Down:

1. Close to temple base. (5)
2. S. (7)
3. $\frac{dI}{dt}$. (5)
4. n, N, $\underset{\sim}{n}$, $\underline{n}$, $\mathcal{N}$, ... (6)
5. $\frac{1}{8}$m for winter coat? (7)
6. Eg. QQQ. (5)
7. C(XT)T, gone. (7)
12. Sigmoid elegance for expertise. (7)
13. Sets of 8. (7)
15. Post above sounds salic. (7)
16. Greek angle set square. (6)
18. Long short signs. (5)
20. $\cup$. (5)
21. Conical rectum. (5)

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