THE NEW ZEALAND MATHEMATICAL SOCIETY

NEWSLETTER

SYDNEY SUMMARIES

HEAVEN WITH HENRY

U.E. MATHS: WHERE NEXT?

CONTENTS

Number 22 December 1981 ISSN 0110-0025

Editorial

With this issue, a stuttering period of sporadic productions of the *Newsletter* splutters to a halt. In placing their official organ in the hands of the dilletante Canterbury team the N.Z.M.S. Council were surely taking a calculated risk. They wanted continued development in the level of service that the *Newsletter* gives to members by an increase in range of material and frequency of publication. Well, in lieu of this, we have given them all we could.

My averred aim has been to take the *Newsletter down-market*. I hope this has been appreciated and that it off-sets the startling, even unwieldly, growth in the sheer bulk of information contained. I trust that some of the new features, notably Centrefold, Cartoon and Crossword are now firmly enough entrenched in our readership's regard that popular demand will ensure their permanent place. For their contributions to these sections I thank the anonymous Maurice Askew of Fine Arts and the eponymous Matt Varnish.

Getting 4 issues per year has been harder to achieve (although it has been if one uses the charitable Roman convention of counting each end-point). Inevitably, the work takes lower priority than class, departmental and university responsibilities for the editors, typists and printers respectively. But more profoundly, there seems to be a natural limitation on the frequency that correspondents can be expected to come up with news and scandals from their departments.

To the others in the team - Ian Coope, Graham Wood, Ann Tindall, Audrey Smythe and Beverley Haberfield - I tender my extreme thanks for their efforts. Also I thank the Honorary Correspondents, particularly Michael Carter for their contributions, and finally I thank Garry Tee for his regular post-card after each issue pointing out the errors, the only real feed-back we get.

Brent Wilson

HONORARY CORRESPONDENTS OF NEWSLETTER

Dr. M.R. Carter	Mathematics Department, Massey University, Palmerston North.
Dr. L. Fradkin	D.S.I.R., Physics & Engineering Labs, Gracefield, Wellington.
Dr. J.F. Harper	Mathematics Dept., Victoria University of Wellington, P.B. Wellington 1.
Dr. D.C. Hunt	School of Mathematics, University of New South Wales, Kensington N.S.W. 2033, Australia.
Dr. M.A. Jorgensen	Biometrics Section, Ministry of Agric. & Fish., P.O. Box 1500, Wellington.
Dr. D.C. Joyce	Mathematics Dept., USP., Box 1168, Suva, Fiji.
Mr. R.S. Long	Department of Mathematics, University of Canterbury, Christchurch.
Mr. J.H. Maindonald	D.S.I.R. - A.M.D., Mt. Albert Research Centre, Private Bag, Auckland.
Mr. Sidney A. Morris	Bulletin of the Aust. Math. Society, Dept. of Pure Mathematics, La Trobe University, Bundoora, Victoria 3983, Australia.
Mr. P.R. Mullins	Dept. of Community Health, University of Auckland, Private Bag, Auckland.
Mr. H.J. Offenberger	School of Maths & Science, Wellington Polytechnic, P.B. Wellington.
Dr. G. Olive	Mathematics Department, University of Otago, P.O. Box 56, Dunedin.
Dr. Ivan K. Reilly	Department of Mathematics, University of Auckland, P.B. Auckland.
Dr. P. Roberts	Fisheries Research Division, P.O. Box 19062, Wellington.
Prof. M. Rosser	Theor. & Appl. Mechanics, University of Auckland, P.B. Auckland.
Dr. M. Schroder	Mathematics Department, University of Waikato, P.B. Hamilton.
Mr. B.R. Stokes	Department of Mathematics, Teachers College, Hamilton.
Dr. G.J. Weir	Applied Mathematics Division, D.S.I.R., P.B. Wellington.
Mrs. H.M. Wily	Dept. of Maths. Education, Teachers College, Secondary Division, P.O. Box 31065, Christchurch 4.

OUTSTANDING SUBSCRIPTIONS

Members (and should-be members) who haven't paid their 1981 dues are implored to send $15 immediately to

Dr. J.L. Schiff, Department of Mathematics,
University of Auckland, Auckland.

News and Notices

THE UNIVERSITY OF AUCKLAND

LECTURESHIPS IN MATHEMATICS (2)

Applicants should have postgraduate qualifications and proven research interests. Applications will be considered from those with qualifications and experience in almost any branch of Mathematics but careful consideration will be given to teaching records. Commencing salary will be within the scale $19,835 - $23,520 per annum. Conditions of Appointment and Method of Application are available from all New Zealand Universities and from the Assistant Registrar (Academic Appointments), University of Auckland. Applications in accordance with Method of Application should be forwarded as soon as possible but not later than 11 December, 1981.

UNIVERSITY OF CANTERBURY

PROGRAMMER-TECHNICIAN (MATHEMATICS)

Applications are invited for the above position in the Department of Mathematics commencing 1 February 1982.

The appointee's duties will include the maintenance and documentation of numerical methods subroutine packages used in teaching; maintenance of student files and records of computer work, programming consultation, setting up and testing of student assignments for all Computational Mathematics courses; preparation of supporting software for instruction; programming support for research projects.

The current salary scale for this position is $10,395 to $13,312 per annum; the commencing salary will be according to qualifications and experience.

Conditions of Appointment may be obtained from the undersigned with whom applications close on 11 December 1981.

W. Hansen, Registrar, University of Canterbury, Private Bag, Christchurch.

NZMS VISITING LECTURER 1982

I would be grateful for any suggestions pertaining to the Visiting Lecturer scheme. Particularly welcome will be suggestions on who might be suitable as the next Visiting Lecturer, together with a current address.

David Gauld, University of Auckland.

THE BNZ SENIOR MATHEMATICS COMPETITION

The Canterbury Mathematical Association conducted this competition for 6th and 7th form students again this year, and was able to persuade the Bank of New Zealand to undertake the role of major sponsor.

The 20 finalists assembled at the University of Canterbury on 30th October. The winners were:

First:	Graham Coop	Auckland Grammar School
Second:	Howard Wong-Toi	Auckland Grammar School
Third:	Andrew Coleman	Christchurch Boys' High School

In the evening the prizes were presented by the General Manager of the Christchurch branch of BNZ, and the audience was entertained and informed by Dr. Brent Wilson who introduced group theory using the Rubik cube. He concluded by solving the cube behind his back, to great applause.

An innovation this year was the issuing of certificates to the "top hundred" entrants on the preliminary round scores. The distribution of these is interesting: Auckland achieved 40 certificates, with eight schools represented, and Auckland Grammar taking 23. Canterbury on the other hand took 38 certificates, with 19 schools represented, no school taking more than five. Otago, Wellington and Hawkes Bay obtained seven certificates each, and the other regions 17 between them. Seventeen certificates were awarded to girls. These certificates should give a target for a wider range of entrants.

D.F.R.

SECOND AUSTRALASIAN MATHEMATICS CONVENTION

University of Sydney, 11-15 May 1981.

There was a good representation of research and teaching institutions of all kinds from secondary level up throughout Australasia, the South Pacific and a few other countries. Seventy-two New Zealanders, including at least 22 school teachers, crossed the Tasman to swell the number of participants to about 450. The main programme took place in the Carslaw building of the University of Sydney, with accommodation available in two on-campus colleges - Women's College (fairly modern, with tight security) and St Paul's College (less security, with an ancient dining hall block).

Mathematical programme: There were eight invited lectures, given by Professor C.T.C. Wall (Liverpool) on *Singularities of smooth mappings* [the New Zealand Mathematical Society Lecture - see NZMS *Newsletter* No. 21 for an edited version of his talk], Professor B. Carter (Observatoire de Paris) on *The uniqueness theorem for black hole equilibrium states*, Professor S. Papert (MIT) on *Topics in mindstorms*, Dr B. Mandelbrot (IBM, NY) on *Fractals, Kleinian groups and iterates of rational functions*, Professor J.T. Stuart (London) on *Hydrodynamic stability, transition and turbulence*, Professor G. Brown (NSW) on *Commutative harmonic analysis*, Professor C. Curtis (Oregon) on *Hecké algebras*, and Dr T.C. Kuo (Sydney) on *Algebraic geometry and singularity theory*.

About 130 contributed papers, including at least 42 by New Zealanders, were read in twelve specialist sections: foundations and history, number theory, algebra and combinatorial theory, analysis, harmonic analysis, geometry and topology, applied mathematics, probability and statistics, computer science and numerical analysis, mathematical education, relativity, and mathematical modelling. Comparing the contributions to each section, mathematical education had the greatest number with analysis next. A noteworthy emphasis at this conference was the importance of modelling and problem solving in the teaching of mathematics, using computers wherever possible.

Social events: An official welcome and sherry party was held in the Great Hall of the University, a magnificent venue, and the conference dinner was held in Women's College. The dinner will remain in the memories of some Kiwis for at least two reasons: (i) the plentiful supply of superb wine, and (ii) the group of New Zealand warriors who sang songs to the accompaniment of a Kiwi pianist. (Who could forget the Editor of this *Newsletter* chanting "Ka mate, ka mate, ..." in the hallowed shrines of Women's College?)

Business meetings: The New Zealand Mathematical Society and the Australian Mathematical Society held their Council meetings and their Annual General Meetings at the conference. The minutes of the NZMS meetings have been printed in the August *Newsletter*. The Councils of the two socieities met informally over lunch.

Success: The Second Australasian Mathematics Convention can be regarded as a great success and a stimulus to mathematics in Australasia, thanks to the excellent organisation and hospitality, and the high calibre of many of the talks.

Next Convention: This will be held in 1985 in Australia, probably in Melbourne. PLAN TO ATTEND IT.

Dean Halford

AUSTRALIAN NEWS

The First Australian Mathematical Society Medal has been awarded to Professor Neil Sidney Trudinger F.A.A. The medal is awarded, normally not more often than every 2 years, to an Australian mathematician not older than forty for his/her contribution to research. Trudinger was appointed Professor of Pure Mathematics at the Australian National University (School of General Studies) at the age of 31 and was elected a fellow of the Australian Academy of Science at 35. His research is in partial differential equations.

Derek Robinson and Leon Simon have been appointed Professors of Mathematics at the Institute of Advanced Studies at the Australian National University.

The proceedings of the 1982 Australian Mathematical Society Applied Mathematics Conference is to be published in the Bulletin of the Australian Mathematical Society. The published Proceedings should include all invited lectures and some of the contributed papers.

Sid Morris.

Local News

AUCKLAND UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE

Dr. Richard Lobb was promoted to a Senior Lecturership to take effect from February 1982.

At the conclusion of an Honorary Visiting Professorship in Auckland, Dr. Graeme Cooper returned to his position at the University of Sussex. While in Auckland for five months until the beginning of August he was carrying out research in Numerical Methods for Ordinary Differential Equations.

Another Numerical Analyst, Dr. Ray Zahar of the Université de Montréal, took up an Honorary Visiting Professorship at the beginning of October. He expects to be with the Department until March 1982.

Dr. Kees Dekker, our Post-doctoral Fellow in Numerical Analysis, returned to the Netherlands after one year with the Department.

Mr. Neil Binnie of Long Bay College and Mr. Alasdair MacLean of Howick College were appointed Visiting teaching Fellows to the department for the 1982 teaching year.

Mr. Ross Gaspard was appointed to the position of Technician/Programmer within the department.

A Continuing Education course for teachers of the new 6th form Computer Studies course will be held late in November. This will repeat the successful course held early in the year and will make use of the Department's microcomputer laboratory.

Seminars:

Prof. R. Mathon (Toronto University) "Boundary approximation methods for the Helmholtz equation" and "On linked arrays of pairs"
Prof. E.M. Luks (Bucknell University) "Isomorphism of graphs of bounded valence can be tested in polynomial time"
Prof. J.J.H. Miller (Dublin University) "Uniform numerical methods for stiff problems"
Dr. E. Schuegraf (Ruakura Research Station) "Information retrieval and associative processing"

As well as the above seminars given by visitors to the department during the year, a regular series of Masters seminars were held and both staff and Masters students participated in these. Finally, several members of the Mathematics and Computer Science Departments held a specialist series of seminars in Numerical Analysis, mainly dealing with stability problems in ordinary differential equations.

H.S.

DEPARTMENT OF THEORETICAL AND APPLIED MECHANICS

Visitors to the Department this year have included Dr. Rodney Stephenson, a past student who served for a month on a temporary lectureship before taking up a position at Carnegie-Mellon Institute at Pittsburgh; Dr. Richard Rosenthal, an Associate Professor of Management Science, University of Tennessee in Knoxville; and Professor Les Woods, a regular visitor to his home country from Oxford University.

Seminars:

Dr. Richard Rosenthal (University of Tennessee) "Interactive computer graphical solutions of constrained minimax location problems"
Don Johnston (M.Phil. student) "James-Stein estimators and applications"
Chris Moore (M.W.D., Wellington) "Major projects: employment and other impacts"
Dr. R.B. Leipnik (University of California, Santa Barbara) "Doubly strange attractors in rigid body motion with linear feedback control"
Professor L.C. Woods (Oxford University) "Non-linear heat conduction in a fluid" and "The bogus axioms of continuum mechanics"
Professor I.F. Collins (Head of Department) "The structure of variational principles" (two sessions)
Dr. D.E. White (U.S. Geological Survey) "Natural geothermal activity in the U.S. and elsewhere, with examples of human influence"
Jonathan Taylor (Ph.D. student) "An integral equation formulation in 3-D elasticity"

M.S.R.

DEPARTMENT OF MATHEMATICS

Dr. Ivan L. Reilly promoted to Associate-Professor. Appointment to take effect from February 1982.

Emeritus Professor Henry J. Forder died on 21 September 1981 aged 92 years. Even though Professor Forder retired from the headship of the department in 1956, he kept in touch with the department and took an active part in its work for many further years. He was a major benefactor to the University especially through his gifts to the library.

Junior Lecturer Gaven Martin, has departed to the University of Michigan to undertake a Ph.D.

Two Lecturership appointments are currently being advertised.

Dr. M. Mrsević, Post-Doctoral Fellow, University of Belgrade arrived in September to take up a 12 month appointment.

Professor Tim Holt returned to the University of Southampton in August.

Drs. Calvert and Dixit returned from leave taken in terms I and II.

Other visitors in the third term have been Professor P. Lambrinos, University of Thrace; Professor E. Cockayne, Victoria University, Canada; Professor H. Morin, University of Laval, Quebec.

Another Z89 Computer has been installed in the department, making a total of four. Academic and office staff are encouraged to familiarise themselves with those machines.

The Mathematics and Computer Science Departments held their Annual Dinner at the Berkeley Lounge, Mission Bay, on Monday, 21 October 1981.

Drs. Smith and Schiff have both been blessed with the arrival of baby daughters in July and October respectively.

Seminars:

Prof. J.N. Crossley (Monash) "How concrete is abstract algebra"
Dr. P. McInerney (Auckland) "Rubik's Cube"
Dr. C.J. Wild (Auckland) 'Unconstrained optimization for statisticians"
Tony Cooper (D.S.I.R.) "Linear regression with biased samples"
Michael Kallay (Hebrew University, Uerusalem) "Algorithms for computing the convex hull of a set of points"
Dr. Marston Condor (Post-doctoral Fellow, University of Otago) "Presentation of Groups"
Assoc. Prof. B. Weir (North Carolina State University) "The statistical analysis of genetical data"
Prof. J.R. Landis (University of Michigan) design effects for differences of proportions and logits"
Drs. Jörg Siekmann and Graham Wrightson (Institute für Informatik, University of Karlsruhe) "State of the art in theorem proving" and "Stage of the art in unification theory"
Prof. L.C. Woods (University of Oxford) "Logical positivism, a threat to applied mathematics"
Dr. L. Kaiser (Auckland) "Line intercept sampling"
Prof. H. Morin (Laval University) "Some practical aspects of survey sampling"
Dr. C.M. Triggs, Applied Maths Division (D.S.I.R.) "A comparison of some algorithms for the analysis of variance"

Seminars in Combinatorics:

Prof. E. Cockayne (Victoria University, Canad) "Multimessage broadcasting in the complete graph" and "Rotation numbers in graphs"
Rueben Sandler "Almost commuting permutations"
Dr. D. Ryan (Auckland) "Perfect and balanced graphs and their applications"

E.D.

D.S.I.R.

Roy Leipnik (Santa Barbara) leaves for Australia in November after completing his Sabbatical at A.M.D. Elizabeth Bradford attended the conference on Numerical Solution of P.D.E.'s in Melbourne, and visited D.M.S., Canberra. Bruce Bensemann attended the International Federation of O.R. Societies in Hamburg, and visited O.R. groups in the U.K. Vicky Mabin has returned from Lancaster to the O.R. section after completing her Ph.D. thesis entitled "Variability in Distribution". Hugh Barr has been appointed to the Social Sciences Subcommittee of N.R.A.C., and is interested to hear of scientific needs in the social sciences. Graeme Edwards has constructed a queueing simulation model of international arrivals at Auckland Airport, and helped reduce passenger waiting time by 25%. Graeme is now operating A.M.D's Auckland O.R. consultancy.

G.W.

DEPARTMENT OF COMMUNITY HEALTH

An Apple Users Group has sprung up at Auckland University with about 25 members. This is more or less centred at the Department of Community Health, and several of its members are interested in software for mathematical education (at all levels, from earliest primary to postgraduate). Anyone who is interested, or may have some contribution to make, can contact Peter Mullins at the Dept. of Community Health in the first instance. There is also great interest in statistical software and games.

Seminars:

Peter Mullin gave a talk entitled "Estimating Relative Risk"

Under the joint auspices of the Mathematics Department and the Dept. of Community Health, Bruce Weir (from North Carolina State University at Raleigh) gave a talk on "The Statistical Analysis of genetical data" and
Dick Landis (from the University of Michigan, Ann Arbor) gave a talk entitled "Design Effects for differences of proportions and logits".

WAIKATO UNIVERSITY

DEPARTMENT OF MATHEMATICS

We still exist! Our honorary correspondent Mark Schroder left in haste last January (1981) on cue from the ILR and didn't appoint a replacement scribe. Thus the onerous task of making out that we are every bit as good as the other New Zealand departments (i.e. creative, brilliant, hardworking and equal to Auckland but nicer) falls to this humble writer.

To a degree, 1981 has been a good year for the department. With RJH and JCT both on leave it was possible for some of the younger members (read middle aged) to reveal their considerable administrative and political skills. For instance we successfully avoided a confrontation with the faculty over the social science statistics requirements, something Jay-Cee was never able to do. It must be said in his defence, however, that he never managed to lose. RJH continued his two year secondment at the S.E. Asian Institute of Technology in Bangkok without his section losing a staff member. He is due back early 1982. Jay-Cee left for North America last May with corrections to the NZMS Statistics publication chasing him off the tarmac. He is now at Warwick. FTM(ark)S attended the Australian Summer Research Institute in Hobart and then headed to Mannheim in West Germany where (among other activities which also will interest you) he bought a camera with an automatic focus facility (jealous?). We have been treated to a regular diet of excellent shots. He is now in Ontario at Queen's University and expected back 365+1 days after leaving. EK was off in August after an addition to his family in July. He went straight to Minnesota and to the indefatigible Willard Miller. Their combined output could raise the figure for New Zealand Productivity a full 1%.

We survived our second year in this noisy/hot/cold/temporary wall to a car park. Present thinking is that we will be here for 10 years. It is infinitely better than nothing (our previous state) but (being a pure mathematician at heart) I wonder just how good that is. Great things are about to happen in the computing area. We have at last convinced the computer committee that we should have an 8 terminal calculating laboratory and convinced the University to buy the symbolic language MACSYMA. These wonders should be on tap to show visiting sceptics (and/or Aucklanders) by early 1982.

Seminars:

Dr. Michael Stenzel (University of Hamburg) "The embedding of topological rings into quotient rings."
Kevin Broughan (University of Waikato) "Analysis for the thoughtful engineer."
Terry Wall (University of Liverpool) "Cubic curves and groups," and "Singularities of caustic curves and surfaces."
Mark Gould (University of Waikato) "Lie Group theory in physics."
Greg Reid (University of Waikato) "Separation of variables."
Sandy McClymont (University of Waikato) "The stability of radiating, conducting plasma in the solar atmosphere."
Ivar Stakgold (University of Delaware) "Classical and modern optimisation" and "Methods of nonlinear analysis."
John Crossley (Monash University) "Computing isomorphisms."
Bruce Weir (North Carolina State Un iversity) "Statistical Analysis of genetic data."
A. Zulauf (University of Waikato) "Solution of a functional equation by use of weighted arithmetic-geometric means."
Paul Lambrimos (Democritus University) "Function spaces and convenient categories of topological spaces."

Douglas Bridges (University College at Buckingham) "Recent advances in constructive mathematics"
Vidar Thomée (Chalmers University of Technology) "Gallerkin finite element methods for elliptic and parabolic problems."

There is an active statistics group in the region consisting of people from Ruakura (Ministry of Agriculture and Fisheries, Meat Research Institute) and this department. They sponsored a successful series of seminars followed by banqueting and revelry. The seminars were as set out below:

WAIKATO STATISTICS GROUP

The group sponsored the above talk by Bruce Weir and also the following:

John Maindonald (DSIR, Auckland) "Some thoughts arising from consulting problems."
Shayle Searle (Cornel University) "The cell-means form of the linear model: a research workers friend."
Tim Ball (DSIR, Wellington) "Pacific Island dental survey."
Francis Cockrem (Ministry of Agriculture and Fisheries) "Philosophy of biometrics derived from Fisher."

K.A.B.

MANAWATU STATISTICS GROUP

The Manawatu Statistics Group got away to a rather sluggish start this year, with no meetings held in the first six months. The action has picked up for the second half of the year with three meetings being held:

Alastair Scott (Auckland) "The Effect of Cluster Sampling on Chi-Squared, and Other Tests"
John Revfeim (Meteorological Service) "Muddles, Middles and Models"
Hugh Morton "Muscle Fibre Size and Malignant Hyperpyrexia"

R.J.B.

MASSEY UNIVERSITY

Kee Teo recently returned from eight months leave in Singapore, where he furthered his research on transformation groups in Lie Algebras, and developed an interest in lattice theory. Most of his time was spent at the National University of Singapore, though he also visited the University of Malaya and Tunku Abdul Rahman College in Kuala Lumpur. Brian Hayman took a short period of leave to attend a conference on distance teaching held in Fiji; this is of course a matter with which Massey is particularly concerned. Ken Palmer has left us to take up an appointment at the University of Miami. Our congratulations go to Terry Moore on being awarded his Ph.D. for a thesis entitled "Efficient Biased Estimation and Application to Linear Models". This is a study of estimators of James-Stein type which are more efficient than the usual estimators when there are more than three parameters.

Douglas Bridges from the University College at Buckingham is visiting us for the summer; he is busy preparing a revision of Errett Bishop's book on the foundations of constructive analysis, but hopes to fit in some touring as well. His many friends here are delighted to see him again, and the fact that his wife hails from Palmerston North gives us cause to hope that this visit will not be the last!

Seminars:

Tony Cooper (AMD, DSIR, Auckland) "Linear Regression with Biased Sampling"
Prof. Ivar Stakgold (University of Delaware, NZMS Visiting Lecturer) "Methods of Nonlinear Analysis" and "Classical and Modern Optimization"
Dr. H.R.H. Reuvers (Netherlands) "Introduction to Galois Theory"
J. Sekhon (N.S.W. Institute of Technology) "New Responsibilities in the Mathematical Education of Engineers"
Dr. Bruce Weir (University of North Carolina) "Statistical Analysis of Genetic Data"
Gordon Knight "Readability in Mathematics"
Pak Yoong "Design and Production of a Multi-Media Learning Package; an Application in First-Year Statistics"
Mike Hendy "Problems Arising from Evolutionary Tree Construction"
Dr. Douglas Bridges (University College at Buckingham) "The Mathematical Description of Consumer Preferences"
Prof. R. Leipnik (University of Southern California) "Double Strange Attractors"

M.R.C.

VICTORIA UNIVERSITY OF WELLINGTON

Andrew Lacy's Postdoctoral Fellowship has been extended until April 1982, after which he will return to Oxford with a Fellowship from the Royal Commission for the 1851 Exhibition.

Applications are being considered at the time of writing for the vacant half-Lectureship in Pure Mathematics; the Department will then have three permanent half-time positions. The other two are occupied by Shirley and Ken Pledger, both of whom recommend the arrangement.

Carl Spencer is a Postdoctoral Fellow in the Institute of Geophysics, having come from Cambridge, England, to work with Jim Ansell on seismology where there are some earthquakes to study.

David Vere-Jones has gone to Rome to give a paper at a conference on earthquake statistics (they are trying to get it called "shakistics"!)

Rob Goldblatt has gone to Singapore to give a paper at the 1st S.E. Asian Logic Conference.

Brian Dawkins has returned from sabbatical leave in London; the intellectual life was good but the cost of living was awful.

Our first-year courses do not satisfy everybody; a subcommittee is busy with alternative syllabuses. The University is also considering alternative arrangements for teaching computing.

Jim Ansell is President of the N.Z. Geophysics Society (and of course incoming Vice-President of NZMS), and also the campaign manager for one of the Island Bay parliamentary candidates.

John Harper is on the N.Z. National Committee for the Lithosphere and the Council of the Wellington Branch of the Royal Society of N.Z.

J.F.H.

CANTERBURY UNIVERSITY

Peter Waylen left in August, on study leave. He will spend most of his time at Cambridge. Bob Broughton is also on study leave, but plans to spend several months in Christchurch. Others due to go on study leave in the near future are Brent Wilson and Graham Wood.

Seminars:

Dr. P. Labrinos (University of Thrace) "Function Spaces and Convenient Categories of Topological Spaces"
Dr. Doug Pitney (Okanagan College, B.C.) "Remedial Mathematics"
Dr. Tadeusz Balaban (University of Warsaw) "Some Mathematical Problems in Statistical Mechanics"

The departmental social committee organised a birthday party for Gordon Petersen at the Cloisters restaurant in the Student Union. The department gave him a set of cups for green tea, and Mr. Muldoon added him to his list of beneficiaries.

A team led by Brent Wilson were flown up to Wellington on election night to analyse voting results as they were announced on radio. Their predictions were better and faster than those of the political scientists and computer programmers on television.

R.S.L.

UNIVERSITY OF OTAGO

Ms Petronella de Roos has been appointed as Assistant Lecturer for 2 years.

Mr. B.F.J. Manly has been promoted to Associate Professor and Mr. J.C.W. Rayner to Senior Lecturer.

Dr. J. Clark will be on leave from September 1981 to May 1982 and will be spending it at both the University of Aberdeen and the University of London.

Dr. M.J. Curran will be on leave during 1982 and plans to spend it at both Oxford University and the University of Notre Dame.

Dr. G. Olive will be on leave for the 1st term of 1982 and plans to visit and give talks at a variety of Institutions (e.g. M.I.T., University of Reading, Oxford University, University of Paris, University of Oregon, California Institute of Technology, Flinders University, University of New England, and University of Newcastle).

Professor Ivar Stakgold, the 1981 NZMS Visiting Lecturer, visited us on July 29. He gave a seminar (listed below) in the afternoon and a talk to High School Teachers and Students on "Optimisation" in the evening.

Seminats:

Mr. W. Schaap (Mineral Technology Department) "Mathematical Statistics and Renewal Theory"
Professor I. Stakgold (University of Delaware) "Bifurcation Theory"
Dr. M.D.E. Conder (our Postgraduate Fellow) "Presentations of Groups"
Dr. J.H. Harris "Existential Claims in Physical Theories"

G.O.

Conferences

*** 1981 ***

December 3-5 (Urbana, Illinois)	*Twenty-fourth Meeting of the Society for Natural Philosophy* Program: Applications of qualitative analysis to nonlinear mechanics. Details from R. G. Muncaster, Department of Mathematics, 1409 West Green Street, University of Illinois, Urbana, Illinois, 51801, U.S.A.
December 11-13 (Victoria, British Columbia)	*Annual Meeting of the Canadian Mathematical Society* Details from C. Robert Miers, Department of Mathematics, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2.
December 14-18 (Gainesville, Florida)	*NSF-CBMS Conference on Bifurcation and Symmetry Breaking* Details from P. J. McKenna, Department of Mathematics, 201 Walker Hall, University of Florida, Gainesville, Florida 32611, U.S.A.
December 21-25 (Madras)	*International Conference on Recent Advances in Uonlinear Analysis and Differential Equations* Details from K. M. Das, Department of Mathematics, Indian Institute of Technology, Madras 600036, India.

*** 1982 ***

January 11-16 (Mexico)	*Fourth International Conference on Universal Algebra and Lattice Theory* Details from Octavio C. Garcia, Institute de Matemáticas, Universidad Nacional Autonoma de Mexico, Ciudad Universitária-Circuito Exterior, México 20, D.F., Mexico.
March 15-19 (Dublin)	*Seventh International Time Series Meeting* Details from Oliver D. Anderson, ITSM Dublin, 9 Ingham Grove, Lenton Gardens, Nottingham NG7 2LQ, England.
March 29 -April 2 (Dundee)	*Seventh Dundee International Conference on Ordinary and Partial Differential Equations* Details from E. R. Dawson, Department of Mathematics, The University, Dundee DD1 4HN, Scotland.
March 30 - April 3 (Bangor)	*Thirty-fourth British Mathematical Colloquium* Details from D. J. Wright, School of Mathematics and Computer Science, University College of North Wales, Bangor, Gwynedd, U.K.
April 22-23 (Pittsburgh, Pennsylvania)	*Thirteenth Annual Pittsburgh Conference on Modeling and Simulation* Details from William G. Vogt, Modeling and Simulation Conference, 348 Benedum Engineering Hall, University of Pittsburgh, Pittsburgh, Pennsylvania 15261, U.S.A.
April 26-28 (Raleigh, North Carolina)	*SIAM Special Conference on Linear Algebra and Applications* Details from Hugh B. Hair, Services Manager, Society for Industrial and Applied Mathematics, 1405 Architects Building, 117 South 17th Street, Philadelphia, Pennsylvania 19103, U.S.A.
April 26-30 (Iowa City, Iowa)	*NSF-CBMS Conference on Automorphism Groups of von Neumann Algebras and the Structure of Factors* Details from Paul Muhly, Department of Mathematics, The University of Iowa, Iowa City, Iowa 52242, U.S.A.
May 5-7 (San Francisco, California)	*Fourteenth Annual ACM Symposium on the Theory of Computing* Details from Walter A. Burkhard, Publicity Chairman, SIGACT-82 Symposium, University of California, San Diego, La Jolla, California 92093, U.S.A.
May 17-19 (Dunedin)	*Seventeenth New Zealand Mathematics Colloquium* Details from Colloquium Secretaries, Department of Mathematics, University of Otago, P.O. Box 56, Dunedin, New Zealand.
May 24-28 (Valencia)	*International Forecasting Conference* Details from Oliver Anderson, IFC Spain, 9 Ingham Grove, Lenton Gardens, Nottingham NG7 2LQ, England.
May 10-12 (Armidale, N.S.W)	*Simulation Society of Australia 5th Biennial Conference* Details from Dr. I.H. Fisher, SSA Conference 1982, Department of Resource Engineering, University of New England, Armidale, N.S.W 2351.

June 21-25 (Les Arcs, France) — *IEEE International Symposium on Information Theory*
Details from Bernard Picinbono, Laboratoire des Signaux & Systemes, École Superieure d'Électricité, Plateau de Moulon, F91190, Gif-sur-Yvette, France.

June 21-25 (Ithaca, New York) — *Ninth U.S. National Congress of Applied Mechanics*
Details from Y. H. Pao, Department of Theoretical and Applied Mechanics, Thurston Hall, Cornell University, Ithaca, New York 14853, U.S.A.

June 28 - July 2 (Clermont-Ferrant, France) — *Eleventh Conference on Stochastic Processes and their Applications*
Details from P. L. Hennequin, Université de Clermont-Ferrand II, Complexe Universitaire des Cezeaux, Départment de Mathematiques Appliquées, B.P. n. 45, 63170 Aubière, France.

June 28- July 3 (Las Palmas, Canary Islands) — *Second World Conference on Mathematics at the Service of Man*
Details from Second World Conference on Mathematics at the Service of Man, Universidad Politécnica de Las Palmas, Casa de Colón, Herrerias 1, Las Palmas de Gran Canaria, Canary Islands, Spain.

August 8-13 (Montreal) — *Tenth IMACS World Congress on Systems Simulation and Scientific Computation*
Details from S. Sankar, Tenth IMACS Congress Chairman, Department of Mechanical Engineering, H 929-12, Concordia University, 1455 Maisonneuve Boulevard West, Montreal, Canada H3G 1M8.

August 9-13 (Sheffield) — *First International Conference on Teaching Statistics*
Details from ICOTS Secretary, Department of Probability and Statistics, The University, Sheffield 853 7RH, England.

August 11-19 (Warsaw) — *International Congress of Mathematicians*
Details from Czeslaw Olech, Institute of Mathematics, Polish Academy of Sciences, Sniadeckich 8, P.O. Box 137, 00-950 Warszawa, Poland.

August 19-27 (Blazejewko, Poland) — *Eighth Conference on Analytic Functions*
Details from Julian Lawrynowicz, Instytut Matematyczny PAN, Oddzial W Lodzi, ul. Kilinskiego 86, PL-90-012 Lodz, Poland.

August 23-27 (Bonn) — *XI International Symposium on Mathematical Programming*
Details from Math. Progr. Secretariat, c/- Institute for Operations Research, Nassestrasse 2, D-5300 Bonn 1, West Germany.

August 19-21 (Cincinnati) — *3rd American Time Series Meeting (8th ITSM)*
Details from Oliver Anderson, 9 Ingham Grove, Lenton Gardens, Nottingham NG7 2LQ, England.

M.R.C.

TWENTY-SIXTH ANNUAL MEETING AUSTRALIAN MATHEMATICAL SOCIETY

The above meeting will be held at the University of Newcastle from 10-14 May, 1982. Further information can be obtained from
Dr. J.G. Couper, Secretary, A.M.S. Meeting, Department of Mathematics,
The University of Newcastle, New South Wales, 2308, Australia.

The invited speakers include
Prof. F.F. Bonsall, FRS, (Functional Analysis), University of Edinburgh, Scotland.
Prof. J.E. Cohen, (Population Modelling), Rockefeller University, New York.
Prof. J.N. Darroch, (Statistics), Flinders University, Sth. Aust.
Prof. T.W. Hawkins, (History of Mathematics), Boston University, Massachusetts.
Prof. G.J. Simmons, (Cryptography, Combinatorics, Sandia Laboratories, Albuquerque, New Mexico.

A HUNDRED YEARS OF ALGEBRA 1830-1930

The LMS two-day meeting in Oxford, 16-17 April 1982, is a special one concerned with the development of algebra and number theory in the nineteenth and twentieth centuries. The speakers will be H. M. Edwards, T. Hawkins, W. Ledermann, R. Taton and B. L. van der Waerden. Accommodation will be available in Queen's College.

For information contact: The Administrative Assistant, London Mathematical Society, Burlington House, Piccadilly, London W1V ONL, England.

Feature Article

UNIVERSITY ENTRANCE MATHEMATICS: WHERE NEXT?

A paper on 6th and 7th Forms developments from the Chairman of the Mathematics Steering Committee, Professor David Vere-Jones.

During the last month the Entrance Board's Steering Committee for Mathematics has sent out a series of requests for information and advice. These requests have gone, not only to University Mathematics Departments, but also to Technical Institutes, University Faculties, the Mathematics Associations, and a number of professional societies. The main prompt for this activity has been the growing hope that, whatever difficulties remain on the political side, the Entrance Board and the Department of Education are finally coming closer to settling their differences over moves to allow the main determination of University Entrance to be shifted from sixth to the seventh forms. At least the signs are sufficiently encouraging to suggest to the Steering Committee that it would do well to consider what changes might be needed in the Bursaries syllabus if the Bursaries Examination were to take on the dual rôle of providing the basis for matriculation as well as for the award of bursaries.

In fact the Steering Committee has had a major reappraisal of the 7th Form syllabuses in view since 1977, when it suggested a two-stage approach to this problem: a series of minor revisions to remove the most acute sources of complaint, and a more thorough-going review in the time bought for that purpose by the first stage. The revisions representing the first stage went into operation in the schools this year, leaving in their wake the salutary lesson that even minor changes can take up to three years to pass through the system of checks, references and approvals required by the Entrance Board and the Education Department together. Even if a formal change to seventh form UE is still some years in the future, it may still not be too early to start considering the syllabus changes implied by such a move. And even without a formal change, the increasing numbers of University courses with restricted entry, which make use of the bursary results as one component in the selection of students, means that the bursary examination is coming to play an increasingly important de facto rôle as a determinant of University Entrance.

The need for revision of the mathematics syllabuses is particularly acute, not only in terms of syllabus content, but also in terms of the structure of the mathematics course. *Should the Bursaries Applied Mathematics syllabus continue to have its three disparate options? Should there, indeed, continue to be an Applied Mathematics course? Should mathematics contract to a single subject, to allow for the development of a computing paper? If there are to be two mathematics papers, what should be the basis of the distinction between them?*

The problems are no easier at the sixth form level, and are inextricably linked here to the wider question of the future of the UE examination. In July, 1980, the Steering Committee reached a position of complete deadlock on the proposals for a revised UE syllabus suggested by the Education Department's Working Party on 6th Form Mathematics. Its report to the September, 1980, Meeting of the Board, which was circulated to University Mathematics Departments last year and (I understand) will shortly be reproduced in the PPTA Journal, analyzed the reasons for this deadlock and traced it essentially to disagreement over the rôle of the UE examination. In broad terms, the teachers on the Committee felt that it was essential to move to a syllabus more in keeping with the interests and capabilities of the pupils actually confronting them in the classroom, while the University representatives pointed to the statutory rôle of the examination in determining a suitable level for University Entrance, and expressed concern that in fact the standards were already declining. The Committee applied to the Board for guidance; but no clear guidance was received. Rather, the Committee was assured that discussions with the Department were continuing, and that the Board would let the present syllabuses stand until the question of 7th Form entry had been resolved. There the matter rests, though many teachers at the 6th Form level face acute difficulties in teaching the present syllabus to classes which include many students who have only a poor understanding of the School Certificate programme, and little interest in the topics emphasized in the UE course.

The origin of these problems is to be found in the massive social changes that have affected the upper secondary schools in the last two decades. Starting in the early 1960's, and continuing through to the present time, there has been a steady increase in the proportion of the age cohort staying on at school into the sixth form and now into the seventh form as well. At the same time the number of girls taking University Entrance has caught up with and now slightly surpasses the number of boys. These are far from forming a purely local syndrome, as similar changes have taken place in most Western countries as well as the Soviet Union. Simultaneously there has been an increase in the proportion of upper school students taking mathematics as one of their subjects. The effects are dramatically illustrated by the following table, extracted from the Steering Committee's report referred to earlier, and showing the numbers of University Entrance candidates offering mathematics as one of their subjects.

Year	Maths Entry	Total Entry	% taking Maths	% Pass
1970	13,151	22,602	58.0	58.4
1971	12,950	23,605	54.9	58.2
1972	10,395	24,572	42.3	58.2
1973	14,971	24,840	60.0	58.1
1974	17,889	24,829	72.1	58.8
1975	19,688	26,143	75.3	58.7
1976	21,944	28,329	77.5	58.8
1977	23,773	30,085	79.0	58.6
1978	25,786	31,901	80.8	58.4
1979	26,893	32,944	81.6	58.5

It is evident that the numbers taking mathematics have increased enormously over the last decade. Despite these changes the Board's policy has been to keep the pass rate constant, while syllabus committees have increased rather than decreased the range of material covered by the examination. Small wonder, then, if a situation of acute tension has arisen in the 6th Form arena. In my opinion, in fact, any decline in the actual performance of students coming on to University is more likely to have its source in these problems than in the merits or demerits of the "New Maths" as against a "Back to Basics" programme. Not that the changes have a purely negative character. On the positive side, they represent a massive raising of the general educational (and, in particular, mathematical) aims and expectations of the majority of the children in New Zealand schools. On the negative side, however, a distressing gap has opened up between the aims, content, and nominal level of the UE syllabus, and the actual performance of the majority of pupils leaving the sixth form with the right to embark on a programme of University study.

Of course it would be idle to suppose that such fundamental difficulties would disappear simply because the Entrance qualifying examination were to be shifted from sixth to seventh form. The sixth form will remain a problem area, catering for students with widely differing interests, abilities, and motivations. Such a move would, however, have the great advantage of freeing the sixth form programme from what has become an increasingly cramping and unrealistic link with the requirements of University study. Moreover there may be a better chance that at the seventh form level the question of University Entrance can be looked at within a more limited framework - not so much determining the whole programme in the sixth and seventh forms, as appending to that programme an additional machinery which can be used to discover those students who have attained a level of preparation appropriate to their commencing a University course.

The period which covered the social changes just referred to has also covered equally if not more far reaching changes within the body of mathematics, and especially in its relations with other disciplines. In the forefront of these changes stands the many-headed Medusa, the Computer. A servant of mathematics perhaps, but a servant whose employment alters the whole traditional pattern of life within the household that employs her. Its advent poses deep questions as to what constitutes mathematics, and especially as to what constitutes that mathematics which is appropriately taught within the school programme. It is evident, for example, that many pupils with mathematical aptitude are looking towards computing, or computer-related work, rather than to mathematics in its traditional guises, as the context of their future activities. Should their requirements be catered for within or outside the mathematics programme? And what are their requirements, in any case? Few of us have any clear idea of what sort of work such pupils will actually end up doing, or what sort of quantitative training will stand them in best stead for the future.

Closely allied with the growth of computing is the increasing quantification of almost all branches of knowledge, and surely of all branches of science and technology. The collection, manipulation, summary and interpretation of large data sets is a mass occupation in government, business, science and industry. Everywhere, information is stored and represented *quantitatively*, by numbers, tables, graphs, indices. What are the implications of these developments for the school programme?

One point at least is clear. The pupils taking mathematics at sixth and seventh form levels have, and will continue to have, a much wider range of interests than was ever previously the case. The numbers heading towards the classical application of mathematics - the hard sciences and engineering - will be, if they are not already, surpassed by those proceeding to quite different areas of study - computing, all branches of commerce and technology, the social and biological sciences, etc. etc. A corollary of this conclusion is that traditional views should not be taken for granted in developing new syllabuses. There is a considerable need for more factual information about just what mathematics is really needed by workers in this wider arena where quantitative methods have come to play a more dominant rôle; and not by the research leaders, but by the general run of those who assist, support and administer. It is possible, for example, that the place of calculus is overvalued in the current programme. Its introduction into the 6th Form programme followed a time when calculus-based techniques were at their very zenith, when the future seemed to be all with physics, engineering, optimization; the analytic approach to quantitative problems. More recent history has seen a splitting up of this monolithic vision; the computer and its packages handle by brute force what the mathematician cannot solve explicitly on paper. The mathematician's rôle is transformed to that of planner, organizer and interpreter, more often in the realm of discrete steps rather than continuous transformations.

Interesting in this regard are the results of a preliminary survey of the actual mathematical concepts made use of in first-year university courses

Summary Results from Questionnaire

Topic (In order of % of departments using it)	% of depts using the topic: Occasional Use		Essential Use	Total Use	% satisfied (of those (using the topic
Basic Arithmetic (including fractions)	50%	+	50%	100%	55%
Graphs	50%	+	45%	95%	25%
Tables	50%	+	30%	80%	60%
Elementary Statistics	45%	+	25%	70%	0%
Histograms	35%	+	25%	60%	45%
Signs	30%	+	25%	55%	55%
Brackets	20%	+	35%	55%	40%
Equations	20%	+	30%	50%	30%
Linear Algebra	20%	+	25%	45%	20%
Σ notation	20%	+	15%	35%	20%
Differential Calculus	20%	+	10%	30%	20%
Trigonometry (functions)	15%	+	15%	30%	0%
Trigonometry (triangles)	15%	+	10%	25%	30%
Integral Calculus	10%	+	5%	15%	50%

Only the hard-core sciences and economics make any use of calculus in their first year courses. On the other hand there is an almost universal demand for, and very little satisfaction with, the ability to handle and interpret graphs and the most elementary statistical concepts. These are basically School Certificate topics, but if one were to add to these two some hands-on experience with a minicomputer, one might have the starting point for an upper school programme that was of interest and relevance to the majority of pupils, while the present programme, at both sixth and seventh form levels, was seen to be of interest to an important but relatively limited group.

The major structural alternatives for mathematics in the seventh form are relatively few in number; in fact they reduce essentially to three:-

1. A single subject, "Mathematics";
2. Two subjects, "Pure Mathematics" and "Applied Mathematics";
3. Two subjects, "Mathematics" and "Additional Mathematics".

When these possibilities were briefly considered by the Steering Committee at its most recent meeting, there was an unexpected degree of support for the first alternative. The main grounds were the extra opportunities this gave to pupils to pursue a range of other subjects; the avoidance of difficult choices; and the advantages to tertiary training of a common level of preparation from the schools. It was assumed in this discussion that computing would be left to develop separately. The Mathematics course would then consist of, perhaps, 30% statistics, an introduction to calculus and its applications at a similar level to the current sixth form programme, the remaining topics chosen from more traditional material on algebra, geometry and trigonometry.

The second alternative represents the status quo, with some evolution. The present trend is for greater independence of the applied maths courses from the parallel course in pure mathematics, and the most likely sequel of the present position would be to drop the mechanics section and make the statistics and computing sections self-contained. It could then cater for a much wider clientele than at present. Correspondingly the pure course could be more specifically directed towards the needs of the hard sciences and the mathematicians themselves. Here might anticipate a greater emphasis on applications, with some mechancis reappearing in the guise of applications of calculus.

The third alternative is perhaps the one which offers the best opportunities for exploring new combinations of material. It is possible that the question of calculus referred to earlier would be the best basis for the division of material. The main mathematics course would concentrate on algebra, geometry, and graphical methods, with a significant section on statistics and another on computing. The additional mathematics course would be aimed specifically at students moving into the engineering, hard sciences, economic areas, and might consist primarily of a course in calculus and applications, with some additional material by way of geometry, trigonometry, and perhaps numerical mathematics.

None of these are questions on which a clear view has been formed to date, and the Steering Committee would welcome comments, alternative views, and any expressions of preference.

To close with a few more personal comments, I suffer from a considerable disquiet over the great difficulty of obtaining any factual information on which to base the continuing sequence of syllabus changes. All of our syllabus decisions seem to be based on committee discussions where rival opinions meet, clash, and are bludgeoned into some kind of unwilling compromise. Committees are swayed by current fashions and beliefs, and there seems to be no point at which one can say "stop; here is a piece of hard evidence which cannot be ignored". On the contrary, one sees the syllabuses of present, past and future as a succession of passing fashions with no clear forward direction and no objective measures of achievement or improvement. Who can really tell, for example, whether the spiral theory much beloved in the school programme is harmful or beneficial to the inculcation of mathematical knowledge? I can readily produce convincing-sounding arguments both in favour and against this proposition, but would prefer some facts to the endless committee discusions. The same goes for almost all the myriad alternatives which presently confront us. The most constructive policies I can think of here are, firstly, to make sure we *do* gather the limited factual material which is available, particularly into the use made at later stages of particular mathematical topics. Secondly, I think it helps to spell out explicit aims, however banal these may sound, as a necessary part of providing a comprehensible legacy for those who follow; at least then one can say if the syllabus changes are needed because the aims have changed, or the given syllabus has not achieved its stated aims.

I am also becoming increasingly sceptical about the importance of syllabus *content*. Syllabus content has not changed all that much even over the last century or more. The binomial theorem; Cartesian geometry; the first beginnings of calculus; the properties of exponential and trigonemetric functions; simple work on sequences and series; even the elements of statistics or the theory of errors; most of these have been on the boundary of what is accessible within the school programme since last century, sometimes in, sometimes out; I find it hard to believe they will be in any very different situation over the centuries to come. What changes more significantly is the attitude towards these topics, the style in which they are taught, the context in which they appear, and the subjects to which they are related. The success with which such features can be reproduced in the classroom is due much more to the level of mathematical culture of the teachers than to the details of what is written in the syllabus prescription. A minimum formal syllabus, giving the teachers maximum scope to develop their own examples, special topics, applications, and cross-links, would come closest to my personal ideal. Instead of this we seem to be caught up in a current of the opposite direction in which everything has to be spelled out to the last detail on bits of paper, and the creative work of the teacher is reduced to an absolute minimum.

This tendency may be partly a New Zealand phenomenon, not unrelated to the fact that we have no less than three State examinations, not to mention a profusion of local certificates, in the last three years of secondary school. What terrible fate could befall us if we did without some of these bits of paper (which must become increasingly redundant as more pupils stay at school longer) and relied more on general common sense? Surely we could hardly be much worse off. Could we even (a totally rebellious thought fed to me by a most respectable colleague) *abolish* such sound institutions as the Entrance Board, subject convenors and all? That at least would be an end to my immediate problems, if not to those of the schools and their pupils!

As a mathematician, if I dare address that label to myself, one of the features that worries me most about the current school programme is the devaluation of the notion of proof. If statistics and computing come to play a more substantial rôle in the programme, there is every chance that this devaluation will be taken still further. Yet it is this very feature, not the fact that computers store numbers rather than colours or smells, that explains the universality of mathematics, and the spreading influence of quantitative methods over all branches of science and industry. It is precision of thought that gives mathematical methods their power. The proof of a theorem is a means of relating the result stated in that theorem to previous results. Our experience of the world suggests that there is no other way of describing the relationship between concepts which is as complete, as succinct, or as relevant to the way the world works. It is also because mathematics describes the relations between concepts, by way of proofs, that it is capable of being applied in such a wide variety of contexts, where the same relationships can be recognized, even though the objects satisfying these relationships are different. Somehow, this fundamental aspect of mathematics, its internal logical structure, needs more illumination in the school programme. It is, of course, present to some extent in any mathematical exercise, or in any description of a mathematical technique, however "cook-book" in character. Any such exercise can be the trigger which suddenly illuminates in a child's mind the relationship between concepts which is the essence of a "proof". But should we not make a more deliberate attempt to isolate and demonstrate such relationships? It is an exceedingly complex and difficult pedogogical problem, but without the idea of proof the whole point of mathematics, the secret of its power and wide applications, is lost.

David Vere-Jones, Institute of Statistics and Operations Research, Victoria University of Wellington.

Problems

Readers are invited to send problems for this section. Some indication should be given of how a problem has arisen and whether a complete solution is known and attribution of sources should be provided for problems that are not original. Attempts at solutions should be sent to the setter or to the Editor.

Comments on Problem 3 (December 1979).

This problem is now nearly 50 years old, and apparently still unsolved. However, substantial progress has been made, as can be seen from a recent paper entitled "The Collatz 3*n* +1 Algorithm" by L.E. Garner in the Proceedings of the American Mathematical Society, Vol. 82 (May 1981), Number 1, pages 19-22.

A. Zulauf

SOME QUESTIONS FROM THE 1981 SENIOR MATHEMATICS COMPETITION

1. Let S be the set of positive integers which leave remainder 13 when divided by 17 and remainder 7 when divided by 19. Find the two smallest members of S, and the number of members of S which are less than 10,000.

2. If $0 < y < \frac{1}{2}$, arrange in order of decreasing size:

$$(1-y)^2, \frac{1-y}{1+y}, \frac{1}{1+2y}, 1-2y .$$

3. The Earth's equatorial circumference is 40,000 km. An aircraft flies round the equator in a westward direction at u km/h. Calculate in hours the apparent length of the day D (e.g. between consecutive times that the sun is overhead).

   Sketch D against u for $-2000 \leqslant u \leqslant 2000$.
   What do negative values for u mean?
   At what value of u is D discontinuous?

   Describe what happens at that speed, and what is seen if u exceeds this critical value. Find the speed that would make a day of 48 hours.

4. A set S of positive integers has the following properties
   (i) If a and b are in S, then so is a.b
   (ii) If a and b are positive integers both not in S, then a.b is in S
   (iii) If a,b are positive integers of which one is in S and the other is not, then a.b is not in S.

   These conditions do not define S completely. Show the following for all such S:
   (a) 1 is in S
   (b) If n is any positive integer, n^2 is in S
   (c) If every prime number is in S then S is the set of all positive integers.

   Also show
   (d) The set of squares $\{1,4,9,\ldots,n^2,\ldots\}$ does not satisfy all of (i) to (iii)
   (e) The set of numbers $4^n q$, where $n \geqslant 0$, n an integer and q is odd, does satisfy all of (i) to (iii).

D. F. Robinson.

Book Review

THE SCIENCE OF MENTAL ARITHMETIC, by S.W. Taylor. An Octanary Publication,
49 Wynyard Road, Mt. Eden, Auckland.

This book is intended to promote the introduction of a style of mental arithmetic in schools. It is attractively produced with two-colour printing and illustrative, mildly humorous pictures. Yet, with its references to Hegelian Philosophy it could scarcely be a text.

Its scope is very limited - to teach a method of multiplying two two-digit numbers by mental arithmetic. No theory is given, this being reserved for a promised later book, but there is a range of worked examples. Indeed, examples are worked first, with the variations caused by different cases, and the full statement of the rule is only achieved near the end of the book.

It is couched in terms far removed from those familiar to mathematicians, and indeed from school children brought up on our present curriculum, with terms like 'objective number' and 'subjective number' and a sprinkling of philosophy, if not mysticism, that will put off anyone used to working with the conventional vocabulary.

This would be unfortunate. It is true that the language is strange, but the method works and seems well adapted to remembering the various operations necessary on the mental arithmetic.

The language emphasises that mental arithmetic should not employ the same means as standard paper calculations, but may need to work in a completely different fashion. It is a set of techniques adapted to the task, as 'long multiplication' with its need to remember several multi-digit partial answers is not. Dr. Taylor (his degrees are M.B., Ch.B.) asserts that the ultimate mental arithmetic is a blind guess that is exactly correct.

In terms understandable to the conventional mathematician, the procedure first calculates the product modulo 11 and modulo 9. These are combined to give the answer modulo 99. The last digit is found from the original multiplicand and the rest of the answer is then known modulo 990. An estimate of the magnitude of the product enables the mental calculator to select the right answer. There is of course no use of the words 'congruence' and 'modulo', everything is in terms of appropriate digit operations. The rules can certainly be learnt, and with practice would prove effective.

It is, of course, a matter of judgment whether the task of multiplying two two-digit numbers in the head is important enough for it to be taught in schools. Once these techniques have been mastered, it would, presumably, be easier to extend the sphere of application to other products and operations. But I can see the children being confused by the addition of these new terms to the mumbo-jumbo of 'numerals naming a number' already inflicted on them.

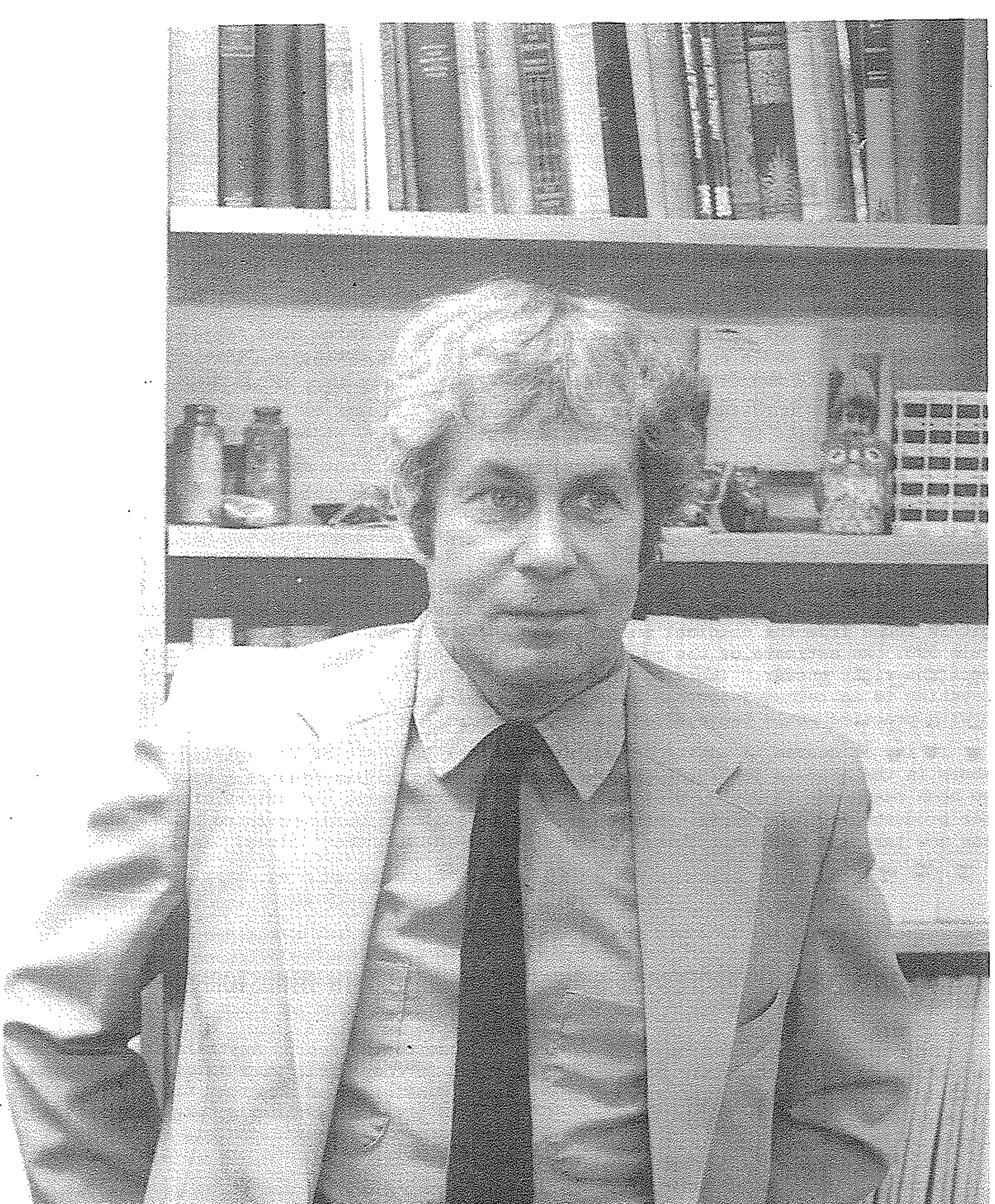
D.F. Robinson

VIDEOTAPE: PROFESSOR S. PAPERT - TOPICS IN MINDSTORMS

This tape of Professor Papert's invited address to the Second Australasian Mathematics Convention in Sydney last May is now available and is compatible with NZ video systems. A small hire fee of $4.00 covers outward postage and packaging and handling. Bookings, accompanied by the hire fee in cash or cheque (made payable to the N.Z. Mathematical Society), should be made with

Dr. W. D. Halford, Department of Mathematics & Statistics,
Massey University, Palmerston North.

Those who missed this extraordinary presentation at Sydney and who are unaware of Seymour Papert's work on computer techniques with children are referred to his recent book *Mindstorms*, which gives the background to the lecture.



Centrefold

PROFESSOR PETER WHITTLE

Churchill Professor of the Mathematics of Operational Research
University of Cambridge

Peter Whittle was born in Wellington on 27 February 1927. He enjoyed considerable success at school, being Dux of Wellington Boys' College in 1944 and winning a Junior University Scholarship. His New Zealand University career was equally distinguished. He graduated M.Sc. with first class honours in 1948. He was awarded a Post-Graduate Scholarship in Mathematics. Other honours to come his way included Senior Scholarships in Applied Mathematics and Physics and a shared Cook Prize in Mathematics.

From 1950 to 1953 he worked at Uppsala University, first producing a trail-blazing thesis *Hypothesis Testing in Time Series*, and then working as a docent in the same University. During his time in Sweden he met and married Kathe, a Finnish girl. They came to New Zealand in 1953 and for six years he worked at the Applied Mathematics Laboratory (later the Applied Mathematics Division), DSIR. He then went to England, first to lecture at Cambridge, and then in 1961 to take the Chair of Mathematical Statistics at the University of Manchester. In 1967 he was appointed to the Churchill Chair of the Mathematics of Operational Research at the University of Cambridge. During his career he has accumulated a number of honours and this process culminated in his being elected a Fellow of the Royal Society in 1978. He has since, in 1981, been elected a Fellow of the Royal Society of New Zealand.

Those are the basic facts: to them must be added first a brief appreciation of his work and then an appreciation of the man himself. His first research was in time series, their specification and certain sorts of hypothesis testing. At the time his research began it would be fair to say that time series as a statistical topic lacked direction. The associated basic probability structure had received the attention of a host of eminent mathematicians, including his supervisor Wold, Doob, Cramér, Loève, Kolmogoroff and many others, but the point of view of the statistical practitioner, the man who ultimately had to find suitable tools for the analysing of data, had not received treatment of a corresponding depth. In his thesis Peter posed and solved in workable terms a body of problems connected with auto-regressive schemes and moving averages that are fundamental in the application of time series analysis to actual data. A reasonable judgment is that the difficulties in these problems were more those of analysis than of concept. The formidable success of the thesis lay, I believe, in the fact that analytical complexity was conquered with elegance and simplicity.

Peter's concern with the fundamental use of his techniques is evident in all his writings. In many of these a very difficult problem has had the difficulty squeezed out of it to the extent that the final translation to workable procedures has become relatively plain sailing. This concern with the effective use of his work was apparent in his early days at the Applied Mathematics Laboratory where he tackled a variety of practical problems, coping with rabbit population growth, plant variability in agricultural trials, seiche record analysis, control of errors in accounting and many others.

His early work branched out into Markoff chains and processes and to more general types of stochastic processes, and he produced many papers adorning this general area. A field which has occupied him most in recent years is that of optimisation and control. In 1963 this interest culminated in a book *Prediction and Regulation by Linear Least Squares Methods*. This field has become the corner-stone of the research he has done during his Cambridge professorship. It is strewn with problems that bristle with the kind of analytic intricacy that Peter is able to handle with ease.

Along with other members of the Applied Mathematics Laboratory, I was privileged to work with him, or at least watch him work, for some years. None of us at the time was really capable of working with him in the sense of being able to keep up with him. A problem could be formulated and looked at round its fringes by those of us who tackled it in a routine manner. While we were groping, Peter would have somehow infiltrated the problem in all its complexity and not only produced an answer, but pointed to future work.

It was soon clear that his abilities would not be constrained within the confines of New Zealand and it was no surprise that he decided to move to England. We followed his career and papers (in so far as we were able) with considerable interest and were delighted that his contributions were rewarded with the accolade of a richly deserved Fellowship of the Royal Society.

This account of his work may suggest that Peter's having wide work interests would also have wide interests outside his work. This is indeed so. For one thing he is most musical. As a boy he sang in his church choir. Later, he learned to play the oboe. When I visited him in Cambridge in 1973 he said his research was suffering because he was learning to play the flamenco guitar, which posed problems as hard in their way as he was used to tackling in mathematics. Of course, the flamenco guitar was conquered and the research did not suffer. He was a keen harrier in his younger days and has been known to run to work in Wellington from his home in Island Bay. He has always taken an active interest in church affairs.

By nature, he is quiet with a keen sense of humour and a ready laugh. He is unassuming and very friendly to everyone, while at the same time being willing and able to hold his own both technically and with dignity at meetings. His family life, with a devoted and charming wife and six children with their many demands and diverse interests must have been a continuing source of strength and pleasure to him.

Perhaps this account is beginning to sound like an obituary. The opposite is intended of course. We can all be very pleased that the life and work process of Peter Whittle is very much an on-going one, by no means stationary but proceeding in a controlled optimum manner to new heights. He has many years left of productive mathematical life that will bring even greater credit to him and greater pride to his fellow New Zealanders.

J.H. Darwin

OBITUARY : H.G. FORDER

Henry George Forder, Professor Emeritus of the University of Auckland, died on 21 September 1981 at the age of 91.

He came to New Zealand from the United Kingdom in 1934 and, apart from remarkably few visits elsewhere, including just one overseas, he remained in Auckland for the rest of his life.

He retired from the University of Auckland in 1955 but continued to teach there for fifteen more years. He survived a Festschrift, a special edition of the New Zealand Mathematics Magazine and a tribute in the Mathematical Chronicle, all on his 80th birthday, some more articles in the Chronicle dedicated to his 90th birthday and, just recently, a centrefold in this newsletter. He lived long enough to see some of his students become colleagues and then retire in their turn from his University.

With his very white hair and alight stoop, he already seemed quite old when I first met him in 1959. Over the 22 years since then he seemed to age only a little, and he never lost his interest in mathematics or his ability to appreciate it.

Peter Lorimer

SEVENTEENTH NEW ZEALAND MATHEMATICS COLLOQUIUM

UNIVERSITY OF OTAGO, MAY 17-19, 1982.

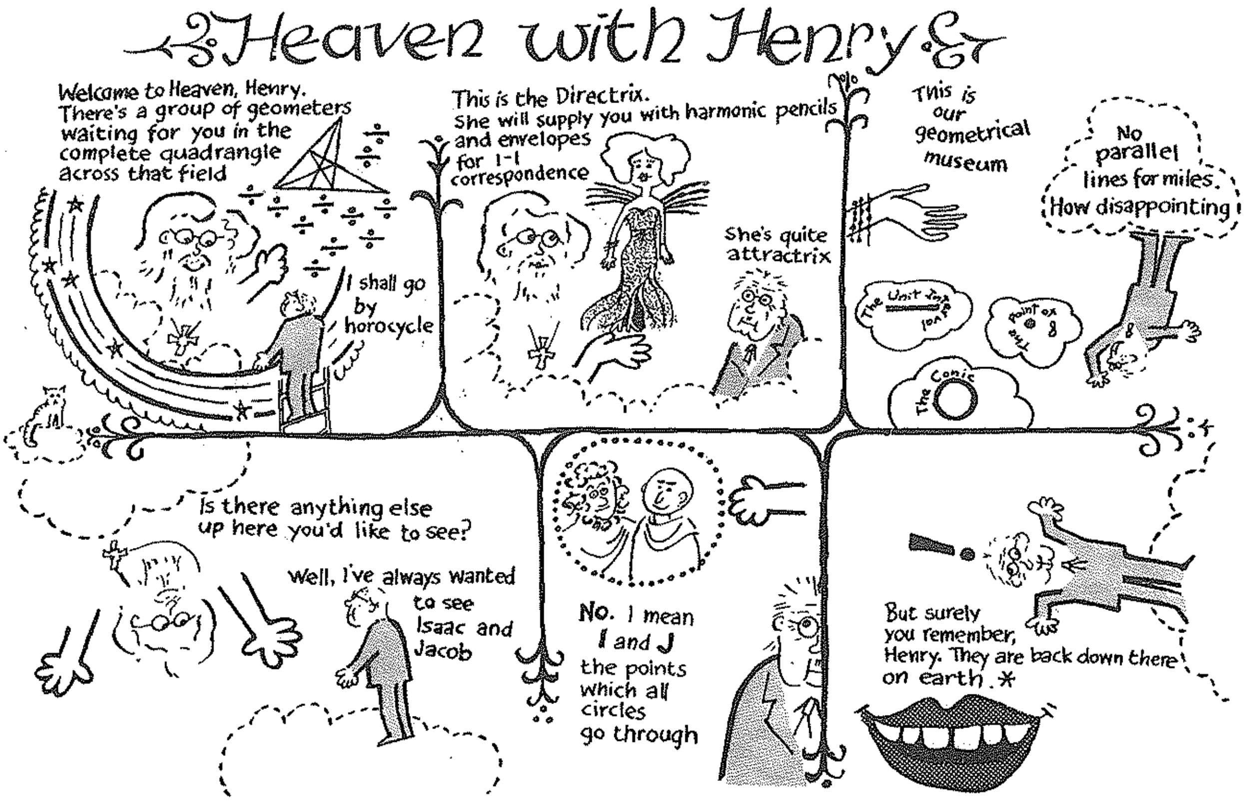
PAPERS The organising committee would like to see an increase in the proportion of papers accessible to a general mathematical audience. We feel that the Colloquium's theme is "talking together" and there should be an emphasis on expository papers, without of course excluding advanced research reports. With this philosophy in mind we hope to include a number of longer survey papers. We invite papers on any topic in pure or applied mathematics, statistics, operations research, mathematical aspects of computing, numerical analysis, mathematical physics, and mathematical education. Also of interest are papers with a substantial mathematical content from other disciplines, e.g. economics, social sciences. Suggestions for invited addresses, survey papers etc. are welcome.

ACCOMMODATION Rooms have been reserved from Sunday 16th to Wednesday 19th May in a university hall of residence, St. Margaret's College, which is on the Campus. The tariff per day will be about $17 for dinner, bed and breakfast, and $20 for full board. There will also be a number of double rooms suitable for married couples and families.

Motel accommodation can also be arrnaged, but this will be at a higher rate (about $35 for a double unit, without meals . Such bookings will have to be confirmed by a deposit prior to the Colloquium.

PRELIMINARY REGISTRATION Forms are available from

Colloquium Secretaries, Department of Mathematics, University of Otago, P.O. Box 56, Dunedin, New Zealand to whom enquiries and suggestions should also be addressed. We expect to distribute the second notice in February.



*'The "Circular Points at Infinity" were discovered at Saratov by General Poncelet, when he was a prisoner of war there after Napoleon's retreat.'
— "Geometry" by H. G. Forder, Hutchinson, 1950, p50. (See also p. 57.) – Ed.

SUMMARIES OF N.Z. PAPERS AT THE SYDNEY CONVENTION

Authors were given the opportunity to provide summaries of papers they gave at the Second Australasian Mathematics Convention.

Something more than an abstract but less than a page was asked for and full coverage of the 40 N.Z. contributions was not expected - some after all will be published here in full. We hope this format is useful to our readers, especially those who weren't able to go to Sydney.

GRAD-FOKKER-PLANCK PLASMA EQUATIONS

Kevin Broughan, University of Waikato.

Thirteen moments were taken of the Boltsmann-Fokker-Planck equation for a multispecies, multitemperature, high-temperature plasma, following the method first developed by Harold Grad for neutral gases. Like the Chapman-Enskog procedure the calculation was extremely lengthy and could not be completed without the help of a symbolic computer language; in this case the language REDUCE developed by Anthony Hearn of the Rand Corporation and implemented at the University of Cambridge on their IBM 360 processor. The collision moments may be combined with the moments of the other terms in the kinetic equation to form a set of thirteen moment equations for the time variation of the density, velocity, viscous stress tensor and heat flow vector for each species. These equations were not elaborated in the paper, but, since the collision moments were calculated with these equations in mind, it is well to consider the motivation for forming them: (1) The equations may be used for plasmas containing several species at different temperatures. (2) By splitting up a single species into several energy groups the equations may be used for plasmas whose distribution functions differ markedly from the Maxwellian. (3) They give precise results for the rate of momentum and energy exchange between each pair of species. (4) They may be compared with other hydrodynamical plasma descriptions.

For the background to the moment method and references, the reader is directed to Liley [2] and Salat [3]. The method was originally developed by Grad [1]. A summary of the moment equations formed by using the so called Krook collision term is given in Tannenbaum [4].

In his paper [3], Salat used Grad's method to obtain values for the collision integrals correct to second order in the relative Mach numbers of the species but to first order in the heat fluxes and viscous stresses. The reasons for the calculation included the development of equations valid for fast electrons with drift velocities near their thermal speeds. This paper was an extension to the work of Salat in that no approximations were made regarding the heat fluxes or thermal stresses other than the assumptions inherent in the basic assumption that the distribution function for each species may be expressed in a 13-moment form. New collision terms were derived including, in particular, bilinear functions of the heat flux vectors and traceless pressure tensors of the interacting species. However the paper did not go as far as the work of Salat in that the approximations were made to first order in the relative Mach numbers. Thus valid motions would be sub-thermal. This being so, the approach adopted will give reasonable results whenever the Boltzmann equation with Fokker-Planck collision term is a good model for the plasma.

The effort involved in including second order Mach number terms, and retaining full order for the heat fluxes and viscous stresses, is about the same, according to the author's estimate, as that involved in taking all variables to full order. This work is about to begin using the symbolic language MACSYMA developed by Joel Moses of MIT and implemented at the University of Waikato on our VAX 11/780 processor. Once these full order collision terms have been derived it should be easier to determine the range of validity of the first order Mach number assumptions and to determine the extent to which the second order Mach number approximation gives rise to, for example, an adequate model for fast electrons.

References

1. Grad, H., *Comm. in Appl. Math.* 2 (1949), 331-407.

2. Liley, B.S., *University of Waikato Physics Research Report* 103 (1972).

3. Salat, A., *Plasma Physics* 17 (1975), 589-607.

4. Tanenbaum, B.S., *Plasma Physics*. McGraw-Hill (1967).

BINOMIAL FUNCTIONS AND GIAN-CARLO ROTA

Gloria Olive, University of Otago

The paper is divided into 3 parts: *"Binomial Functions"* (BF), *"Applications of the Rota-Roman paper* [2] *to BF"*, and *"Applications of BF to the Rota-Roman paper"*.

The first part consists of definitions, examples and properties of BF. Assuming that c and k are non-negative integers, $B_k(c) = B_c^k$ is called a *(non-trivial) BF* if

$$B_{c+1}^k = \sum_{j=0}^{k} B_c^j B_1^{k-j} \quad \text{and} \quad B_0^k = \delta_{k0} \quad \text{(Kronecker-delta)} .$$

Simple examples include $c^k/k!$, $\binom{c}{k}$, $c!\ S_c^k/k!$ and $c!s_c^k/k!$ where S_c^k and s_c^k are *Stirling numbers* of the first and second kinds, respectively.

If $B_1^0 = 1$ and $B_1^1 \neq 0$, then the *BF* $B_c^k = p_k(c)/k!$ when $p_k(c)$ is a "polynomial of binomial type" of Rota. If B_c^k is a *BF* with $B_1^0 = 0$ and $B_1^1 \neq 0$, then it is said to have the "Stirling property" and is called a *BFSP*. (The last two examples given are *BFSP* and hence show where the name "Stirling property" came from). If B_c^k is a *BFSP*, then the infinite matrix B with B_c^k in the $(c+1)^{st}$ row and $(k+1)^{st}$ column is called a *BFMSP*.

The second part consists of modified definitions and results from [2], and some of their applications to *BF*. The three basic definitions are

(1) If $p_k(x)$ is a polynomial in x of degree k as well as a *BF*, it is called a *BFP*.

(2) The symbol $\langle L|p(x)\rangle$ represents the action of the linear functional L on the polynomial $p(x)$. If $\langle L|1\rangle = 0$ and $\langle L|x\rangle \neq 0$, L is called a *delta functional* (ΔF)

(3) If $\ell_k(x)$ is a polynomial of degree k and L is a ΔF such that $\langle L^c|\ell_k(x)\rangle = \delta_{ck}$, then $\ell_k(x)$ is called a *(normalised) associated sequence* for L.

Then, Rota-Roman [2] yields "A ΔF L has (normalised) associated sequence $\ell_k(x)$ iff $\ell_k(x)$ is a *BFP*", and Krouse-Olive [1] yields "Each *BFP* can be expressed as a linear combination of a sequence of *BFP* in which the coefficients are *BFSP*". It turns out that each *BFSP* corresponds to an ordered pair of ΔF [since if B_c^k is given, we can find ΔF M and L such that $B_c^k = \langle M^c|\ \ell_k(x)\rangle$ when $M = \sum_{j\geq 1} B_1^j L^j$ and $\ell_k(x)$ is the (normalised) associated sequence for L]. This result provides a key for establishing that the collection of *BFMSP* form a group with respect to matrix multiplication.

In the third part, the *"Bell BF"* $B_c^k = B_c^k(x_1,\ldots,x_k)$ [in which $B_1^j = x_j\ (1 - \delta_{j0})/j!$] is used to obtain the *"Bell polynomial"* $b_k(x)$ of Rota-Roman. Then, results in [1] are used to show that $b_k(x)/k!$ is a *BFP* and hence that

$$\left[\sum_{k\geq 0} b_k(1)\ \frac{t^k}{k!}\right]^c = \sum_{k\geq 0} b_k(c)\ \frac{t^k}{k!}$$

in the formal sense. The Bell *BF* is also used to establish the exponential generating function for the Bell numbers (which count the number of partitions of a finite set) and the Bell-Hat numbers (which count the number of surjections on a finite set).

Finally, the answer to the *Chinese Dinner Problem* ("How many different ways can k people choose their dinner in a Chinese restaurant that offers x_j different choices for j people eating together?") is given as $b_k(1)$.

References

1. D.P. Krouse & G. Olive, Binomial functions with the Stirling property, *J. Math. Anal. Appl.* (to appear in 1981).

2. G.-C. Rota & S.M. Roman, The umbral calculus, *Advances in Math.* 27 (1978), 95-188.

THE HERITAGE OF CHARLES BABBAGE IN AUSTRALASIA

G.J. Tee, University of Auckland.

(The text of the lecture, with many illustrations, is to appear in *Annals of the History of Computing.*)

Mount Babbage, in northwest New South Wales, was named in honour of Charles Babbage (1791-1871) by the explorer Sir Thomas Livingstone Mitchell on 26 June 1835; Babbage Island (off Carnarvon) was named in his honour by Sir George Grey on 6 March 1839. A Canadian river and a lunar crater are also named after him.

Charles Babbage's eldest son Benjamin Herschel Babbage (1815-1878) migrated to Adelaide in 1851, and became the first Surveyor-General of South Australia. In his 1856 expedition to Central Australia he named Mount Hopeful, which was re-named as Mount Babbage after him, in 1857.

B.H. Babbage's eldest son Charles Whitmore Babbage (1841-1923) was born in Somersetshire, and in 1880 he came from South Australia to New Zealand, farming first near Hawera and then near Wanganui. Charles Babbage's youngest surviving son Major-General Henry Prevost Babbage (1824-1918) attempted to continue his father's work on the Difference Engine and the Analytical Engine, and he sent very many relics of his father (and other ancestors) to his nephew C.W. Babbage at Wanganui. Many of those relics of Charles Babbage are now held by his numerous descendants in New Zealand and Australia, and others are kept in several museums and libraries.

The Wanganui Regional Museum has a large collection of papers of Charles Babbage, including the manuscript of his very strange memoir *Passages from the life of a philosopher* (Longmans Green, London, 1864). There are a few letters to him, many cards for him to attend meetings of the British Association and other societies, newspaper clippings about him and several legal documents (including the Royal Charter of the Borough of Totnes in Devonshire, sealed by Queen Elizabeth in 1596). There is also a large bundle of documents of H.P. Babbage. A photograph of a ragged little barefoot newsboy, brandishing a poster for the *Pall Mall Gazette* announcing the DEATH OF MR BABBAGE, seems to have been posed at Wanganui sometime around the beginning of the 20th century.

Mrs Jean Babbage (in Auckland) has a fragment of the Difference Engine, assembled by H.P. Babbage in 1879, with operating instructions. There is B.H. Babbage's drawing of the 1833 fragment of the Difference Engine, which has been reproduced in many publications by and about Charles Babbage. There are hundreds of letters, from Ada Augusta, George Cayley, George and Edward Scheutz, Dionysius Lardner, H. Wilmot Buxton, the poet Samuel Rogers, the actor Macready, Babbage's mother, his sons et alia. There are many cards and invitations to Babbage, with diplomas and gold medals awarded to him. There are genealogical items and legal documents of the Babbage family, from the 17th century onwards. There are inscribed copies of publications by Charles Babbage and his sons.

Dr Stuart Barton Babbage (in Sydney) has a huge album of diplomas awarded to Charles Babbage, many letters to him, portraits, photographs and inscribed copies of almost all publications by him, together with many publications about him.

Dr Neville Francis Burton Babbage (of Sydney) has portraits of Charles Babbage and his fiance (painted in 1813) and of his father, a bronze medal awarded to Charles Babbage, inscribed copies of some of his books, and other items. He has compiled an extensive family tree of the Babbages.

The MacLeay Museum (University of Sydney) has 10 loose parts of the Difference Engine and of the Analytical Engine, presented by Dr N.F.B. Babbage. (Those 10 parts were displayed at the lecture.)

Two (or three) trunks of Babbage papers are also held by members of the Babbage family in Australia.

The Auckland Public Library has 3 letters from Babbage to Sir George Grey.

One or two Babbage documents each are held by the Basser Laboratory (University of Sydney), Fisher Library (University of Sydney), Mitchell Library (Sydney), and the Alexander Turnbull Library (Wellington).

Thus, all future studies of the life and work of Charles Babbage will need to take into account his very extensive heritage in Australasia.

DETERMINATION OF THE METRIC TENSOR FROM COMPONENTS OF THE RIEMANN TENSOR. I : EXAMPLES

W.D. Halford, Massey University and C.B.G. McIntosh, Monash University.

Consider a Riemannian or pseudo-Riemannian manifold having a curvature tensor **R** with components $R^{\mu}{}_{\nu\alpha\beta}$ and let **g** with components $g_{\mu\nu}$ be the corresponding metric tensor. Suppose **R** is known, but **g** is not. The problem: *find all metric tensors* **g** *which have the same given* **R** has been investigated by Hlavatý (1959) and Ihrig (1975) in particular, who used the holonomy group of the manifold in their discussion. The purpose of this paper is to explain and illustrate Ihrig's method and our extension of it, the full details of which appear in our forthcoming paper, McIntosh & Halford (1981b). A joint companion paper read by Colin McIntosh at this Convention relates our method to curvature collineations.

The method is purely algebraic, no differentiability or continuity conditions being assumed, the calculations being carried out at a point in the manifold. In most cases the method determines the metric **g** up to a conformal factor from the given **R**. The Schwarzschild metric is an example. However, there are cases in which the method will determine the metric, but not up to a conformal factor; an example is the pp-wave metric which has two arbitrary functions. Furthermore, in some cases the method must be extended by introducing the covariant derivatives of **R** in order to determine the metric up to a conformal factor; an example of this kind is the vacuum type N plane-fronted rotating metric. On the other hand, there are cases in which the method will not determine **g** from a given **R**, an example being Newtonian gravity for which there is no 4-dimensional metric.

Method

This is based on the skew-symmetric property $R_{\nu\lambda\alpha\beta} = -R_{\lambda\nu\alpha\beta}$ of the curvature tensor, which may be written

$$g_{\mu\nu}R^{\mu}{}_{\lambda\alpha\beta} + g_{\mu\lambda}R^{\mu}{}_{\nu\alpha\beta} = 0 ,$$

all Greek letters taking values 0 to 3. A vector space X is constructed, spanned by 10 orthonormal vectors $x_{\mu\nu}$ $(= x_{\nu\mu})$, and with inner product $(x_{\mu\nu}, x_{\alpha\beta}) = \delta_{\mu\alpha}\delta_{\nu\beta}$.

Step 1: In X form as many linearly independent vectors $\mathbf{v}_a$ $(a = 1,2,\ldots,m|m \leqslant 9)$ as possible from the expressions

$$\psi(x_{\mu\nu}R^{\mu}{}_{\lambda\alpha\beta} + x_{\mu\lambda}R^{\mu}{}_{\nu\alpha\beta}) ,$$

where ψ is a scalar chosen to give the $\mathbf{v}_a$ simple forms.

Step 2: Find the vector **w** in X orthogonal to all the $\mathbf{v}_a$.

Step 3: Calculate the $g_{\mu\nu}$ from the inner product $\lambda g_{\mu\nu} = (\mathbf{w}, x_{\mu\nu})$, where λ is a scalar.

Examples

Given the components of **R** corresponding to each of the Schwarzschild, vacuum pp-wave and Newtonian gravity cases, the calculation of the respective **g** components is displayed (on transparencies - for printed details see our 1981b paper). It is important to note that we need nine independent vectors $\mathbf{v}_a$ in order to determine the metric up to a conformal factor. If one calculates the dimension of the bivector space B spanned by the $R_{\cdot\cdot\alpha\beta}$ he finds that in the Schwarzschild case $\dim B = 6$, while in the pp-wave case $\dim B = 2$. In McIntosh & Halford (1981a), we have shown that for our method to determine **g** up to a conformal factor we must have $3 < \dim B \leqslant 6$, which improves Ihrig's result. If $\dim B \leqslant 3$, then the metric as determined by our method will contain up to four arbitrary scalars - see Hall & McIntosh (1981).

References

Hall, G.S. & McIntosh, C.B.G. (1981) Preprint.

Hlavatý, V. (1959a) *J. Math. Mech.* 8, 285.

Hlavatý, V. (1959b) *J. Math. Mech.* 8, 597.

Ihrig, E. (1975a) *J. Math. Phys.* 16, 54.

Ihrig, E. (1975b) *Int. J. Theor. Phys.* 14, 23.

McIntosh, C.B.G. & Halford, W.E. (1981a) to appear in *J. Math. Phys.*

McIntosh, C.B.G. & Halford, W.D. (1981b) *J. Phys. A* 14, 2331.

CRITICALITY IN HOMOGENEOUS EXOTHERMIC CHEMICAL REACTIONS

A.A. Lacey, Victoria University.

An exothermic chemical reaction takes place within a well-stirred fluid. The system, which is taken to be homogeneous, can be modelled by the system of differential equations:

$$\rho \frac{dT'}{dt'} = c'^n BH\, exp[-E/RT'] - k(T' - T_a'),\ T'(0) = T_a' ,$$

$$\frac{dc'}{dt'} = -c'^n B\, exp[-E/RT'] ,\quad c'(0) = C' .$$

The former equation describes the rate of change of temperature T' in terms of the chemical heating, $c'^n BH\, exp[-E/RT']$, and the heat loss to the outside of the reaction vessel, $k(T' - T_a')$. t' and c' are the time and reactant concentration; ρ, B, H, E, R, k and T_a' are the volumetric specific heat, rate constant, heat of reaction, activation energy, gas constant, heat transfer coefficient, and ambient temperature respectively. The latter equation models the consumption of the reactant.

The corresponding dimensionless equations are

$$\frac{d\theta}{dt} = c^n\, exp\left|\frac{\theta}{1+\varepsilon\theta}\right| - \theta ,\qquad \theta(0) = 0 ,$$

$$\frac{dc}{dt} = -\varepsilon\Gamma c^n\, exp\left|\frac{\theta}{1+\varepsilon\theta}\right| ,\qquad c(0) = C ;$$

or

$$\frac{dc}{d\theta} = -\varepsilon\Gamma / \{1 - \theta c^{-n}\, exp[-\theta/(1+\varepsilon\theta)]\} ,\qquad c(0) = C . \tag{1}$$

We consider the case of $^1/_\varepsilon$, the dimensionless activation energy $(= E/RT_a')$, very large, with $^1/_\Gamma$, the dimensionless heat of reaction, of order one. Taking $\varepsilon \to 0$ we find an asymptotic expression for the critical initial concentration $c(0) = C_{cr}$ of the form

$$C_{cr} \sim exp(-^1/_n) + 2.946\,[\varepsilon^2\Gamma^2/_n\, exp(^1/_n)]^{1/3} + (4\Gamma\varepsilon/9)\ln\eta\varepsilon + O(\varepsilon). \tag{2}$$

This is a refinement of that found in [1].

For our discussion of criticality we consider four possible definitions. These are based on the appearance of a point of inflexion in the c/t graph, on a point of inflexion in the c/θ graph [2], seeking the most sensitive dependence of the maximum θ_M on C, and finding the value of C at which the time t_M to this highest temperature has a local maximum as a function of C.

In each case the algebraic and logarithmic terms in the asymptotic expansion of C_{cr} are as in (2), but the individual critical concentrations differed by amounts of $O(exp[-K/\varepsilon])$ for some positive order one K.

The terms in the series (2) are found by matching between an initial region where $c \sim exp(-1/n)$ with θ and $1-\theta \sim O(1)$ and a "critical region" of $\theta \sim 1 + O(\varepsilon^{1/3})$, $c \sim exp(-1/n) + O(\varepsilon^{2/3})$. We choose our solution to (1) to have a special behaviour in this critical region which results in it entering an outer or "slow" region (see [3]) near the upper branch of $c^n = \theta\, exp(-\theta)$. The solution may depart from the outer region near some temperature $\theta_D > 1$. θ_D depends extremely sensitively on C:

$$\ln|C - C_A| \sim -\frac{1}{\varepsilon\Gamma n}\int_1^{\theta_D} (1-1/s)^2 s^{1/n}\, exp(-s/n)ds \tag{3}$$

for a value C_A such that if $c(0) = C_A$ then at some time we have $c \sim O(\varepsilon)$ while $\theta \sim -n\ln\varepsilon + \ln(-\ln\varepsilon) + O(1)$. C is greater (less) than C_A for a reaction which has a temperature which increases (decreases) rapidly from θ_D.

It is found that the first two criteria result in two distinct values of θ_D with $C = C_{cr} < C_A$ in (3). The other two definitions indicate that
$$\ln|C_{cr} - C_A| \leqslant -\frac{1}{\varepsilon\Gamma n}\int_1^{\infty} (1-1/s)^2 s^{1/n}\, exp(-s/n)ds .$$

References

[1] D.R. Kassoy & A. Liñan, *Q.Jl. Mech. Appl. Math.* 31 (1978) 99.
[2] J. Adler & J.W. Enig, *Combustion and Flame* 8 (1964) 97.
[3] B.F. Gray, *Combustion and Flame* 21 (1973) 317.

CRITICALITY IN A MODEL FOR THERMAL IGNITION IN THREE OR MORE DIMENSIONS

G.C. Wake, Victoria University.

An explicitly resolvable model which was first introduced in a previous paper (Bazley and Wake (1978)) is used to obtain exact behaviour of its bifurcation curves. This model closely approximates the true Arrhenius law for an exothermic reaction in a spherical vessel of reacting material in three (or more) dimensions, in that the non-linear dependence of a dimensionless variable θ, which is proportional to the temperature use over the ambient temperature, is given by the true Arrhenius formula

$$exp(\theta/(1+\varepsilon\theta)) \approx exp(\theta/(1+\varepsilon\theta_m)) ,$$

where θ_m = maximum of θ over the region. Here $\varepsilon = RT_a/E$ is a dimensionless parameter dependent on the activation energy E and ambient temperature T_a. Usually $\varepsilon << 1$. The non-linear eigenvalue problem is then

$$\nabla^2\theta + \delta\, exp(\theta/(1+\varepsilon\theta_m)) = 0$$

in a sphere, with spherical symmetry, with zero value on the boundary. The solution of the problem

$$(P)\quad \begin{cases} \dfrac{d^2\theta}{dr^2} + \dfrac{j}{r}\dfrac{d\theta}{dr} + \delta\, exp(\theta/(1+\varepsilon\theta_m)) = 0 , & 0 < r < 1 , \\ \text{with } \theta'(0) = \theta(1) = 0 , \end{cases}$$

is expressible explicitly in terms of a function ψ studied by Chandrasekhar (1957) and tabulated by Chandrasekhar and Wares (1949). Here j is one less than the dimension, and so has value 2 in most practical cases. This function satisfies the related initial value problem

$$\frac{d^2\psi}{d\xi^2} + \frac{j}{r}\frac{d\psi}{d\xi} = e^{-\psi} , \quad \xi > 0 , \qquad \psi'(0) = \psi(0) = 0 .$$

Extending this problem to general values of j by writing the appropriate function as $\psi_j(\xi)$ we can write the solution for the bifurcation curves of the problem (P) as

$$\delta = (1+\varepsilon\theta_m) \left\{\psi_j^{-1}\left(\frac{\theta_m}{1+\varepsilon\theta_m}\right)\right\}^2 exp(-\theta_m/(1+\varepsilon\theta_m)) .$$

Using this explicit form (unusual for such nonlinear eigenvalue problems) we can show that the bifurcation diagram has the properties:

(1) For large θ_m, δ varies linearly with θ_m the slope being dependent on ε.

(2) There is a decreasing sequence (ε_n) of countable values of the parameter ε, at which the multiplicity of the solution of (P) changes. Specifically, for $0 < \varepsilon_{n+1} < \varepsilon < \varepsilon_n$, $n \geqslant 0$ there exists at most $2n+3$ solutions and for $\varepsilon \geqslant \varepsilon_0$ there exists exactly one solution for all values of δ.

Calculation for three dimensions gives $\varepsilon_0 = 0.124$, $\varepsilon_1 = 0.018$ and asymptotically for large n

$$\frac{\varepsilon_{n+1}}{\varepsilon_n} \sim e^{-2\pi/\sqrt{7}} = 0.093 .$$

(3) For fixed θ_m we find that δ is monotonic increasing with ε, and so the curves are layered for increasing ε, and all lie above the curve for the classical case of Gel'fand's equation $\varepsilon = 0$.

This problem is unusual in that it admits explicit determination of its bifurcation curves. Most nonlinear eigenvalue problems do not. The problem also is an example of the "third catrastrophe" at $\varepsilon = \varepsilon_0$ where criticality disappears, and this parameter is a considerable interest. Its value for the true Arrhenius equation has been given in Fenaughty, Lacey and Wake (1982).

The purpose of this investigation is to attempt to model the behaviour of the solution of the true Arrhenius nonlinearity by means other than putting $\varepsilon = 0$ as in Joseph and Lundgren (1973). The full details of the paper is included in Bazley and Wake (1981). It appears that the behaviour of the solution of this model shares many of the features of the true model which is, at present, unsolved except for small ε by asymptotic methods.

References

Bazley, N.W. and Wake, G.C., The disappearance of criticality in the theory of thermal ignition *ZAMP*, 29, 971, (1978).

Chandrasekhar, S., *An Introduction to the Study of Stellar Structure*. Dover, 1957.

Chandrasekhar, S. and Wares, G.W., The isothermal function, *Astrophysics J.* 109, 551, (1949).

Fenaughty, K.F., Lacey, A.A. and Wake, G.C., The disappearance of criticality for small activation energy with arbitrary Biot number, to appear in *Combustion and Flame* (1981-2).

Joseph, D.D. and Lundgren, T.S., Quasilinear Dirichlet problems driven by positive sources *Arch. Rational Mech. Anal.* 9, 241, (1973).

Bazley, N.W. and Wake, G.C., Criticality in a model for thermal ignition in three or more dimensions, to appear in *ZAMP*, vol. 32, (1981-2).

SOME RECENT RESULTS ON QUASICONFORMAL UNKNOTTING

David Gauld, University of Auckland.

At the First Finnish-Polish Summer School in Complex Analysis, F. W. Gehring listed ten problems which had attracted his interest "for quite some time", [5]. His problem 10 is the following:

Suppose D is a Jordan domain in $\overline{\mathbf{R}}^n$ and suppose that each $x \in \partial D$ has a neighbourhood U which is quasiconformally equivalent to the unit ball. Is D quasiconformally equivalent to the unit ball?

Gehring himself had answered the question in the affirmative for $n = 3$ in [6]. At the Colloquium on Complex Analysis held at Joensuu in 1978, the speaker described a procedure for answering Gehring's question in the affirmative for all $n \neq 5$ [1], although the procedure had a missing link (related to Gehring's problem 9) which was removed in a subsequent joint paper with J. Väisälä [4]. In fact [4] answered much more; if $X_k \subset S^n$ is a locally qc-flat topological k-sphere then (S^n, X) is qc equivalent to (S^n, S^k) provided $k \neq 4$, $(n,k) \neq (4,2)$ and, when $k = n - 2$, X is homotopically unknotted. The case $k = n - 1$ leads to the answer to Gehring's problem.

Motivated by [4], one is led to enquire what happens when $k = n - 2$ and X is knotted and also what happens when X is replaced by any other submanifold.

The first question has been answered (with even more elusive dimensions!) in [2] in the following way: if $X_{n-2} \subset S^n$ is a locally qc-flat topological $(n - 2)$-sphere then there is a neighbourhood N of S^{n-2} in S^n and a qc embedding $(N, S^{n-2}) \to (S^n, X)$ provided $n \neq 4$, 5 or 6.

The second question has been harder; so far positive results have been obtained only in codimension 1. In [3] the following is established: if M and X are homeomorphic compact, connected, orientable, handlebody, codimension one, locally qc-flat submanifolds of S^n then M and X have neighbourhoods N and Y in S^n such that (N,M) and (Y,X) are qc equivalent, provided $n \neq 5$. Further if (S^n,M) and (S^n,X) are topologically equivalent then we may take $N = Y = S^n$ provided further that $n \neq 4$.

The basic procedure underlying all of these proofs is an enlargement of one of the local collars until it engulfs all of the submanifold. To make this enlargement work, one makes piecewise-linear (hence quasiconformal) approximations which is the main reason for the dimensional restrictions.

References

1. David B. Gauld, "A technique for extending quasiconformal embeddings", *Complex Analysis Joensuu* 1978, *Springer Lecture Notes* v.747, 136-143.
2. David Gauld, "Lipschitz and quasiconformal tubular neighbourhoods of spheres in codimension two", *Ann. Acad. Sci. Fenn., Series A.I. Math.* 5 (1980), 175-178.
3. David Gauld, "Lipschitz and quasiconformal flattening of manifolds in codimension one", *Univ. Auck. Math. Dept Report Series* #172 (March 1981).
4. D.B. Gauld and J. Väisälä, "Lipschitz and quasiconformal flattening of spheres and cells", *Ann. Acad. Sci. Fenn., Series A.I. Math.* 4 (1978/9), 371-382.
5. Frederick W. Gehring, "Some problems in complex analysis", *Proc. First Finnish-Polish Summer School in Complex Analysis at Podlesice, part II* (1978), 61-64.
6. F.W. Gehring, "Extension theorems for quasiconformal mappings in n-space", *J. d'Anal. Math.* 19 (1967), 149-169.

ADAPTIVE CONTROL OF BILINEAR SYSTEMS

R.S. Long, University of Canterbury.

In this paper we prove the convergence of certain algorithms for the control of systems which can be represented by an equation of the form

$$y(t+1) = ay(t) + bu(t) + c + nu(t)y(t) \qquad t = 0,1,2,\ldots \tag{1}$$

Here a,b,c,n are constants, and u,y are scalar functions. This is the simplest example of a <u>bilinear system</u>. If $n = 0$ the equation reduces to the linear form so we assume $n \neq 0$.

Some examples of bilinear systems are: inhibition of cell growth by drugs, temperature regulation of the body, nuclear fission, population dynamics, distillation columns, treatment of industrial waste water.

The representation (1) is in the discrete or difference form, rather than the more familiar differential equation form. This is particularly convenient when y and u are measured at discrete intervals of time, rather than continuously. The sequence $\{y(t)\}$ is the output of the system, $\{u(t)\}$ is the input or control sequence, and a,b,c,n are the parameters of the system. We assume that $y(t)$ can be measured for all t, and that $u(t)$ can be assigned any value within prescribed bounds, i.e. we must have $|u(t)| < \alpha$, where α is given.

The specific problem we consider is that of "tracking". Here we are given a specified output sequence $\{y^*(t)\}$, and wish to choose $\{u(t)\}$ so that the actual output $\{y(t)\}$ agrees with $\{y^*(t)\}$ as closely as possible. Even if $\{y^*(t)\}$ is constant, this can be quite difficult to achieve. A further complication arises if some of the parameters are unknown or known only approximately. It is then necessary to estimate them from the measured values of the output, and to continually improve these estimates as more data becomes available. This constitutes what is known as adaptive control.

Let θ_0 denote the (unknown) column vector of parameters, i.e. $\theta_0 = [a,b,c,n]^T$ and let $\hat{\theta}(t)$ denote our estimate of θ_0 at time t. Also write $\phi(t) = [y(t),u(t),1,u(t)y(t)]^T$, and let $\hat{y}(t+1)$ be the estimate of $y(t+1)$, based on $\hat{\theta}(t)$. Then (1) can be written $y(t+1) = \phi(t)^T\theta_0$, and we also have $\hat{y}(t+1) = \phi(t)^T\hat{\theta}(t)$. At time t we know all quantities except $u(t)$ and θ_0. If we knew θ_0 we could choose $u(t)$ to satisfy the equation $y^*(t+1) = \phi(t)^T\theta_0$ (provided this gives $|u(t)| < \alpha$), so ensuring $y(t+1) = y^*(t+1)$ which we desire. Since θ_0 is unknown we proceed as follows (i) choose $u(t)$, as far as possible, to satisfy $y^*(t+1) = \phi(t)^T\hat{\theta}(t)$, so that $y(t+1) = \hat{y}^*(t+1)$. If it turns out that $|u(t)| > \alpha$, then instead choose $u(t) = \pm a$, as appropriate. Apply $u(t)$ to the system. (ii) The system now generates $y(t+1) = \phi(t)^T\theta_0$, which we measure. (iii) Using some standard algorithm calculate $\hat{\theta}(t+1)$ e.g.

$$\hat{\theta}(t+1) = \hat{\theta}(t) + \frac{\phi(t)}{1+\|\phi(t)\|^2}\,[y(t+1) - \hat{y}(t+1)] \tag{2}$$

(iv) Return to (i) with $t+1$ replacing t.

Simulation studies on a simple model of waste water treatment showed that the algorithm worked well, that is, $y(t)$ converged to the desired $y^*(t)$ even when the parameter values and $y^*(t)$ were caused to change abruptly from time to time.

In the paper, we prove these observed convergence properties. We first make the situation more realistic by rewriting equation (1) as $y(t+1) = \phi^T(t)\theta_0 + \omega(t)$, where $\omega(t)$ is a noise term representing measurement errors, computer round-off etc. This term is not treated by the usual stochastic methods, but instead we assume $|\omega(t)| < \Delta$, all t, where Δ is a constant and the convergence results are expressed in terms of Δ. We are then able to prove results such as $\limsup |y(t) - y^*(t)| \leqslant 2\Delta$, as $t \to \infty$.
This is a global convergence result; it is not dependent on a "good" initial estimate.

As yet, we have no theoretical results concerning the rate of convergence. Simulation studies indicate that this is strongly dependent on the particular parameter estimation algorithm used.

It is perhaps worth mentioning a feature of the parameter estimation algorithms, such as that given by equation (2). One might expect that the estimate $\{\hat{\theta}(t)\}$ would converge to θ_0, but in fact this rarely happens. We almost invariably find that $\{\hat{\theta}(t)\}$ converges to some vector $\theta_f \neq \theta_0$. Theoretically, all we can prove is that $\{\|\theta(t) - \theta_0\|\}$ converges, but this is enough to establish the convergence property of $\{y(t)\}$.

References

1. G.C. Goodwin, R.S. Long and B.C. McInnis Adaptive control of bilinear systems, *Technical Report EE8017*, Department of Electrical and Computer Engineering, University of Newcastle, N.S.W., Australia, August, 1980.

2. G.C. Goodwin, P.J. Ramadge and P.E. Caines Discrete-time multivariable adaptive control, *IEEE Trans. Auto. Control*, vol. AC-25, 449-456, June 1980.

SUBNORMAL INDICES AND NORMAL PRODUCTS OF GROUPS

D.J. McCaughan, University of Otago.

A subgroup H of a group G is said to be *subnormal* in G if there is a finite chain $H = H_0 \triangleleft H_1 \triangleleft \ldots \triangleleft H_r = G$ of subgroups of G, each normal in its successor, linking H to G. If such chains exist the least r which can occur is called the *subnormal index* of H in G.

We are concerned with the class B_r of groups (finite or infinite) whose subnormal subgroups have subnormal index at most r. A group is called a B-group if it is a B_r-group for some r. Notice that the infinite dihedral group D_∞ with presentation $\langle x,y : x^2 = (xy)^2 = 1\rangle$ has subnormal subgroups of arbitrarily large subnormal index - consider $\langle x, y^{2^n}\rangle$ - so that D_∞ is not a B-group. On the other hand B_r contains the class N_r of groups having a central series of length r, so that B contains N, the class of all nilpotent groups. This connection between B and N is further illustrated by the following difficult theorem of Roseblade (1965): there is a function ρ defined on positive integers such that if G is in B_r and in N then G is in $N_{\rho(r)}$.

It is well known that the direct product of two nilpotent groups is again nilpotent, and a classical result of Fitting (1938) shows that the same is true of "normal products": to be precise, if $G = UV$ where $U,V \triangleleft G$ with U in N_r and V in N_s then G lies in N_{r+s}. An unpublished result of D.J.S. Robinson asserts that the direct product of two B-groups is again a B-group - curiously, the proof relies on Roseblade's result mentioned above. It is natural then to ask whether the normal product of two B-groups is necessarily a B-group.

This question is answered in the negative by the following example. Let A denote the set of functions from the integers $\mathbf{Z}$ to $\mathbf{Z}$ with pointwise addition. Define automorphisms x and y of A as follows:

$$\alpha^x(n) = \alpha(n-1) \qquad \alpha^y(n) = -\alpha(n+1) .$$

Note that $xy = yx$ as automorphisms of A, so that the natural semi-direct product $G = A\langle x,y\rangle$, with the usual action, is a normal product $\langle A,x\rangle\langle A,y\rangle$. Then it can be checked that $\langle A,x\rangle$ and $\langle A,y\rangle$ both lie in B_2, but G has a normal subgroup which in turn has D_∞ as a homomorphic image, from which it follows that G is not a B-group.

References

Fitting, H., Beiträge zur Theorie der endlichen Gruppen. *Jber. Dtsch. Math.-Ver.* 48, 77-141 (1938).

Roseblade, J.D., On groups in which every subgroup is subnormal. *J. Algebra* 2, 402-412 (1965).

SOME SPECIAL COLLINEATION GROUPS OF FINITE PROJECTIVE PLANES

Peter Lorimer, University of Auckland.

The huge study of finite simple groups undertaken over the last twenty years has provided a body of knowledge that has applications wherever finite groups are found. Here is a case. In 1956, D. E. Hughes found that "almost all" finite projective planes known at the time could be classified into certain "types" depending on the way that their collineation group acts. Among these are the planes of type $(6,m)$ of which no examples were known until I found one of type $(6,2)$ in 1973. I have now been able to show, applying results of M. E. O'Nan, that a plane of type $(6,m)$ cannot exist for $m > 3$. As others have proved that the plane of type $(6,2)$ is unique, this leaves only the possibility that there are planes of type $(6,3)$, a problem which has not been solved.

To be a little more precise, a plane of type $(6,m)$ is one on which a group G of collineations acts in the following way: every collineation of G must fix each point and each line in a particular subplane of order m and G must act regularly (or sharply transitively) on the lines which pass through no point of the subplane and on the points which lie on no line of it. The plane of type $(6,2)$ has order *16* and the group is the simple group $PSL(3,2)$ of order *168*. If a plane of type $(6,3)$ exists, it has order *81* and the group is the simple group $PSL(3,3)$ of order *5616*.

O'Nan's results can also be applied to planes of Hughes types $(4,m)$ and $(5,m)$ and I have followed that through to the identification of the collineation groups.

THE STABILITY OF SOLAR AND STELLAR CORONAL LOOPS

I.J.D. Craig, University of Waikato.

The solar corona is now recognised as a highly inhomogeneous plasma that comprises a complex network of individual arch and loop like structures [j]. This has led to the recent idea that an isolated "coronal loop" may form a basic building block of a quiescent solar and stellar atmospheres [2]. Theoretical studies [3],[4] however, suggest that isolated loops, far from being stable, quasi-static structures as the observations would indicate, are radiatively unstable and hence are prone to collapse on the radiative timescale of the plasma. The basic aim of the paper is to reconcile the *existence* of coronal loops with the disruptive effect of the radiative instability.

Firstly, we apply a standard normal mode analysis to the equilibrium configuration of the loop. Two classes of lower boundary conditions are considered: in one case the foot-point is held at a fixed chromosphere temperature $(\delta T_b = 0)$; in the other the heat flux is assumed to vanish at the base $(\delta T_b' = 0)$. By constructing analytic solutions for the states of marginal stability we show that the loop is *neutrally stable* when the footprint temperature is fixed, but unstable when the heat flux vanishes at the base. In all cases of interest however, the instability is found to be *physically insignificant* in the sense that the linear growth-time is appreciably longer than the lifetime of the loop.

Finally, these results are interpreted in terms of a finite-amplitude perturbation analysis of the plasma [5]. The evidence suggests that the radiative instability is generally physically insignificant in the context of the coronal loop model.

References

[1] Orral, F.Q., *The Skylab Workshop on Active Region,* (Editor), 1981.

[2] Craig, I.J.D., McClymont, A.N. and Underwood, J.H., *Astron and Astrophys* 70, 1-11, 1978.

[3] Antiochos, S.K., *Astrophys. J.* 232, L125-130, 1979.

[4] Hood, A.W. and Priest, E.R., *Astron and Astrophys* 87, 126-131, 1979.

[5] Craig, I.J.D., Robb, T.D. and Rollo, M.D., *Solar Phys* (In press), 1981.

APPROXIMATIONS FOR OVERFLOW STREAMS OF TELEPHONE TRAFFIC

Donald C. McNickle, Economics Department, University of Canterbury.

A standard method of estimating the effects of overflow (that is, non-Poisson) telephone traffic is to approximate the overflow stream by one with a simpler rational Laplace transform. The approximating process is selected by matching the first few moments of the number of lines the overflow traffic will use. One such approximation is the interrupted Poisson process.

We consider a Poisson process with parameter λ which is interrupted by a random switch. $1/\gamma$ is the mean on-time of the switch and $1/\omega$ is the mean off-time. The on and off-times both have negative exponential distributions. With three parameters, λ, γ and ω, we can fit the first three factorial moments of the overflow process. The resulting approximations have been compared numerically with the exact overflow processes, for telephone systems by Kuczura (1973), and more recently, for systems involving storage, by Rath and Sheng (1980).

In both cases the interrupted Poisson process turns out to give remarkably accurate approximations to the exact distributions. We can show that in fact the approximation is a lower bound to the exact distribution, in the sense that it underestimates all the remaining moments, and that this partially explains the accuracy of the results.

References

Kuczura, A. (1973) The interrupted Poisson Process as an overflow process, *Bell Syst. Tech J.* 52, 437-448.

Rath, J. and Sheng, D. (1980) Approximations for overflows from queues with finite waiting room, *Operations Research,* 27, 1208-1216.

COMPACT CONVEX SETS AND MIXTURE IDENTIFIABILITY

G. R. Wood, University of Canterbury.

A *simplex*, S, in $\mathbb{R}^n$ is the convex hull of $n+1$ or fewer affinely independent points. So a point, a closed line segment, a triangle and a tetrahedron are examples of simplexes. Their best known feature is the following "uniqueness of representation" property: given x in S ($S = co\{x_1,\ldots,x_{n+1}\}$ say) there exists a unique $(n+1)$-tuple $(\lambda_1\ldots, \lambda_{n+1})$ with $0 \leq \lambda_i \leq 1$ for each i and $\sum_1^{n+1} \lambda_i = 1$, such that $\sum_1^{n+1} \lambda_i x_i = x$. Roughly speaking, each point in a simplex is uniquely expressible as the centre of mass of a (probability) weighting of the extreme points. We call $\lambda = (\lambda_1\ldots,\lambda_{n+1})$ a *representation* of x, and in general denote the set of all representations of x by Z_x^+ (see Alfsen (1)).

Note that if a point in some set should have two representations, λ and μ, then every convex combination of λ and μ again forms a representation. A statistical problem which arose through considering the identifiability of mixtures of distributions suggested that we search for all compact convex sets D in $\mathbb{R}^n$ with the following property: some point in D has such a segment of representations, while all points in D have either a segment of representations or a unique representation. If we dub these sets *duplexes* we readily find:

THEOREM: D is a duplex in $\mathbb{R}^n$ if any only if $D = co(S_\cup\{z\})$ where S is a simplex in $\mathbb{R}^n$, $z \in$ aff S (the affine span of S) and the extreme points of S and z are convexly independent.

Consideration of m-plexes, K (where dim $Z_x^+ \leq m$ each $x \in K$) leads to a classification of all polytopes (convex hulls of finite sets) in $\mathbb{R}^n$.

In order to state the analogous result in a metrizable locally convex space E we need the following definitions:

1) A compact convex set D in E is a *duplex* if

   i) $\dim Z_x^+ = 1$ for some x in D,

   and ii) $\dim Z_x^+ \leq 1$ for all x in D.

2) A subset D of E is a *point-duplex* if $D = co(S_\cup\{z\})$ where

   i) S is a simplex

   ii) $z \in$ aff S

   iii) $\partial_e D = \partial_e S_\cup\{z\}$ ($\partial_e D$ is the extreme points of D).

3) A compact convex set K in E is said to contain a *maximal simplex* if we can find a simplex $S \subseteq K$ such that i) aff S = aff k

   and ii) $\partial_e S \subseteq \partial_e k$.

Then we have

THEOREM: i) Every point-duplex is a duplex
ii) Every duplex containing a maximal simplex is a point-duplex.

An example (due to Thomas E. Armstrong) of a compact convex set not containing a maximal simplex is known. So duplexes have the finite dimensional form (of point-duplexes) precisely when they have the finite-dimensional property of possession of a maximal simplex.

As in the finite-dimensional case, this result can be extended in a natural way to m-plexes. These results can then be applied to the problem of mixture identifiability.

Reference

[1] E. M. Alfsen, *Compact convex sets and boundary integrals*, Ergebnisse der Math. 57, Springer-Verlag, Berlin - Heidelberg - New York, 1971.

TESTING FOR CATEGORISED EXPONENTIALS

J.C.W. Rayner and D.J. Best, University of Otago.

Suppose observations of a continuous random variable are rounded, as is always the case in practice. What affect does this have on the usual tests of significance? We posed this question in the particular case when the distribution was assumed to be exponential, and assessed both one and two sample problems for the rate parameter λ.

For the one sample problem a random sample $X_1,\ldots,X_m$ is taken from a population with probability density function $\lambda exp(-\lambda x)$ for $x > 0$, zero otherwise, where $\lambda > 0$. The observation space is partitioned into k categories $C_1,\ldots,C_k$, and U_i is the number of observations (from the random sample of m) that fall in C_i. $U = (U_1,\ldots,U_k)^T$ is multinomial with parameters m, and π where $(\pi)_i = P(C_i)$. Under the null hypothesis $\lambda = \lambda_0$, $\pi = p$. For the two sample problem similar definitions apply: for a random sample of n the observation space is partitioned into $C_1^*,\ldots,C_{k*}^*$, V_i counts the number of observations in C_i^*, and V has the multinomial distribution with parameters n and π^*.

If, due to rounding, the most powerful test based on $\sum_{i=1}^{m} X_i$ is not available, then tests based on $\sum_{i=1}^{k} e_i U_i$ arise naturally. $e = (e_1,\ldots,e_k)$ gives the scoring for each class. For the one sample problem the parameters to be considered are the sample size m, significance level α, null parameter value λ_0, scoring e, category probabilities p, and number of categories k. Obviously the two sample problem is more complicated.

Our method has been to calculate powers and asymptotic relative efficiencies. With such a multitude of possibilities, such an investigation is limited, and the results that may be presented must be restricted also. Powers, where possible, were calculated exactly, but more usually a normal approximation ($T = \Sigma e_i U_i$ is approximately normal with mean $= mL(e) = m\Sigma e_i \pi_i$ and variance $= m\,C(e) = m\{\Sigma e_i^2 \pi_i - L^2(e)\}$) was used. Simulation confirmed the adequacy of the normal approximation.

Broadly speaking our results showed a remarkable robustness to the choice of parameter. Three scorings were investigated: they were virtually indistinguishable and included the most powerful possible given the categorisation. We recommend using a median probability scoring,

$$e_i = -\{\ln(P_{i-1}^{\lambda_0} - p_i/2)\} / \lambda_0 ,$$

where $P_{i-1} = p_i + p_{i+1} + \ldots + p_k$. Different forms of the p_i were investigated: for example $p_i = 2i/\{k(k+1)\}$ and $p_i = 1/k$. Powers seemed to vary by at most 3% or 4%. The affect of increasing k was interesting. Fruitful gains in power or ARE were observed for k increasing from 2 to 10. Thereafter gains were small and certainly negligible after $k = 50$. So for most purposes one decimal is adequate and two more than enough.

NEW RESULTS IN POTENTIAL THEORY

Ü. Kuran and J. L. Schiff, University of Auckland

It is well known that a nonnegative superharmonic function $s(z)$ defined on a Jordan domain D can tend to zero as z approaches a point on the boundary, ∂D, of D. The Green's function for D is just such an example. However, we demonstrate in this paper that $s(z)$ cannot tend to zero "too rapidly" as z approaches any point on ∂D, unless $s \equiv 0$. In fact, if $s(z)$ is nonnegative superharmonic in D and satisfies

$$\liminf\ s(z)/\mathrm{dist}(z,\partial D) = 0 \quad as\ z \to \zeta_0,\ z \in D$$

for any point $\zeta_0 \in \partial D$, in a neighbourhood of which ∂D is at least Dini-smooth, then $s \equiv 0$ in D.

This result can be extended as follows: If ∂D makes an angle of aperture $\kappa\pi$, $0 < \kappa \leq 2$, at $\zeta_0 \in D$, then the analogue of the preceding theorem requires the modification

$$\liminf\ s(z)|z|^{1-1/\kappa}/\mathrm{dist}\,(z,\partial D) = 0 \quad as\ z \to \zeta_0,\ z \in D$$

where in a neighbourhood of ζ_0, ∂D consists of two Dini-smooth arcs emanating from ζ_0. Whence $s \equiv 0$ in D.

The first result is established via the Riesz-Herglotz representation for nonnegative superharmonic functions. This result can also be extended to domains $D \subseteq \mathbb{R}^n$, $n \geq 3$, where ∂D is locally C^1.

KERNEL DENSITY ESTIMATION AND PREDICTION INTERVALS IN MEDICAL DIAGNOSIS

P.D. Hill, Waikato University.

Prediction intervals (normal ranges) are widely used in preliminary diagnosis of disease. A normal range of variable values is constructed on the basis of healthy patients in such a way as to exclude a specified small proportion of healthy patients and an unspecified (much) larger proportion of diseased patients.

Practitioners in this context have been confused as to the relative merits of the distribution free (sample percentile) approach and the normal distribution-dependent approach to constructing prediction intervals. A method of construction using a kernel estimate of the density function of the "healthy" variable values is suggested as an alternative to these two approaches.

A kernel estimate of a density $f(x)$ is

$$\hat{f}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\lambda} \, exp\left[-\tfrac{1}{2}\left(\frac{x - x_i}{\lambda}\right)^2\right].$$

This estimate averages normal kernels centred at the data points with a fixed smoothing parameter λ. Fryer (1976) showed how to obtain a value for which λ is optimal by an integrated mean squared error criterion if the data are normally distributed. Habbema et al (1974) devised a criterion which involves a jack-knife type of modification to a maximum likelihood approach. Unlike Fryer's method, it is a data dependent, operational method for choosing λ.

Having obtained such density estimates a one-sided prediction interval with $.95$ probability content (cover) would be of the form $(-\infty, U)$, where $\hat{F}_n(U) = .95$. A simulation study on normally distributed data compared 3 estimates of the population percentile; the sample percentile and solutions of $\hat{F}_n(x) = .95$ using the Fryer and the Habbema smoothing parameters. The study also compared the covers of the consequent one sided normal ranges. Sample sizes studied were $n = 19(20)99$ and 100 experiments were simulated at each sample size. (There is a need for further studies with a wider range of "parameters", particularly with non-normal distributions.) The three methods were compared in terms of empirical bias, variance and mean squared error of the percentile estimates and normal range covers.

The two kernel methods exhibit a consistently positive bias in estimating the population percentile, leading to covers between 1 and 2% greater than the nominal 95%. On the other hand the variability of the sample percentile estimate is consistently greater than that of the two kernel methods. The main conclusion is, that in terms of mean squared error of the coverage distribution, the kernel methods are superior to the sample percentile method. The improvement gained by the smoothing is not great, however.

A major practical problem in establishing normal ranges is that they need to be based on patients who are known to be healthy. If patients of unknown condition could be used the data base would be much larger. In practice, data may be available on three categories of patient: healthy, unhealthy and unknown condition. Following Murray and Titterington (1978) the category can be treated as a discrete variable for which a suitable discrete kernel can be chosen together with a continuous kernel for the data itself. A marginal kernel estimate for the healthy category density can then be derived and a normal range for healthy people is obtainable by inverting the marginal kernel distribution function as described above. This use of the unknown category data should result in a much improved normal range.

References

Fryer, M.J. (1976) *J. Inst. Maths. Applic.* 18, 371-380.

Habbema, J.D.F. et al (1974) In compstat 1974. *Physica Verlag.*

Murray, G.D. and Titterington, D.M. (1978) *Appl. Stats.* 27, 325-334.

MINIMAL GENERATING PAIRS FOR CERTAIN PERMUTATION GROUPS

Marston Conder, University of Otago.

In a finite group G, a pair of elements x and y which together generate the group, and are such that x has order k, and y has order ℓ, and their product xy has order m, is called a *(k,ℓ,m) - generating pair* for G. In such a case, the group G must be a quotient of the *triangle group* $\Delta(k,\ell,m)$, that is, the group with presentation

$$\Delta(k,\ell,m) = \langle a,b \mid a^k = b^\ell = (ab)^m = 1 \rangle .$$

Suppose further that whenever r,s,t are positive integers with $r \leqslant s \leqslant t$ and such that G has an (r,s,t) - generating pair, then $k \leqslant r$, and if $k = r$ then $\ell \leqslant s$. Then we shall call (x,y) a *minimal generating pair* for G.

The triangle group $\Delta(k,\ell,m)$ is finite precisely when $\frac{1}{k} + \frac{1}{\ell} + \frac{1}{m} > 1$, and the groups which arise in this case are the following:

$\Delta(1,m,m) \cong C_m$, cyclic of order m

$\Delta(2,2,m) \cong D_{2m}$, dihedral of order $2m$

$\Delta(2,3,3) \cong A_4$, tetrahedral

$\Delta(2,3,4) \cong S_4$, octahedral

$\Delta(2,3,5) \cong A_5$, icosahedral .

If $\frac{1}{k} + \frac{1}{\ell} + \frac{1}{m} = 1$, then the group $\Delta(k,\ell,m)$ is infinite but soluble: the commutator subgroup is free Abelian of rank 2 (with cyclic quotient).

Our interest centres on insoluble permutation groups, and therefore we look for (k,ℓ,m)-generating pairs with $\frac{1}{k} + \frac{1}{\ell} + \frac{1}{m} < 1$. Any finite group which possesses such a generating pair is representable as a group of automorphisms of a compact Riemann surface of genus g, where

$$2g - 2 = |G| \, . \, (1 - (\tfrac{1}{k} + \tfrac{1}{\ell} + \tfrac{1}{m})) .$$

From this formula it is evident that $|G| \leqslant 84(g-1)$, the maximum bound being attained only if G is a *Hurwitz group*, that is, a quotient of $\Delta(2,3,7)$.

Using coset diagrams for the latter triangle group, we can show that all but 64 of the finite alternating groups A_n are Hurwitz groups. The technique used can be adapted to achieve the following generalization:

Theorem

Given any integer m greater than 6, all but finitely many alternating groups A_n occur as quotients of the triangle group $\Delta(2,3,m)$. Moreover, if m is even, then all but finitely many symmetric groups S_n occur as well.

Finally, we can prove that the group of order 43,252,003,274,489,856,000 associated with Rubik's cube possesses a $(2,4,1260)$ - generating pair.

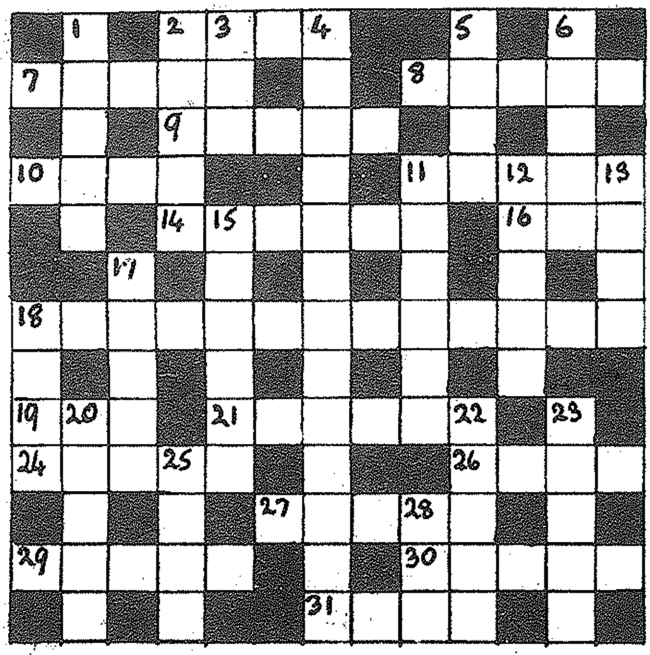
A full version of this paper appears in *Quarterly Journal of Mathematics* (Oxford, 2nd series), 32 (1981), 137-163. Also, since the time of the Sydney conference, the author has found a minimal $(2,4,12)$ - generating pair for the group of Rubik's cube.

HONORARY AUDITOR 1981

The Society expreses its gratitude to Assoc. Prof. D.M. Emanuel, Accountancy Department, University of Auckland, for accepting the appointment of Honorary Auditor.

Crossword

N^o 5 BUVOS KOCKA by Matt Varnish



CROSSWORD N^o 4 SOLUTION

The letter a is not used in the clues and answers

Across:

1. Figure of eight, 8. Round, 9. Cheques, 11. Ephor, 13. Tomb, 15. Idle, 16. El-bowing, 17. Onto, 18. Lend, 20. Corridor, 21. Took (to O.K), 23. Oozy, 24. Jones, 27. Twosome, 28. Inigo, 29. Pieces of eight.

Down:

1. First footstep, 2. Gourmet, 3. Rude, 4. Ouch, 5. Ever, 6. Gourd, 7. To speed by foot, 10. Homilize, 12. Poolroom, 14. Block, 15. Igloo, 19. Eroding, 22. O-zone, 24. Joke, 25. Nero, 26. Size.

It 14a(6) from the mind of 25d(4) 8a(5). The 4d(9,4) with its 18a(13) 4×10^{19} positions, the wonder of our 3d(3), has swept the 1d(5). In each of the 24a(5) there is a 7a(5) and the 31a(4) pieces 2a(4) around these each on a sort of 28d(3). The 24a(5) are 10a(4), 16a(3), 12d(5), 22d(5), 29a(5) or orange, and 23d(5). Some have 20d(5) theirs to make the 2d(5) smoother. Just the thing for the mathematical 19a(3). He can 11a(5) about the 2d(5), find the 30a(5) of each 26a(4), verify his 15d(6), and then even with these 18d(4) 17d(5) hours following a 27a(5) until finally with a cry of 21a(6) he solves it. From then on he will 5d(4) with smugness and 9a(5) his expertise as you stand in frustrated 6d(5) 11d(6) at your incompetence. No wonder some get 13d(4) (pardon the pun) just thinking about it.

* * * * *

The Newsletter is the official organ of the N.Z.M.S. The last 5 issues have been produced in the Mathematics Department (thanks due to Professor Petersen for his support!) and the Printery of the University of Canterbury. We trust that they have evoked some interest, even amusement, amongst readers and just wish that more response was evinced. In consigning you all to new editors we implore you to send in material for publication and wish everyone a Merry Xmas.

The official address of the NZMS is:
The New Zealand Mathematical Society (Incorporated)
c/- The Royal Society of New Zealand
P.O. Box 12-249,
WELLINGTON.

However, correspondence should normally be sent direct to the secretary at the address overleaf.

Registered at the Post Office as a magazine.