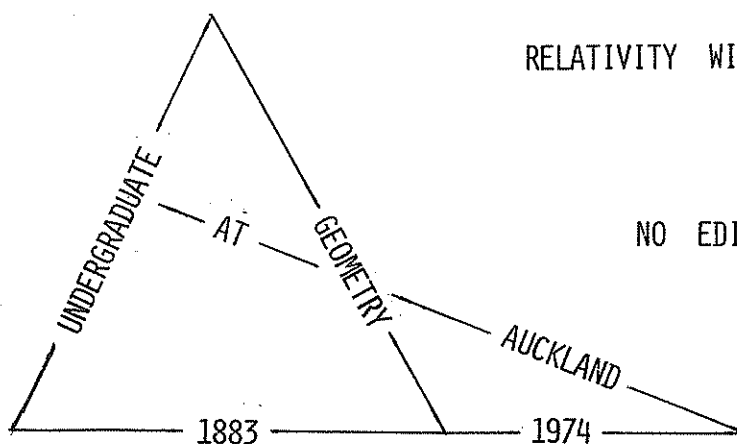


# THE NEW ZEALAND MATHEMATICAL SOCIETY



## NEWSLETTER



RELATIVITY WITH ROY

NO EDITORIAL

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LATE NEWS : SUB-EDITORS WORK TO RULE

## MATHEMATICAL DIRECTIONS: A POLICY COMMITTEE?

*A white paper for members to consider, from the President, Dean Halford.*

### SOME CONCERNS

The present economic situation and the static nature of university rolls have made it necessary to re-examine the funding of teaching and research at the tertiary level. It is clear that the cuts in education spending in real terms made by Government must affect the future direction of mathematics in the universities and polytechnics, with a flow-on to the job market. But the process goes beyond this. Constraints placed on government research establishments must have some long term effects in several areas, not just on the establishments themselves. And the same may be said about teachers' colleges. What is the future of mathematics in these places?

We must ensure that the excellence in mathematical research which has been built up by the universities and research establishments continues to be promoted and achieved. Given the limited resources of this country and a traditional reluctance to give education a higher place in the political scheme of things, I believe that strenuous efforts will be required to give tertiary mathematics the place it should have in the minds of those making top-level decisions about New Zealand's future.

I am also motivated by a concern to map out some of the issues which I see as requiring immediate attention. Recommendations which spring from these will determine a direction for mathematics in New Zealand.

### SOME QUESTIONS

Are we using the existing channels as much and as effectively as we could? Or is a new approach necessary?

The Subject Conferences in mathematics are useful in bringing university concerns to the attention of the Vice-Chancellors. But these conferences are held irregularly and (if past experience is a guide) very infrequently. What is the outcome? The Royal Society of New Zealand maintains a brief over mathematics through the member bodies representatives, and maintains an international link through the National Committee for Mathematics. Could we be of more assistance to the RSNZ as a voice for mathematics? The Department of Education controls course development in the polytechnics, but how accessible is the Department? To what extent does its planning for polytechs reflect the future needs of NZ, and to what extent does the Department consult with staff at the polytechnics when assessing these needs? There is also a Consultative Committee on Mathematics within the structure of the Department of Education, but what is its constitution and brief?

Which bodies are keeping a continual review of, say, the strengths and deficiencies in mathematical research in NZ and making recommendations (to whom?) in the light of such information? What is the potential for using mathematics in business and industry? Several mathematical organisations exist in this country - how familiar are we with their activities? Shouldn't closer links between them be forged?

If we are to make a concerted effort and thrust for mathematics in the future, perhaps we should create a new national policy committee on mathematics representative of the various mathematical bodies interested in teaching and research at tertiary level. Such a policy committee could tackle such general matters as I have mentioned above plus more specific issues of the kind which I list next. These matters could and should be the concern of the New Zealand Mathematical Society, but I believe they would be more appropriately and more effectively placed within the brief of a wider policy committee.

## SOME ISSUES

The following seem to be significant issues worthy of immediate research:

- (1) The growth in the number of graduates of NZ universities, polytechnics and teachers' colleges who have majored in any branch of mathematics, and implications therefrom.
- (2) Employment opportunities and the need for mathematics in employment.
- (3) Strengths and deficiencies in mathematical research in NZ.
- (4) The availability of funds for research in mathematics.
- (5) The development of inter-disciplinary courses requiring a mathematical input, in particular the relationship of computer technology to mathematics.
- (6) Contacts with mathematicians in the South Pacific and South East Asia, and the special needs of countries in that region.
- (7) The understanding and further development of relations between existing bodies representative of mathematics in NZ.
- (8) Public awareness of mathematics.

## SOME SUGGESTIONS

- (a) With regard to (1), there is at the University of Canterbury a mountain of data on graduates of all NZ universities and the destination of these graduates. It should be relatively easy to tap this system for information about mathematics graduates. Is there a similar data bank for graduates of polytechnics?
- (b) Of special significance for the future of tertiary mathematics is its relationship to business and industry. Available data should be processed showing trends in the numbers of New Zealand graduates entering business or industry. Mathematicians as a group should be more dynamic in their contact with the world of business or industry. For example, a special effort should be made to involve people from this section of the work force in the annual NZ Mathematics Colloquium.
- (c) The New Zealand Mathematical Society has shown initiative in publishing the handbook "Employment Opportunities in Mathematics" for those about to graduate. However, more research is needed on item (2).
- (d) We must do all we can to bring before Government the value of mathematical research to NZ, and to be aware of ways in which our existing research strengths can be fully deployed. The special needs of this country may point up deficiencies in our mathematical research; we must provide the expertise to fill these gaps, ensuring that sufficient funds are available to do so without undermining the efforts in other worthwhile research areas.
- (e) On public awareness of mathematics, we should think of ways to get society at large "on board" with mathematical developments. We should line up some good speakers for the 1981 science programme on television to demonstrate significant achievements and developments. Maybe a regular column in the Listener, either entertainment or serious, could be started. Perhaps national radio could feature a regular slot on "what's happening in mathematical science in NZ". How about whipping up interest (at high school?) with the sale of buttons proclaiming such things as "*The logical thing to do is mathematics*" or "*Mathematics is changing your world*" or "*Mathematics: the universal language for all time*"?

I urge you to ponder these issues and to feed back your comments to the Society. We must be demonstratively vigorous in laying out the future for mathematics.

Dean Halford

## News and Notices

### OPPORTUNITY IN COMPUTING, GRAPHICS

Applied Mathematics Division, Department of Scientific and Industrial Research is seeking a staff member, with a special interest in computer graphics, to work in its Computing Section.

The primary tasks will be: advising and assisting computer users, especially with computer graphics applications; the support of graphics and related packages; scientific applications programming; systems programming relating to graphics; advising the Division on graphics hardware and software selection.

Qualifications: science or engineering degree (preferably honours) with computing and mathematics. While experience is not required, an interest in computer graphics is essential. Particular interest in numerical mathematics or programming languages would be welcome and a knowledge of digital electronics would be an added recommendation.

*Students graduating this year will be considered.* Training will be given and there are opportunities for further study.

Salary: \$10 126 to \$13 810 according to qualifications and experience. Promotion to the next grade (maximum \$16 849) depends only on satisfactory performance. Excellent opportunities exist for promotion beyond this.

Please address enquiries to the Director, H.R. Thompson, P.O. Box 1335, Wellington.

### VACANCY: PHYSICS AND ENGINEERING LABORATORY, DSIR

There are vacancies for research scientists in geomagnetism and atmospheric physics at the Geophysical Observatory in Christchurch and the PEL Auroral Station at Lauder in Central Otago. Applicants should have a good honours degree in physics or mathematics and research experience for a Ph.D. or M.Sc. Appointment will be to the scientist grade in the NZ Public Service. Salary up to \$19 360 according to qualifications and experience.

R.S. Unwin, Officer in Charge,  
Geophysical Observatory,  
P.O. Box 2111, Christchurch.

### NZMS MATHEMATICS SYLLABUS SERIES

The New Zealand Mathematical Society is preparing a set of three booklets to cover the new 7th form Bursary Applied Mathematics Syllabus. Their titles will be 'Probability and Statistics', 'Computing and Numerical Mathematics', and 'Mechanics'.

Each booklet supplies definitions, formulae, worked examples, problems, and historical notes, together with Tables and selected questions (with answers) from past examination papers. It is planned to publish these very early in the new year, and it is hoped to set the price at \$5 per booklet.

Advance orders would be appreciated. Please send these and any requests for further information to Dr. G. Wake, Department of Mathematics, Victoria University, Private Bag, Wellington.

## REPORT ON EIGHTH AUSTRALIAN CONFERENCE ON COMBINATORIAL MATHEMATICS

This Conference was conducted by the Combinatorial Mathematics Society of Australasia and held at Deakin University, Geelong, Victoria, Australia, from Monday 25 August to Friday 29 August 1980. It heard 37 speakers deliver 42 papers including 12 invited addresses. Ten countries were represented with 47 participants including 9 invitees:

Dr Brian A. Alspach (Simon Fraser University),  
*Hamiltonian paths and cycles in vertex-transitive graphs.*

Dr Chuan-Chong Chen (National University of Singapore),  
*Strongly hamiltonian abelian group-graphs.*

Dr Ronald L. Graham (Bell Telephone Laboratories),  
*Long lattice paths in subsets of  $\mathbb{Z}^n$ ; Distance matrices of trees.*

Professor Jun-Shung Hwang (Academia Sinica),  
*Complete stable marriages and systems of I-M preferences.*

Professor Peter J. Lorimer (University of Auckland),  
*The construction of finite projective planes.*

Professor Ronald C. Read (University of Waterloo),  
*A survey of graph generation techniques.*

Professor Johan J. Seidel (University of Technology, Eindhoven),  
*Two-distance sets.*

Dr John Sheehan (University of Aberdeen),  
*Finite ramsey members.*

Professor Ralph G. Stanton (University of Manitoba),  
*Further results on covering computations; Construction of H-systems;  
Computation of number theoretic coverings.*

The talks touched most areas of combinatorics with perhaps graph theory dominating.

Accommodation was provided on campus. Social activities were a games night, half-day excursion (tour of Fort Queenscliff and smorgasbord lunch), theatre night (Evita), and Conference Dinner. Any idle moment seemed to be occupied by Ron Graham demonstrating the subtleties of the Rubik (alias Hungarian Magic) Cube.

The refereed proceedings will be published in a volume of the Springer-Verlag series 'Lecture Notes in Mathematics'.

*Kevin McAvaney (Director, CMSA)*

### FROM THE PRESIDENT OF THE IMU

At the International Congress of Mathematicians in Warsaw 1982, the IMU expects to be able to award Fields medals in mathematics, as has been a tradition since 1936. The medals should be awarded for outstanding achievements in mathematics and the medallist should be at most 40 years old in 1982. It should be emphasized that mathematics should be interpreted in a wide sense and also include applied areas.

The decision on recipients of the awards is taken by a committee where I, as president of the Union, am chairman. The composition of the committee is confidential until the Congress 1982. In order to ensure complete coverage of all possible candidates, the Executive Committee of the IMU has decided to invite all National Committees to suggest candidates. The suggestions should contain complete bibliographies and short motivations. They should reach me no later than January 1, 1981, and be sent under the address: Institut Mittag-Leffler, Auravägen 17, 182 62 Djursholm, Sweden.

*Lennart Carleson, Djursholm,  
April 25, 1980*

## APPLIED MATHEMATICS SYMPOSIUM

Wellington, August 1980

During one of the wettest days of the winter, about thirty applied mathematicians gathered in Wellington from all over New Zealand for a two day working symposium on applied mathematics. This was held in recognition of Professor Cecil Segedin's contribution to applied mathematics in New Zealand [Professor Segedin retires from the Chair of Theoretical and Applied Mechanics, University of Auckland at the end of 1980].

The symposium was held as part of the joint seminar series of the Applied Mathematics Division, DSIR and the Mathematics Department of Victoria University of Wellington and was supported by the NZ Mathematical Society as well as DSIR and Victoria University.

A full two day programme of talks was organised by Dr. Alex MacNabb of DSIR. The major event was a fascinating invited address by Professor Segedin entitled "*Forty-four years of Applied Mathematics*" in which he summarised his own career and the factors which influenced the course of events. This address was introduced by Mr. Merv Rosser from Auckland who underlined some of Professor Segedin's own contributions and a warm vote of thanks was moved by the Chairman of the Victoria Mathematics Department, Professor Wilf Malcolm. A full list of the talks given is reprinted below. Almost all of these involve some contact with Professor Segedin - either as former students, or past or present collaborators, or just friends. Judging from the tired faces on the second morning, the dinner the night before, at the Mount Cook Cafe - hosted by Professor Terence Nonweiler - was a great success. A book containing all the talks is being edited by Dr. Alex MacNabb of the DSIR.

The specialist talks given at the Symposium were as follows:

- Selwyn Gallot, AMD, DSIR: *Stochastic models for extreme loading on structures.*
- John Harper, VUW: *Stokes flow between parallel planes, due to the tangential motion of an end wall.*
- Mary Fama, WU: *An approximate solution to the stress near the top of a shallow slope.*
- Graham McVerry, PEL, DSIR: *Structural identification from earthquake records.*
- Jim Ansell, VUW: *The separation of compressional and shear waves in an elastic medium.*
- Alex MacNabb, AMD, DSIR: *Mechanics in gymnastics.*
- Brad Imrie, VUW: *Design and development of oxy-fuel gas cutting nozzles for improved performance.*
- Brian Williams, Hamilton Science Centre, MOWD: *Design of a water reticulation system using network analysis.*
- Chris Rutherford, Hamilton Science Centre, MOWD: *Aspects of water quality modelling.*
- Robin Wooding, AMD, DSIR: *On transient flow in stratified aquifers with high horizontal permeability.*
- Robert McKibbin, AU: *Some aspects of nonhomogeneity in the "fluid saturated porous layer" model of a geothermal field.*
- Graeme Wake, VUW: *An analytic criterion for the disappearance of criticality.*
- Peter Bryant, CU: *Two dimensional waves in shallow water.*
- Brian Woods, CU: *Recollections of hypersonic flow theory.*
- Geoff Mohr, AU: *Quadratic finite elements with particular emphasis on shell elements.*
- Don Nield, AU: *Natural convection from a time heat source in a porous medium.*
- Terence Nonweiler, VUW: *Internal wave instability in open-ended slender tubes.*

G.C. Wake

### CORRECTION

The Council minutes (page 44 of the last issue) were in error in that the sponsor of the Employment Brochure was actually Burroughs Ltd., whom we thank for their generosity.

# Local News

## AUCKLAND UNIVERSITY

### DEPARTMENT OF MATHEMATICS

Professor George Seber's term as Head of Department ends on 31 January 1981, and Associate-Professor Gordon Hookings has been appointed as Head of Department for 1981 (or until the vacant Chair of Mathematics is filled).

Dr. David Gauld has been promoted to Associate-Professor.

Dr. Mila Mršević, a topologist from Belgrade, has been appointed as a post-doctoral fellow for 1981.

Professor Alistair Scott has been elected a Fellow of the Institute of Mathematical Statistics.

Professor Charles Rees has returned to the University of New Orleans.

Dr. Jeff Hunter has returned from Virginia Polytechnic Institute.

Dr. Ivan Reilly has returned from his visit to the UK.

Chris King has returned from his tour of the USA.

The NZMS (Inc.) provided lunch for the people attending the one day seminar on finite geometries, held on Saturday 2 August. The participants were Dr. Reuben Sandler (Auckland), Professor Johan Seidel (University of Eindhoven), and six students taking the Stage 3 course on finite geometries.

### Seminars:

Professor S. Chatterjee (New York University):

"Exploratory Analysis of Olympic Running Times".

Dr. Alan J. Lee (University of Auckland):

"Incomplete U Statistics".

Dr. Harold Henderson (Ruakura):

"Building Multiple Regression Models Interactively".

Dr. G. Ross Ihaka (University of Auckland):

"An Identification Problem with EEG Data".

Dr. Norman Bayley (University of Cologne):

"Bifurcation of Periodic Solutions of a Nonlinear Wave Equation".

Dr. Johan Seidel (University of Eindhoven):

"2-Graphs".

### DEPARTMENT OF COMPUTER SCIENCE

Dr. Bruce Hutton and Dr. Kevin Burrage have been appointed Lecturers. Dr. Hutton is currently a Junior Lecturer in our Department, and Dr. Burrage is currently a Researcher at the University of Sussex. It is hoped that 3 Junior Lecturers can be appointed for 1981.

Dr. Kees Decker, a numerical analyst from Amsterdam, is here as a Post-Doctoral Fellow.

Professor John Butcher attended the Melbourne session of the IFIP 1980 Conference in October (which followed the Tokyo session).

Garry Tee spoke on "L.J. Comrie and A.C. Aitken" at the First Australian Conference on the History of Mathematics, held at Monash University in November. Much interest was aroused by the replica of John Napier's pioneering calculating machine, his Promptuary for multiplication (1617), which was also demonstrated at the Australian National University, CSIRO DMS (Canberra) and the University of Melbourne. Dr. Jock Hoe's invited address on Zhū Shìjié" was regarded by many as the highlight of the Monash conference.

The first N.Z. Subject Conference on Computer Science was held here from the 25th to the 27th of August. It was attended by 32 people, including almost everyone teaching computer science at N.Z. universities. The delegates found that everyone was experiencing similar difficulties, with only a very few people in each Department to cope with many hundreds of students clamouring for courses in computer science. Furthermore, no Department has adequate departmental computing equipment (Auckland had none whatsoever), and the several university Computing Centres cannot provide the specialized services required for courses in Computer Science. The delegates agreed that each Department of Computer Science (or Information Science, et cetera) requires now many more people and much more equipment.

Since that Subject Conference, our Department has begun to receive some equipment. Ten Z89 microcomputers have arrived, and items on order include an LS11/23 minicomputer and 2 Data Royal printers.

## Seminars:

Dr. Peter Albrecht:

"On the order of stability of multi-stage multi-step methods for ordinary differential equations".

Professor E.K. Blum (University of Southern California):

"On a generalization of the homomorphism concept and its application to computer science".

Dr. Dick Cooper (University of Canterbury):

"Direct execution of intermediate languages".

Dr. Heinz Zemanek (IBM, Vienna):

"Theory of System Design".

Professor Mike Larkin (Queen's University, Kingston):

"A new method for finding simple zeros of functions".

A series of seminars on Stability has been given by Professor Butcher, Dr. Decker, Robert Chan and Dr. Graham Baird.

G.J.T.

## DEPARTMENT OF COMMUNITY HEALTH

Mr. David Eaton joined the staff as System Manager/Computer Programmer on November 10, replacing Chris Lovell, who left the Unit in July to gather some overseas experience.

## Seminars:

The following seminars have been presented to the Medical School Biometrics Group in the Department.

Dr. Brian McArdle (Zoology):

"Multivariate analysis of Bird song".

Mr. Alistair Stewart (Biostatistics Unit):

"Survival Analysis: A Case Study".

Professor Arthur Veale (Community Health):

"Investigation of a Genetic Model".

Professor Samprit Chatterjee (New York):

"Near-miss episodes and the sudden infant death syndrome: A study of breathing and sleep patterns".

P.R.M.

## WAIKATO UNIVERSITY

Kevan Broughan returns after Christmas from a year at Cambridge.

Peter Hill is on leave at Glasgow till May.

Mark Schroder leaves for parts unknown all next year.

Raoul Cornwell's stay here ran out on November 15, and he gave a farewell seminar, 'For The Record; Some Findings and Opinions Arising from a year as Visiting Teaching Fellow in the Mathematics Department', shortly before going.

Mary Fama has resigned, to return to the Mines Department. The one-year vacancy arising from this will be filled by Dr. M.D. Gould (Adelaide), who has a post-doc at Flinders and interests in Lie groups and plasma physics.

In addition, Dr. A.N. McClymont (a graduate of Glasgow) will be here on a post-doc for the first term, to improve our knowledge of solar and numerical MHD.

Professor S.D. Smith (Professor of Natural Philosophy, Herriott-Watt University, Edinburgh) gave the only other remotely mathematical seminar this term on August 22, on "The Optical Transistor".

M.S.

## MASSEY UNIVERSITY

Peter Thomson has resigned and will be taking up a post at Victoria University in the New Year.

Giovanni Moretti, our recently-appointed computing technician, has also resigned; he is moving to the Department of Computer Science at Massey.

Terry Moore, who is in the process of completing a Ph.D. in statistics, has been appointed to a three-year contract lectureship.



This year's promotions gave great pleasure to our whole department. Dean Halford (the current NZMS President) was promoted to reader, and Kee Teo to Senior Lecturer. Kee has just left for Singapore, where he will spend eight months sabbatical leave.

Several members of staff have been overseas for short periods recently - Professor Brian Hayman to Adelaide for the annual meeting of the AMS in May; Howard Edwards to the 5th Australian Statistical Conference in August; Gordon Knight, during August and September to the 4th International Congress on Mathematical Education (Berkeley) and a conference on mathematical education in Cambridge; and Chin Diew Lai to the University of Melbourne in November to work with Professor Williams.

#### Seminars:

Professor J.J. Seidel, from the University of Eindhoven, paid us a brief visit and spoke on "The Soccer-ball", giving suggestions for improved design.

David Johnstone, one of our M.Sc. students, spoke on "Theoretical Models in Ecosystem Modelling".

Ken Palmer, who recently joined the department, asked "Is there a Floquet theory for almost periodic systems of linear differential equations?"

Peter Thomson gave us a farewell talk on "Speech Recognition: an Application of Time Series Methodology".

Sirimathie Wenvala, another M.Sc. student, spoke on "Design and Analysis of Mixture Experiments".

We took advantage of David Gauld's presence at Massey for an NZMS regional council meeting, to hear him speak on "Something for nothing: some profound topological consequences of  $1 = 1 + (-1+1) + \dots = (1-1) + (1-1) + \dots = 0$ ".

M.R.C.

## VICTORIA UNIVERSITY

Peter Thompson has joined us from Massey University as a Senior Lecturer; we are glad to welcome him.

Megan Clark is soon to rejoin us from England as a Lecturer. (How many of us could get reappointed to our previous jobs if we resigned to face the international competition?)

John Burnell and Sharleen Forbes have been appointed Junior Lecturers.

Lindsay Johnston, Ken Pledger and Philip Rhodes-Robinson have all been promoted over the Senior Lecturers' bar - congratulations!

David Spence paid us a brief visit from Oxford to give a seminar about his biharmonic boundary value problems and commiserate with ours.

Normal Bazley is here for a month (Nov. - Dec.) as a Visiting Fellow from Cologne to work on bifurcation theory.

There are at the time of writing 6 Student Community Service Programme people in the department, mainly computing for various staff members on their research problems and on development of teaching programs for next year's classes.

There will be a seminar for secondary teachers on Wed. 29 April 1981 on Mathematics at Work in Other Subjects.

J.F.H.

## WELLINGTON POLYTECHNIC, SCHOOL OF MATHS & SCIENCE

John Offenberger, Head of School since its inception, left us for a year. He began a one-year contract at Anoka Ramsey Community College, Coon Rapids, Minnesota, in September. He expects to pick up ideas on tertiary education in the U.S.A. The College runs courses in jogging and belly-dancing, so we may see some changes on his return.

Bruce Phillips returned from a period of secondment with the Department of Education to take over John's position.

#### Recent Developments:

We have begun a series of short courses on microcomputer applications in EDP and have been experimenting with microcomputer applications to education.

M.C.

## D.S.I.R.

Robert Davies has returned from eight months at Berkeley, California, on a Fullbright grant.

*H.S.R.*

## CANTERBURY UNIVERSITY

Bob Long has returned from study leave in Australia and John De la Bere from study leave in the U.K.

Easaw Chacko has gone on leave to Chapel Hill, North Carolina. He is currently the only member of the Mathematics Department who is absent on leave.

Dr. David J.N. Wall has been appointed to a Lectureship in Mathematics. He comes to us from the Department of Engineering Mechanics at Ohio State University. Dr. Wall earned his Ph.D. in the Department of Electrical Engineering here at Canterbury, under the supervision of Prof. R.H.T. Bates. He has since held a Postdoctoral Fellowship in the Department of Mathematics at the University of Dundee, Scotland, and a visiting position at the Chalmers University of Technology in Sweden.

Bill Taylor has been promoted to Senior Lecturer.

## Seminars:

Dr. Susan J. Byrne (Massey University):

"A quadratic programming method".

Dr. S.K. Shum (Chinese University of Hong Kong):

"Order Ideals and Dilworth Elements".

Professors Josephine Mitchell and Lowell Schoenfeld, from Buffalo, New York, will visit us briefly in January 1981. She works in several complex variables, and he in analytic number theory.

Dr. David Spence, Reader in Theoretical Mechanics at the University of Oxford, has been offered an Erskine Fellowship to visit us during the first term of 1982.

*N.A.W.*

## OTAGO UNIVERSITY

Professor S.P.H. Mandel returned in August from his leave, which he spent at both University College, London and the University of Vermont in the U.S.A.

Mr. B.F.J. Manly will be on leave in 1981 and will spend most of it in the Department of Experimental Statistics of Louisiana State University (U.S.A.). He will also be visiting the Department of Mathematics at the University of Salford and the Department of Zoology at the University of Manchester (both in the U.K.).

Dr. Chris Meaney, who has been our Postdoctoral Fellow will be a Research Associate in the School of Mathematics at the University of New South Wales in 1981.

## Seminars:

Dr. David Hill:

"Embedding in Sequence Spaces".

Dr. John Clark:

"Rings of Small Order".

Dr. Chris Meaney's talks on

"Compact Groups" included the following topics: representations of compact groups; Frobenius reciprocity; Fourier series on compact homogeneous spaces; spherical harmonics on the  $n$ -dimensional sphere; and the Fourier algebra of absolutely convergent Fourier series.

*G.O.*

## UNIVERSITY OF THE SOUTH PACIFIC

Visitors this year have included:

Richard Cooper (University of Canterbury, New Zealand) who gave useful advice on computing courses and facilities;

Colin Meek (Ministry of Education, Solomon Islands) who discussed research in mathematics education;

Michael Larkin (Queen's University, Canada) who contributed to ED 314 (Numerical Analysis) and discussed research and teaching in numerical analysis and computing;

Ted Phythian (Open University, UK) who contributed to ED 314 and advised on distance teaching as well as computing courses and facilities;

David Vere-Jones (Victoria University, New Zealand) who made his final visit as external assessor and once again acted as a catalyst for worthwhile discussions on our teaching and research activities (this year he also visited Samoa and held discussions with staff at Alafua and the Extension Centre).

Donald Joyce gave an invited address, on "The Electronic Teacher" to the Annual Conference of the Fiji Mathematics Association in August, then flew to Sydney for the Australian Statistical Conference and the inaugural meeting of the East Asian and Pacific Regional Committee of the Bernoulli Society, giving a paper on "Statistics Training in the USP Region". Ananda Rao helped with the organization of the Annual Conference of the Fiji Mathematics Association.

Staff have been involved in a variety of consultancies, including the following: construction of the lecture timetable, preparation of computerized examination results, review of secondary mathematics curricula in Fiji and Tuvalu, statistical analyses, calculation of mortgage rates, selection of computing equipment. Research projects on "The Use of Microcomputers and Calculators in Teaching and Research" and "Cross Cultural Studies into Mathematical and Logical Thinking" have been initiated.

The most significant equipment purchases this year were of two microcomputer systems: a 48K Apple with dual disk drives, thermal printer, colour monitor and language system and a 32K Commodore with dual disk drives, impact printer and cassette recorder. Although initially expensive (about \$3000 each), these systems will soon "pay for themselves" in saved computer charges (which exceeded \$1000 in semester 1, before the microcomputers arrived).

Seminars:

Donald Joyce:

"Mathematics as an Experimental Science".

Bill Miller (Physics):

"Transcendental Numbers" and "Derivatives Revisited".

Julia Norton:

"A Simple Model for Basal Body Temperature"

Geoffrey Whittle:

"Zeno's Paradoxes" and "Discrete Models for Space and Time".

Joseph Ha:

"A Parabolic-coherence Model for Wave Propagation" and "Mathematical Fallacies".

David Hassall (Biology):

"Numerical Taxonomy"

Peter Sanders (Education):

"Teaching Aids in Mathematics"

Ananda Rao:

"Ancient Hindu Mathematics".

Colin Meek (Ministry of Education, Solomon Islands):

"Implications of Research into Mathematics Education conducted in Papua New Guinea".

Michael Larkin (Queen's University, Canada):

"Simple Root Finding by Divided Differences".

Ted Phythian (Open University, UK):

"Mathematics Applied in a Developing Country".

D.J.

## ACKNOWLEDGEMENT

The splendid photograph of Professor Butcher in the last centrefold was reproduced by courtesy of the Auckland Star.

## UNDERGRADUATE GEOMETRY AT AUCKLAND 1883 - 1974

- Peter Lorimer -

For many years I have had the wish to improve the education that our mathematics students are getting. In a limited way I suppose I have had some success: I have taught my courses with the interests of my students at heart and once or twice I have had a small influence in designing a new course or redesigning an old one. But it gradually became clear to me, or it was forcefully pointed out to me, that I really have little influence on the way that mathematics develops in my University. The ability to make changes rests very much with the people in our Mathematics Departments who are Professors; the rest of us are left with the right to make suggestions which may, of course, be taken up, but may also be rejected out of hand or shelved by being delayed.

This essay is another attempt on my part to influence the evolution of mathematics teaching in our Universities. When decisions are made by authority and the authority has no formal need to defend his decisions, he must expect that others will put them under scrutiny. My object is to examine the education that our students have received in the past to see whether these authorities have carried out their duties in a competent manner. This has a bearing on the future. If the present constitutions of our Universities are such that our students have had a very good education in the past then we may be satisfied to continue in the same way; but if this education has been less than adequate, then we must examine our plans for the future in the light of that finding. He who refuses to learn from history is doomed to repeat the mistakes of the past.

My conclusions are that the education that our undergraduate students have been receiving in geometry at Auckland has been inadequate. It is difficult for me to say where the blame for this lay in the early days, because all exams were set in Britain then, but recently it has been the Auckland Professors who have had the responsibility. My experience suggests that we must somehow get away from concentrating the decision making powers in a few hands. In our Universities today the rank of a person is correlated more closely to his age than anything else and the situation seems to be that the present distribution among the ranks is likely to be very static for many years ahead. The strength of our Mathematics Departments is being concentrated among the Senior Lecturers and we must look for a way to move the right to make decisions where it belongs. Unless something is changed we can look forward to the present imbalance becoming further away from level.

The most obvious of the practical solutions to this problem is to loosen the hierarchy by changing to a structure like that in North America and parts of Europe where a larger fraction of the staff are Professors. However, my hope is that we can go in the other direction and abolish ranks: the present system, which is exceeded in rigidity only by the army, seems out of place in a University with pretensions to academic freedom and so on.

I ought to say something about why this essay is restricted to geometry. In order to draw the conclusions I want about undergraduate mathematics I should really survey all of it, not just a part. However the magnitude of this task daunts me: it is something that would have to be done over a period of years. In the meantime I hope that this lack will not be used as an excuse by those in a position to make decisions to ignore this report.

### WHAT IS POSSIBLE?

A mathematics education ought to have a purpose. There should be some reason for young people to come to the University and spend a large part of their time studying mathematics, and this reason should be explained to them. I want to investigate now what the study of geometry can offer.

It is necessary to begin with something of the history of the subject. My aim here is not completeness; rather I want to pick out the highlights that are relevant to the subject of this report.

All my historical remarks are based on Morris Kline's book "*Mathematical Thought from Ancient to Modern Times*".

Geometry began with the formalization of the Egyptian rules for surveying land by the early Greeks. Its early development, notably by the schools of Pythagoras and Eudoxus, culminated in the publication of "*The Elements of Geometry*" by Euclid about 300 B.C. The study of Euclidean geometry was continued after Euclid with Apollonius, Archimedes and Pappus being eminent. This activity left a large body of knowledge on what we would now call elementary geometrical objects, the line, plane, circle, sphere, conics, some quadrics and some more complicated curves. It is justly described, in Stephen Leacock's words, as primitive geometry. This is not to belittle it; it was geometry of its time, it is evidence of the genius of men who produced it and it has had an enormous influence on the mathematics that came after it. The period of classical Greek mathematics was from about 600 B.C. to about 400 A.D.

The study of perspective in Renaissance Art led to the development of projective geometry by men such as Desargues and Pascal in the seventeenth century. At much the same time the work of Fermat and Descartes gave us co-ordinate or analytical geometry.

Also belonging to the seventeenth century is the discovery of calculus by Leibniz and Newton. This had an enormous influence on all the mathematics that came after, not excepting geometry. Up until this time geometry had been concerned with things that were in some way simply presented; lines, planes, conics and quadrics have polynomial equations of degree at most two for example; but calculus gave a method for attacking more general objects, a method much more powerful than anything which came before.

Newton is also famous for his study of gravitation and cosmology, topics not at once recognized as geometrical, but Einstein remarks in his 1953 preface to the book "*Concepts of Space*" by Max Jammer, that

*"space must be introduced as the independent cause of the inertial behaviour of bodies if one wishes to give the classical principle of inertia (and therewith the classical law of motion) an exact meaning. To have realized this fully and clearly is in my opinion one of Newton's greatest achievements".*

The true role of non-Euclidean geometry was discovered in the early nineteenth century by Gauss, Bolyai and Lobatchevsky. Among Gauss' contributions were not only that non-Euclidean geometry was the mathematical equal of Euclidean but also two other fundamental things: Euclidean geometry is not necessarily the geometry of space; other geometrical objects, particularly surfaces, had their own intrinsic geometry and could be regarded as geometries in their own right. Those ideas were taken up by Gauss' student Riemann, who extended them to higher dimensions and explicitly made speculations about the geometry of the space in which we live.

Tied up with all this was the beginning of differential geometry which, on its lowest level, is the application of calculus to geometry.

The early 19th Century saw a revival of projective geometry, beginning with Monge and continuing with others, principally Poncelet.

Two other events of the late nineteenth Century were the Erlanger Programm in which Klein linked certain types of geometries with certain groups and the publication by Hilbert of "*The Foundations of Geometry*" which finally put Euclidean geometry on a proper axiomatic basis.

The last hundred years has seen the development of topology which has concentrated on different geometrical aspects of things and has helped in understanding a new range of objects such as knots, the Klein bottle, and so on.

In 1905 Einstein published his first paper on special relativity. This was followed by Minkowski in 1908 with an interpretation which removed space as a basic physical object and replaced it by space-time. In general relativity, first propounded fully in print by Einstein in 1916, gravitation is also incorporated into geometry in a very fundamental way.

Two other things worth mentioning from the twentieth century are the invention of finite geometries and the development of the theory of differentiable manifolds.

This, then, is a rough outline of the major parts of geometry and some of the influences on it. Let us now consider what an education in geometry might be like.

Geometry is, first of all, the science which investigates the underlying structure of the world in which we live. Until the days of Gauss and Riemann, Euclidean geometry was thought to be the geometry of space; they raised doubts. Minkowski moved the emphasis from space to space-time and from three to four dimensions. In Einstein's general theory of relativity, space-time is curved by gravitational fields.

Those things could form a basis of one aspect of an education in geometry: an aim could be to develop in the students a feeling for what space and space-time are like. First there is the traditional Euclidean geometry, but not only is there the traditional way of studying it, there are also some special skills which can be used to explore it: I think of surveying, painting, some aspects of astronomy and navigation (although the latter is really a part of the non-Euclidean geometry of the sphere). To the extent that it enables calculations to be done with triangles, trigonometry is also a proper part of this geometry. The mathematics needed for the special theory of relativity is quite simple and this is a proper topic for undergraduate geometry. A.S. Eddington wrote in 1921 that

*"I am convinced that the relativity outlook will in time lose its strangeness and become a commonplace of educated thought, just as the Copernican system has done".*

It could for our students. The mathematics of the general theory of relativity is probably beyond the undergraduate, but there are many things about it which could serve as an introduction to it: for example, the quote of Einstein given earlier is worth investigating.

The intrinsic geometry of many objects such as the 2-dimensional and 3-dimensional spheres is also a proper topic. Many things about those objects can be approached quite easily without much mathematics, but a proper appreciation of them requires calculus: in my opinion a study of the differential geometry of surfaces from a geometrical point of view should go hand in hand with a student's study of the more usual parts of calculus. A major aim should be to give the students a feeling for what these are like as spaces.

A study of geometrical objects, things which exist as part of a space, is also a proper study. Those things range from the traditional objects such as the line, circle, ellipse and so on to others such as the regular solids (traditional but not widely studied in courses), crystals, higher dimensional objects such as the 4-dimensional tetrahedron and cube, many of the objects from topology and configurations of points and lines. There are also the finite geometries which are both geometrical objects in this sense and spaces of a type in their own right. The spaces in the last paragraph can also be regarded as geometrical objects in this spirit.

It was once supposed that one of the advantages of studying the geometry of Euclid was that it gave the student training in logical thinking. As this view is now largely discredited, if only because the logical reasoning in Euclid was not as strict as supposed, I do not wish to dwell upon this point. (See, for example, Bertrand Russell writing in the *Mathematical Gazette*, 1902).

What can a student hope to gain by studying geometry? First of all he can expect to gain some knowledge, but if that were all, it would not be worth the trouble for most of us. It is generally reckoned that education has other benefits and some of these can be made explicit in the case of geometry.

First, a student might expect to gain some intuition or feeling for the space he lives in and the objects that inhabit it. Here is a test of intuition in a common puzzle: a long strip of paper is twisted a few times and the ends joined together (as in a Moebius strip); if it is now cut down the middle, how many pieces will result and how will they be twisted? Some people will be able to solve this by visualizing what happens, an ability which is worth developing. To the extent that special relativity is a model of the real world, we inhabit a space-time which is governed by its laws. Can we get students to the stage that they inhabit this space-time just as naturally as the ordinary one? This seems

a worthwhile result to get from a study of geometry.

Along with intuition of what is, goes imagination of what might be. What is a 4-dimensional cube like? Whatever facility for imagination a student has, it can be developed.

Many geometrical things are very appealing, some to the extent that we would call them beautiful. People like dealing with geometrical things particularly if they have a high degree of symmetry or are made from an appealing substance; for example solid objects from wood and crystals of some minerals.

The book par excellence for things connected with intuition, imagination and perhaps beauty is "*Geometry and the Imagination*" by David Hilbert. This book comes from lectures that Hilbert gave at the University of Göttingen in 1920-21.

On a more practical level the solving of problems should occupy a large part of a student's time in geometry as elsewhere in mathematics.

An undergraduate education takes three years of a person's life. After it he will have hoped to have matured considerably, somewhat more than if he had not come to the University. It is reckoned by many that the greater part of education is gained outside the classroom: nonetheless we should keep the overall development of our students in mind.

It is unfortunately true that very few of our students will become professional mathematicians: our teaching is not of the same nature as that in the medical school or the school of engineering. None the less, when they leave the University they have to find a job and what they find will determine their future life to a large extent. The University cannot be indifferent to the future of its students: what they will need in life should have a large influence on what is selected for their education. The relevance here is that when there is something geometrical and useful it could well displace something just geometrical; for example the geometrical background to linear programming has a pressing case to be included in our courses.

This may all sound like a song in praise of geometry but that is not its purpose. It is meant to set a standard by which we can judge what has actually been going on in undergraduate geometry in Auckland.

#### WHAT HAS IT BEEN LIKE?

The source of the information in this section is the calendars of the University of New Zealand (NZU), Auckland University College (AUC) and the University of Auckland (AU), which the college became in 1958, together with the exam papers set over the years by these institutions. Nowadays the information about courses in our calendars is notoriously unreliable but this has not also been so; in the old days the course prescriptions governed the exams which were set in England and the lectures prepared students for the exams. The calendars and exam papers that I used are the collection in the library of the University of Auckland.

The University of Auckland began its life in 1882 when an Act of Parliament established Auckland University College as a college of the University of New Zealand. The University of New Zealand was the degree granting institution; it determined the syllabus for the college and conducted the official exams. According to the 1884 AUC Calendar there were 11 subjects that could be studied for the Bachelor of Arts degree, one of which was Mathematics. To gain a B.A. a student had to pass the UNZ exams in five subjects of which two had to be Latin and Mathematics: he was examined in two or three subjects of which one had to be Latin or Mathematics at the end of two years study and in the rest after another year. To qualify to sit an exam in a subject he had to "keep terms"; this normally involved, among other things, attending lectures at the college and sitting internal exams there.

The 1884 AUC Calendar has mathematics as three papers: elementary geometry and trigonometry; algebra; elementary mechanics and hydrostatics. The prescription for geometry states simply

*Euclid, Books I., II., III., IV., and VI.,  
together with the definitions of Book V.*

Judging from the exam papers, the geometry was about half the paper it shared with trigonometry.

This prescription remained the same until 1904, the only change in the NZU calendar entry being a note stating that

*In the Euclid examination easy riders will be set; and no candidate will be allowed to pass unless he exhibits a fair knowledge of both Geometry and Trigonometry.*

The 1904-5 NZU Calendar contained a new definition of Elementary Geometry, to be valid from 1905. It is reproduced here as Figure 1. There is considerable expansion here from the earlier prescription. Firstly, much of the solid geometry comes from Books XI and XII of Euclid and secondly, the work of authors later than Euclid is included. For example the facts about the sphere at the end of schedule D are due to Archimedes and polar co-ordinates, mentioned under Practical Geometry originated with Newton.

Figure 1

## Extract from the 1904-5 NZU Calendar

NOTE.- In and after 1905 the definition of Elementary Geometry shall be as follows -

(a) *Elementary Geometry.*

- (i) Practical Geometry, and the Theoretical Geometry of angles at a point, parallel straight lines, triangles and rectilinear figures, areas, loci, and the circle as defined in the Statute "Matriculation," section 1, subsection 4.  
 (ii) Practical Geometry, as in Schedule C below.  
 (iii) Theoretical Plane and Solid Geometry, as in Schedule D below.

## SCHEDULE C.

*Practical Geometry.*

Determination by measurement of the ratio of the circumference of a circle to its diameter.  
 The area of a circle.  
 To draw a normal to a plane from an external point.  
 Projections of a point on three planes at right angles.  
 Determination of a point by means of its coordinates  $(x, y, z)$ , referred to three rectangular axes, and by means of its polar coordinates.  
 Projection of a straight line on a plane making a given angle with it.  
 Projection of a plane figure on a plane making a given angle with it.  
 Development of the right prism, and of the right pyramid.  
 Determination of the surface, the base being a regular polygon, of the right prism and right pyramid.  
 Volume of the prism and pyramid.  
 The generation of the right circular cylinder, right circular cone, and sphere by resolution,  
 Development of the right circular cylinder, and right circular cone; the surface of each.  
 Volume of the cylinder, cone and sphere.

## SCHEDULE D.

*Theoretical Geometry (Plane).*

If a straight is drawn parallel to one side of a triangle, the other sides are divided proportionally; and the converse.  
 If two triangles are equiangular, their corresponding sides are proportional; and the converse.  
 If two triangles have one angle of the one equal to one angle of the other, and the sides about their equal angles proportional, the triangles are similar.  
 The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle, and likewise the external bisector externally.  
 The ratio of the areas of similar triangles is equal to the ratio of the squares on corresponding sides.  
 The ratio of the areas of similar polygons is equal to the ratio of the squares on corresponding sides. In equal circles (or in the same circle) the ratio of any two angles at the centre or of any two sectors is equal to the ratio of the arcs on which they stand.  
 The rectangle contained by the diagonals of any quadrilateral is less than the sum of the rectangles contained by its opposite sides - excepting in the case of a cyclic quadril-

ateral, when the inequality becomes an equality.

The area of a regular polygon = the area of a rectangle whose base is half the perimeter and whose altitude is the apothem.  
 The area of a circle = the area of a rectangle whose base is equal to half the circumference and whose altitude is the radius.

*Solid Geometry.*

The intersection of two planes is a straight line.  
 Through three points not colinear only one plane can be drawn.  
 If a straight line be perpendicular to each of two straight lines at their point of intersection it is perpendicular to the plane passing through them.  
 All normals to the same plane are parallel to one another.  
 The projection of a straight line on a plane is a straight line.

Planes which have a common normal are parallel.

Every plane passing through a normal to a plane is perpendicular to that plane.

The plane angles which contain any solid angle are together less than four right angles.

The locus of a point which is equidistant from two fixed points is the plane bisecting at right angles the join of the two fixed points.

The locus of a straight line perpendicular to a given straight line at a given point therein is a plane.

The locus of a point equidistant from two fixed planes consists of the two planes passing through the intersection of the fixed planes and bisecting the angles between them.

The section of a prism or a pyramid by a plane parallel to its base is similar to the base, and in the case of a prism is equal to the base.

The section of a right circular cylinder or cone by a plane parallel to the base is a circle, and in the case of the cylinder is equal to the base.

The section of a sphere by a plane is a circle.  
 The intersection of two spheres is the circumference of a circle.

The intersections of a sphere and a right circular cylinder or cone whose axis passes through the centre are circles.  
 The cylinder or cone circumscribing a sphere touches the sphere along the circumference of a great or small circle respectively.

Parallelepipeds (or prisms, or pyramids, or cylinders, or cones) on the same base and of the same altitude are equal in volume.

A prism on a triangular base can be divided into three triangular pyramids equal in volume; hence

The volume of any pyramid (or cone) = one-third of the volume of the prism (or cylinder) on the same base and of the same altitude.

The volume of a sphere =  $\frac{2}{3}$  of the volume of the circumscribing cylinder.

The surface of a cone =  $\frac{1}{2}$  circumference of the base  $\times$  length of the slant side.

The surface of a sphere =  $\frac{2}{3}$  of the surface of the circumscribing cylinder = 4 times the area of a great circle of the sphere.



In 1906 it became possible for a student to repeat one subject in a B.A. at a higher level. The repeated subject for mathematics students contained pure and applied mathematics and he had a choice of sitting the papers for Senior Scholarship or both the Stage II and III engineering mathematics papers. In that the Senior Scholarship papers were the so-called Honours papers which were also sat by M.A. students the situation must have been complex. Anyway, the geometry prescription for Honours was

*Elementary geometry. As for B.A. with the addition of Elementary modern geometry, plane co-ordinate geometry and conic sections. Deductions and examples will be set to some or all of the propositions given.*

I don't know what the word "modern" means here. The exam papers of the period suggest only a study of Euclidean geometry including conics treated without co-ordinates.

The Senior Scholarships present a problem. There was one of those available for competition among undergraduates in Mathematics. It was awarded on the results of the Honours papers which were also sat by M.A. students; presumably the Honours lectures were attended by both sets of students, though what a bright student did for an M.A. if he had already done well in the exams I don't know. Significantly, one of the Honours papers was on Calculus so the brighter undergraduates would probably have gone to lectures on that subject. Because it would get me too far from the standard undergraduate courses I don't want to pursue the topic of Senior Scholarships.

In 1913 the compulsion to do some Mathematics for B.A. was dropped: from then on it was only compulsory to study one of Latin and Greek!

Practical geometry as such was dropped in 1914. The 1913-14 NZU calendar announced a change which put algebra in the first paper, geometry in the second and trigonometry in both: judging from the relevant exam papers, the three subjects had equal weight. The prescription for geometry became

*as defined in Matriculation and in the Schedules C and D below*

Schedules C and D are reproduced in Figure 2 and seem to be substantially the same as the earlier ones in content. Notice the last sentence

*Candidates will be expected to show a knowledge of the practical geometry implied in the propositions included in the above schedules.*

(the spelling of propositions is corrected from later calendars).

Figure 2

<p style="text-align: center;">SCHEDULE C.</p> <p><i>Plane Geometry.</i> A straight line parallel to one side of a triangle divides the other two sides in the same proportion and the converse. If two triangles have the angles of one respectively equal to the angles of the other, the lengths of their corresponding sides are proportional; and the converse. If two triangles have an angle of one equal to an angle of the other and the sides about their equal angles proportional in length, the triangles are similar. The bisectors of an angle of a triangle divide the opposite side, internally and externally, in the ratio of the lengths of the other two sides. Triangles of a given altitude have areas proportional to the length of their bases. If two parallelograms (or two triangles) have an angle of one equal to an angle of the other, the ratio of their areas is equal to the product of the ratios of the lengths of their sides (about the equal angles). The ratio of areas of two similar triangles is equal to the square of the ratio of lengths of corresponding sides; and the corresponding theorem for similar polygons. If two similar polygons have their corresponding sides parallel, the lines joining corresponding vertices are concurrent. If two circular sectors have the same radius, the ratio of their areas and the ratio of the lengths of their arcs are equal to the ratio of their angles. If two circular sectors have the angle of one equal to the angle of the other, the ratio of the lengths of their arcs is equal to the ratio of their radii; and the ratio of their arcs is equal to the square of that ratio.</p>	<p>A circle is equal in area to a rectangle of which the sides are equal in length to its radius and to its semi-circumference. The area of the rectangle under the diagonals of a cyclic quadrilateral is equal to the sum of the areas of the two rectangles under the two pairs of opposite sides.</p> <p style="text-align: center;">SCHEDULE D.</p> <p><i>Solid Geometry.</i> Propositions on intersections, parallelism and perpendicularity, for planes with straight lines and with planes. There is in general one straight line which intersects each of two given straight lines and is at right angles to both. A system of parallel planes divides all transverse straight lines in the same proportion. The section of a pyramid by a plane parallel to its base is a polygon similar to the base. The section of a right circular cone by a plane parallel to its base is a circle with radius proportional to the distances of the plane from the vertex of the cone. The section of a sphere by a plane is a circle. Parallelepipeds which have the same base and are equal in altitude are equal in volume; and the corresponding propositions for prism (and cylinders) and pyramids (and cones). The volume of a prism is jointly proportional to its altitude and the area of its base. The volume of a pyramid is one-third that of a prism which has the same base and equal altitude. Propositions on the volumes and superficial areas of right circular cylinder, right circular cone, and sphere. Candidates will be expected to show a knowledge of the practical geometry implied in the proportions included in the above schedules.</p>
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By 1915 the repeated subjects had their own prescriptions. Pure Mathematics became separate from Applied and the geometry required was

*The fundamental theorems of modern Plane geometry and of Conic Sections. Elementary analytical treatment of Plane Geometry so far as to involve the equation of the first degree and simple forms of the equation of second degree.*

Again the exam papers give no hint as to what was meant by "modern". The new prescription for Pure Mathematics contained no calculus.

In 1920 it was changed again. The two levels for B.A. were renamed as Pass and Advanced, Pure and Applied Mathematics were reunited at the advanced level and it became possible to study two advanced subjects. The advanced prescription for geometry became

*Pure geometry including the elements of projective geometry and conic sections; plane analytical geometry, so far as to include conics referred to their principal axes; geometrical applications of the calculus.*

These prescriptions remained in force until 1927 when a restructuring of the B.A. was announced. In that year the system of Pure Mathematics I, II and III came into being, the I, II and III referring to first, second and third year courses respectively. A student was required to pass 9 units for his degree, with restrictions on the numbers of units from different sections that could be taken. Each of the new units was examined in two papers. The old Pure Mathematics became Pure Mathematics I, one paper of Pure Mathematics II was on geometry and trigonometry and one of the stage III papers was on Geometry alone. The prescription for Stage II geometry was

*Harmonic section; poles and polars; radical axis; inversion; analytical Geometry of the straight line and circle; simple properties of conics deduced by Pure Geometry or from the conical equations*

and for Stage III

*Geometry: Elements of Projective Geometry.*

*Analytical Plane Geometry: Elementary treatment of the equation of the second degree, and tracing of conics from the general equation.*

*Analytical Solid Geometry: The straight line, plane, and sphere.*

The changes in 1920 and 1927 were a real departure from what had been taught before. Up until this time, geometry was firmly rooted in the book of Euclid, already 2300 years old. Now came some new ideas, not exactly modern, but not ancient either. The new mathematics here comes from many sources. The study of conics has been a constant theme in geometry, beginning before Euclid, continuing with Appolonius and Archimedes, then Desargues, Fermat and Descartes in the 17th century and Monge and Poncelet in the 19th. Conics will be a feature of the prescriptions at AUC until 1974; they will draw from all these eras, treating them at a level judged appropriate for the particular stage. Three-dimensional analytical geometry was developed in the early 18th century: the straight line, plane and sphere are the simplest 3-dimensional objects which can usefully be treated analytically.

The next change came in 1938. The stage I paper in geometry and trigonometry became three-fifths geometry and the prescription became

*Plane geometry, including properties of triangles and Theorems of Menelaus and Ceva. Solid geometry as usually defined.*

Menelaus and Ceva were two more Greeks, both living after Euclid. Although this was the first time their theorems had been mentioned in prescriptions, they were presumably taught before, because questions on them had previously been asked in exam papers. Judging from the exam papers based on this prescription, the rest of it meant much the same as Schedules C and D in Figure 2. The Stage II prescription became

*Harmonic section; poles and polars; coaxial circles; inversion; simple properties of the conic sections treated by the methods of pure and analytical geometry.*

There is no substantial change here: although the analytical geometry of the straight line and circle were dropped from the prescription, questions were still asked about them in exams. The stage III prescription was changed to

*Projective Geometry, elements, including cross-ratios, the use of the circular points and applications to the theory of conics.*

*Analytical Geometry: two-dimensional, including conic sections and the general equation of the second degree; three dimensional, the plane, the straight line and the sphere.*

This seems to be a restatement of the previous prescription.

Things continued in the same way until 1949 when there was a fundamental change in the University of New Zealand. Up to 1949, UNZ was responsible for the prescriptions and for the official examinations in the country; the students were taught in the colleges and sat internal exams there, but what really counted were the exams set by UNZ. Now the colleges took responsibility for their own stage I and II exams. This began a series of constitutional changes which saw the responsibility for the stage III exams being passed a few years later and finally the University of Auckland, an independent University, came into being in 1958.

As it happened, this didn't make much immediate impact on the teaching of geometry. The 1949 prescription in Pure Mathematics I, now in the AUC calendar, was

*Plane geometry. The circle, triangle and simple properties of the parabola and ellipse. Solid geometry as usually defined.*

The major innovation here is that the study of conic sections was included in the first year course for the first time. The Stage II prescriptions was transferred word for word from the NZU Calendar, and the stage III prescription, still in the NZU Calendar became

*Analytical Geometry of two dimensions.*

*The general conic; reduction of conic to its principal axes; tangential equations, applications; pencils of conics.*

*Analytical Geometry of three dimensions; the plane; straight line, sphere, quadric cone.*

*Projective Geometry: the elements, including properties of cross-ratios, the use of circular points, involution and application to conics, e.g. Pascal's Theorem, poles and polars.*

*Inversion.*

There are one or two new things here: the study under "general conic" looked at conics in a deeper way first undertaken by Fermat, Kepler and Descartes; the quadric cone is perhaps the simplest 3-dimensional object after the sphere; much of the projective geometry is due to Poncelet who worked in the first half of the 19th century.

The next changes came in 1957 when the prescriptions for geometry became

*Stage I: Properties of lines, circles, conics obtained by co-ordinate methods; fundamental notions in solid geometry.*

*Stage II: Transformations of co-ordinates in the plane; further treatment of the conic; invariants. Co-ordinate solid geometry (simple loci; planes; spheres). Vectors.*

*Stage III: Plane geometry: The projective plane; lines; conics; homogeneous co-ordinates.*

*Solid geometry: Quadrics in three dimensions (co-ordinate methods).*

The stage I and II sections were each examined in a half paper and the stage III in two half papers.

Quadric surfaces of revolution were studied by Archimedes who is supposed to have burnt the sails of Roman ships by concentrating the sun's rays on their sails with a mirror

shaped as a paraboloid of revolution. The co-ordinate geometry of quadrics was established by Monge and Clairaut in the 18th century.

By 1962 projective geometry was dropped from Stage III.

In 1967 a special first year course for Engineering Intermediate students was begun: this sort of course will be ignored here.

Also in 1967 the Stage III syllabus was changed to only

*Co-ordinate solid geometry (simple loci, planes, lines, spheres).*

In 1969 a new system foreshadowing the paper system for B.A. and B.Sc. was set up in the Mathematics Department. Each of the units was split into its component papers and the students were free to arrange them into units to suit themselves, subject to a complicated set of rules. Among all the resulting papers geometry has only one mention: there is a first year paper called "Calculus and Geometry" and the geometry prescription is

*properties of lines, circles and conics treated by co-ordinate methods; fundamental notions in solid geometry.*

This is the formula used in Stage I for the previous 12 years. There were no longer second and third year papers which included geometry as such.

In 1970 one of the first year papers was renamed "Elementary functions and co-ordinate geometry", the geometrical part of which was defined as

*rudiments of plane analytical geometry, including locus problems, polar co-ordinates, and geometry of complex numbers; introduction to three dimensional analytical geometry.*

The other principal first year paper, called "Introductory Calculus" had *Calculus in Euclidean Geometry* as part of its syllabus. The second year calculus course included *solid analytical geometry*.

In 1971 the first of these three was changed to *transformations in the plane, conic sections* while the other two remained the same.

In 1974 conic sections was dropped.

Here is where I want to leave this little history. Since then the main stream courses have remained much the same but two new geometry courses have been introduced at Stage III. I don't want to go into the details of them, partly because I was involved in one of them and partly because they have been peripheral to the main teaching of the Mathematics Department.

This completes the details of the undergraduate courses which contain geometry as a dominant component. It is clear though, that many other places have it as a subordinate yet essential topic of study, for example trigonometry and calculus, and my object now is to review other mathematical courses for the geometry they contained. As there is rarely explicit mention of geometrical topics in the prescriptions beyond what I have already detailed, the evidence will come from the relevant exam papers.

Trigonometry, or the study of triangles, seems to have been on about an equal footing with geometry in the early days, the 1888 paper (a) asking five questions on geometry and four on trigonometry. It continued to be a significant part of at least Stage I mathematics until 1969. Its geometrical content seems to have been of a rather routine nature. For example a question asked in 1918 was

*In a quadrilateral field ABCD, the side AB = 120 ft., BC = 160 ft., CD = 140 ft., angle ABC = 110°, and angle BCD = 132°. Find the area of the field.*

and in 1940

*If O, I be the circumcentre and incentre of a triangle ABC, prove that*

$$OI = 4R \sin \frac{B}{2} \sin \frac{C}{2}$$

and hence, or otherwise that  $OI^2 = R^2 - 2Rr$ ;  
 where  $R, r$  are the radii of the circumcircle and incircle.

From the point of view of geometry these are properly described as questions on the use of trigonometry in Euclidean geometry.

The first significance of calculus in geometry is that it gives a powerful method of studying geometrical objects. It gives a general method for calculating areas and volumes, maxima and minima, describing curves given by equations and so on. This seems to be the level on which geometry is treated in the calculus courses.

Calculus was not a part of the undergraduate curriculum until 1927 when it was first taught at Stage II and III. The stage II prescription is

*Elements of differential and integral calculus involving no geometrical application going beyond the above geometry*

"the above geometry" presumably refers to the geometry then being taught at Stages I and II; this is detailed earlier. The prescription for stage III is

*Differential and integral calculus with applications to plane curves.*

A question from the 1930 Stage III exam is

*Find the angle between the radius vector and the tangent at a point on the curve  $r = f(\theta)$ .*

*Find the orthogonal trajectory of the family of curves  $r^2 = A \cos \theta$  where  $A$  is an arbitrary constant. Sketch the two families of curves.*

Here is a proper application of calculus to geometry, studying ideas different from those studied in the geometry courses, and curves having more complex equations.

In 1940 the Stage II students were asked to

*Prove that for the curve  $\frac{y}{c} = \frac{1}{2} (e^{x/c} + e^{-x/c})$*

*we have  $\frac{s}{c} = \frac{1}{2} (e^{x/c} - e^{-x/c}) = \tan \psi$*

*where  $s$  is the length of the arc from a suitable point, and  $\psi$  is the inclination of a tangent to the axis of  $x$ .*

*If the curve is rotated about the  $x$  axis, find the volume generated between the curve and the planes  $x = 0, x = h$ .*

Stage III in 1940 contained questions on radius of curvature and envelopes.

In 1972 there was a first year paper in Calculus: it contained questions on routine aspects of the application of calculus to sketching curves; areas and volumes. The stage II had a question which considered tangent planes to the surface with equation  $x^{2/3} + y^{2/3} + z^{2/3} = a^{2/3}$  ( $a > 0$ ) and the stage III had a question asking for the points on a particular ellipse (in a 3-dimensional space) which are closest and furthest from the origin.

It seems fair to say that while these courses gave the students a fair grounding of the more or less routine ways of applying calculus to geometry, they did not go any deeper into the ideas of Gauss and Riemann.

Linear algebra became a second year subject of study in 1969. This is important as a modern development from co-ordinate geometry which permits spaces of higher dimensions than 3 to be studied comfortably. In general, the linear algebra courses have been strongly algebraic with the applications to geometry being fairly trivial. For example, in 1973 the students were asked:

*If  $U$  is a subspace of the finite dimensional vector space  $V$ , show that there is a subspace  $W$  of  $V$  such that  $V = U + W$*

the corresponding geometrical fact, while adding to a novice's knowledge of finite dimensional affine spaces, is at a superficial level.

## A CRITIQUE

The geometry taught at Auckland over the years came almost exclusively from four sources: the ancient Greeks; the analytical geometers of the seventeenth and eighteenth centuries; and the projective geometers of the seventeenth and nineteenth centuries.

The geometry in the first year courses came exclusively from the ancient Greeks and principally from Euclid himself until 1949 when the College took over responsibility for its own prescriptions. This means that 2000 year old mathematics ruled this prescription for the first 65 years of the life of the college, a period which ended only 30 years ago. The innovation in 1949 was to introduce plane co-ordinate geometry, then more than 300 years old and apart from one or two things such as polar co-ordinates and the geometry of complex numbers which surfaced briefly, nothing new was added before geometry more or less faded away.

The strict adherence to Euclid and his immediate successors was not a part of the second and third level courses even when they were first possible in 1906. Co-ordinate plane geometry was the feature of the repeat course then and remained so until it too was dropped in 1969.

A third level became possible only in 1927 and it featured co-ordinate plane and solid geometry and projective geometry. The projective geometry was dropped in 1967 and the rest in 1969.

In a previous section I have mentioned what I thought students might gain from a study of geometry. I now want to examine how well they have been served over the years.

The most tangible thing they could gain was knowledge. It is clear from the preceding few paragraphs that they have not fared well in this regard. What they have missed out on are the advances made after the discovery of calculus, principally by Gauss, Riemann and Einstein, and the development of topology. The sort of education they have been given might be compared with a medical education that ignored Pasteur and his successors, an education in English that had nothing in it of Shakespeare or after or a chemistry education without the periodic table.

If the tangible benefits are not there, what of the intangible ones? As I mentioned earlier, the study of Euclid was supposed to give the students some benefits because of its logical development from axioms, and so on, but as I also pointed out this view has been discredited and I mentioned an article by Bertrand Russell dated 1902. From our point of view we can see that the study of triangles and circles, and that is what the early syllabus amounted to, was essentially limiting when compared with the vast number of other geometrical objects that could have been studied. If the early Professors thought there was some benefit to be gained from studying Euclid, we can only say today that they were wrong. Interestingly enough, the early syllabus was simply the first six books of Euclid but only the definitions from Book 5. However it does appear that Book 5 may have been the most important one of all: according to Morris Kline "*Book V, based on Eudoxus' work, is considered to be the greatest achievement of Euclidean geometry*". It could be objected that Book V is properly a part of algebra today: but then the corresponding algebra was not taught either. It seems that, even with their reverence for Euclidean geometry, they missed the brightest gem of all. By the way, the definitions in Book V are such things as

1. *A magnitude is a part of a magnitude, the less of the greater, when it measures the greater.*
18. *A perturbed proportion arises when, there being three magnitudes and another set equal to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes.*

I wonder what the students, or the lecturers for that matter, made of those!

None the less, I don't want to say that no benefits would have come from a study of Euclidean geometry. It would have helped develop the student's imagination and intuition, but only to a limited degree compared with what else was possible. It is a shame that it occupied such a prominent position and that so much time was spent on it. In the long run it was proper that it was superseded by co-ordinate geometry, a superior way of doing much

the same things, but clearly, a knowledge of Euclidean geometry is something that every mathematics student should gain.

The case for projective geometry is somewhat different. I do not think it can be claimed that a knowledge of it is an essential part of an undergraduate education in mathematics, for a start it duplicates a lot of the facts of Euclidean geometry, but there is a case to be made for its intangible benefits. It is something that can be developed from axioms and it can stimulate the student's imagination and intuition. But again I would say that too much emphasis was put on it.

Moreover, the benefits that can be obtained from Euclidean and projective geometry can also come from the more contemporary geometry I have mentioned; even more so.

Geometry at Auckland has had a strange independence from what was going on in the mathematical world. Gauss' paper on the geometry of surfaces appeared in 1827, 54 years before the founding of AUC; Riemann's significant lecture on the hypotheses which lie at the foundations of geometry was given in 1854; Einstein's most important papers were published in 1905 and 1912. Topology began around the 1860's and is still developing. All these things had no effect whatsoever on the teaching of undergraduate geometry in Auckland.

Its independence from social events was even more striking though perhaps not so surprising. The Anglo-Maori wars were fought not long before the founding of the college; the First World War was thirty years after; then came the depression, the build up to the Second World War and the war itself. These events, all major influences in the shaping of our country seem to have had no effect whatsoever on the mathematics in our University.

One event that might have had a beneficial effect was the exodus of many mathematicians from Germany during the 1930's. From being the leading mathematical country in the world, Germany was reduced by Hitler's policies to a much lower level. As Jews and others lost their jobs and were forced out of Germany they spread to other countries taking with them their skills. As far as I can tell, none came to New Zealand, the main reason being the restrictive immigration policies of the government.\* These people were refugees, desperate for a place to live and a job, but we didn't take them. Apart from the moral aspect of closing our doors on many people who ended up being murdered, we lost a wonderful opportunity.

I have often heard that we should "bring back geometry". To the extent that this means bringing back the geometry that was taught in former days there is no case. However I do believe there is a case for devoting a large part of our syllabus to a modern study of the subject, inspired by calculus, principally Gauss' work, and topology with Euclidean and projective geometry playing a minor role. In particular geometry should be an underpinning of our courses in calculus and algebra.

And one thing that should be learnt from Gauss is that geometry is local, not global.

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\*[Some scientists (not mathematicians) came: the names of Fraenkel, Pappé and Popper spring to mind. - Ed]

\* \* \* \* \*

In the Carroll universe all objects have zero velocity although they may have non-zero momentum. Carroll was a pure mathematician who had already foreseen this possibility in 1871: (L. Carroll, *Through the Looking-glass, and what Alice found there*, Macmillan, London, 1871.)

"A slow sort of country," said the Queen, "Now, here, you see, it takes all the running you can do, to keep in the same place."

But his mathematical colleagues once again missed an opportunity by failing to take him seriously.

# Centrefold

PROFESSOR HENRY FORDER

When I first met Professor Forder he had already retired from his position as Professor of Mathematics at the University of Auckland, or Auckland University College as it was then. It was 1959 and I studied Plane Projective Geometry with him in a third year class. Nobody forgets his lectures. I consider this to be one of the best two or three courses that I ever studied, though not all students found it so: if you could keep up with the lectures they were brilliant, but if you got behind you could be terribly lost. He used to pass out cyclostyled notes that were almost impossible for us to follow. In later years these were replaced by direct Xerox copies of his lecture notes which he kept in pocket-size notebooks. He would Xerox two openings of his notebook onto one sheet of paper and pass the result out to the students, but with his tiny obscure handwriting, his brevity, the copying process, sometimes his notation and finally his pagination, we were mystified as often as we were enlightened: I felt thankful that by this time I was no longer taking his course for credit.

Professor Forder's reputation rests largely on his books, particularly "The Foundations of Euclidean Geometry" and "The Calculus of Extension". Everybody knows what the first of these is about. What he calls Grassman's Calculus of Extension we would now call Linear Algebra; thus an extensive of step one is an abstract vector, a spread is a vector space, and so on. It is dated as a book for students but it has a great permanent value for the sheer amount of interesting information that it contains. The book shows Professor Forder as a very learned man in his subject, Geometry, and it is this quality that made his lectures so valuable.

Let me quote a couple of things from the book.

*The angle in a semicircle is a right angle.* For  $[(p-a)|(p-b)] = (p-\frac{1}{2}(a+b))^2 - (\frac{1}{2}(a-b))^2$ . Get it? The second part is the proof! ( $[u|v]$  denotes the inner product of  $u$  and  $v$ , while  $v^2 = \|v\|^2$ .) Any of his students will recognise the sparse style. Here is something else, not quite so familiar.

*If  $p, q, r, s$  be the centres of squares described externally on the sides of any quadrilateral  $abcd$ , then the intervals  $pr, qs$  are perpendicular and equal in length.*

For  $p = \frac{1}{2}(a+b) + \frac{1}{2}i(a-b)$ , and so on. Hence, using  $||v = -v$ ,

$p - r = \frac{1}{2}(a+b-c-d) + \frac{1}{2}i(a-b-c+d)$ ,  $q - s = \frac{1}{2}(b+c-d-a) + \frac{1}{2}i(b-c-d+a) = i(p-r)$ .

( $|v$  denotes a vector of the same length as  $v$ , but rotated anticlockwise through a right angle.)

His knowledge is vast and he passed a lot on to us. We can also be thankful that his judgement did not allow his other interest to seduce him into giving dry lectures on the logical foundations of his subject: we gained an empirical knowledge and an intuitive understanding.

This is not the first tribute that has been paid to Professor Forder. On his eightieth birthday Douglas Robb wrote

*Ave! Sed Nondum "Vale".*

On his ninetieth birthday Gavin Ardley added

*Sapiens senescit, non segnescit.*

What can I, who do not speak Latin, add?: perhaps only the words of Henry George Forder himself:

*Henry Forder, Henry Forder,  
Comparative of Henry Ford,  
The superlative is wanting,  
For his mercies praise the Lord.*

Peter Lorimer

[P. Brown's drawing is reproduced by kind permission of the Auckland Science Librarian, Maxine Watt.]





## SURVEY: SEVENTH FORM MATHEMATICS

During second term, this year, Raoul Cornwell of Waikato University surveyed mathematics teachers at 45 Waikato high schools. Under these teachers were 625 students doing seventh form mathematics or applied mathematics. We print his findings.

### INTRODUCTION

The teachers surveyed were the Head of the Mathematics Department, the form seven mathematics teacher and the form seven applied mathematics teacher (one or more of these posts may be occupied by the same teacher). Some comments on the results are appended, along with some conclusions reached by the writer based on past experience, contact with first year students this year, and results of the survey.

### GENERAL COMMENTS

Most seventh form teachers do not enforce the doing of homework. Most set and mark assignments (mean figure 10 per year). The survey did not ask if these were compulsory, but from other comments made one suspects not. It appears that the attitude is very much non-insistence but gentle persuasion. They are seventh formers now and must learn to be independent. Typically the pure maths class contains 9 boys and 5 girls and the applied maths class 8 boys and 3 girls. In only 7 of the 45 schools surveyed do the pupils receive less than 4 hours per week. Thus it can be seen that most have the opportunity for lots of individual help. Indeed much class time is devoted to "doing examples" with the teacher supplying individual help as requested.

Some teachers experiment with university-type methods (lectures, tutorials etc.) but this is not easy with class sizes so small. Many schools run study skills programs (e.g. using SHEIK materials) in forms 5-6 but instruction of seventh formers in what to expect at university is largely left to individual teachers.

Mechanics has died a natural death. (Only two schools out of thirty-seven teach it in preference to another option, although some larger schools try to cover all three.)

Pure mathematics teachers "prove" (or justify) most of the results they use but generally only require such proofs from the "brighter pupils" i.e. there is a widespread acceptance that many form seven maths students are not capable of deductive reasoning.

Teachers were asked to comment on whether the seventh form year was a social one and whether the low monetary value of the bursary had any effect on this. Opinion varied as to whether it was a social year and some respondents concentrated on the second aspect of the question and did not answer the first part. About half of those answering the first part admitted that it is a social year for some students, and that this in itself is not necessarily a bad thing. Almost all saw the monetary value of the bursary as an unimportant factor. Of far more importance were side benefits such as entry to halls of residence. The relevance of the course was also considered important, and here applied mathematics scored over pure mathematics.

Only 12 of the 45 heads of department criticised the bursary syllabuses. The major criticism was that the syllabuses were designed for the university bound student and (particularly pure mathematics) not relevant for other students. Some criticised the emphasis on vector geometry and the disproportionate amount of time taken up by calculus. Others felt that the fullness of the pure mathematics syllabus prevented them spending time developing manipulative ability.

A question asking for reasons why students with reasonable bursary passes demonstrated glaring weakness in manipulative ability at university drew a good response. Most blamed the school certificate syllabus where

manipulative ability is not needed. Then in forms 6 and 7 syllabuses are overfull and there is no time to develop it. With so many topics and a wide choice in the examination, mastery of individual topics is not being achieved. Many blamed "new maths" and some saw the wide choice of options in the junior school as reducing the total time spent on mathematics. Syllabuses were designed for the mathematically able - those who pursue mathematics for its own sake. The rest, the vast majority - the "users", just slip through the system on the strength of their other subjects. As one teacher put it, "the emphasis in secondary school is on "concepts" rather than competence in applying concepts". We do not emphasise mastery of techniques and accept 50% as a pass. One teacher spoke of students scoring 20-30% in class during the year and getting 50% in the examination as a result of "exam coaching" (how many of us have had this experience?)

Eight heads of department mentioned the removal of University Entrance to form seven as desirable (it was not a question asked of them) and some mentioned the automatic award of Higher School Certificate as a factor in the difficulty of motivating some seventh formers.

In commenting on the transition from school to university, some teachers made the point that very little information comes down to them about the actual content of university courses. Information on content contained in university calendars is minimal. The main shock is in the transition from being an important member of a small select group to "just another face on a university I.D. card". Little could be done about this other than to prepare the students beforehand, preferably with a visit to the university. One teacher actually takes his class along to a randomly selected lecture. Some teachers viewed "free time" given to students at school as important preparation for the transition. Others felt that it depended on how this "free time" was used because there is in fact, very little "free time" for a university student doing eight courses, particularly if some are Science or Computer Science courses involving labs.

Another teacher instanced team teaching in form six as preparation for university.

#### CONCLUSIONS

1. Syllabuses in forms 6 and 7 are too full. Most teachers are aware that they are going too fast for many of their customers.
2. Teachers are not coping well with the two "strands" in their classes - the university bound and the rest.
3. The pendulum has swung too far in the direction of freedom of action for seventh formers. University students do have deadlines to meet and so must seventh formers. Perhaps we need an internally assessed component of Higher School Certificate.
4. More attention needs to be paid to mastery of a topic before moving on.
5. We must provide for our future sixth and seventh formers adequate training in algebra skills even though these are not examined in the School Certificate paper because of the nature of that examination. Perhaps the "levels" concept is the solution here although one suspects that our secondary mathematics teachers, unlike their primary counterparts, are not good at providing for different levels within the same class.
6. Current syllabuses are not developing skill in deductive reasoning. Deduction must be practised to be perfected, but because it is too difficult for the masses who sit School Certificate, our good students are not being trained in it. (These same "masses" are often very good at coming up with strategies in computer games.)
7. Too much spoonfeeding is occurring, not in the sense of setting and enforcing standards, but by teachers hovering over students as they complete repetitive exercises (N.B. the popularity of a certain book of exercises in form seven mathematics.)

Each time a new twist comes up, the student asks the teacher (or his neighbour if the teacher is encouraging independence!) who shows the student what to do. Then if the exercises are sufficiently well graded, this keeps the student going for another six or seven exercises whereupon the process is repeated. The only bit of real mathematics that has occurred, coping with the "twist", has been done by the teacher. Consequently in a couple of months time when this problem crops up again, the student can't do it. Syllabuses must be pruned to provide time for students to sweat over twists like this (and it goes without saying that they must be relevant so as to provide realistic problems).

8. The amount of time spent on calculus in forms six and seven needs to be looked at. The limit concept is too subtle for all but the best students. If we are teaching it as a "tool" subject, does the number of clients justify it, particularly in view of the mounting claims of finite mathematics? (Universities teach it all again anyway.)
9. Are our seventh forms too small? Should neighbouring schools amalgamate senior classes? Should we cut down on supervised time in senior forms to prevent the spoonfeeding mentioned above? There is scope for much research here.

Footnote: In 1979 the correlation between bursary pure mathematics mark and Waikato 05.102 (calculus) final grade was 0.67. Good work habits developed in form seven obviously pay dividends at university.

## FIRST YEAR COURSES IN CALCULUS AND ALGEBRA

Responding to the need in high schools for information about first year university courses, Raoul Cornwell assembled the following material.

### INTRODUCTION

The recent questionnaire of Waikato schools (see previous article) revealed that most seventh form mathematics teachers have little idea of what is taught in first year mathematics courses at university. Information in calendars is sketchy so I wrote to the various university mathematics departments for details. The information I received varies in detail and in the case of Canterbury's algebra (math 111) course has been condensed (my apologies!). Most are taken from 1980 course reports although little change is expected for 1981. Some universities also supplied details of service courses that they teach which are terminating courses and pre-requisites in other subject areas. For simplicity I have left these out of this compilation. (It is worthy of note, however, that these tend to be very applications orientated, particularly toward economics, and point up a deficiency of this type of application in our sixth and seventh form courses.) I have not attempted to survey applied mathematics/mathematical models courses.

Courses generally consist of 2 hours per week of lectures and 1 hour of "tutorial". First year students normally take 8 courses.

### UNIVERSITY OF OTAGO

#### Math 101 Algebra

##### Vectors (19 lectures)

Geometry and algebra of vectors in 3 dimensions; modulus and distance, position vectors, parallel and coplanar vectors; coordinate systems and number triples; the dot product and its properties, angle, projections; equations of lines and planes in various forms, distance and angle between lines and planes, intersections of lines and planes; the vector product and its properties, the triple scalar product and triple vector product, skew lines, linear independence, volume and the triple scalar product, changing the coordinate system.

##### Matrices and Linear Equations (9 lectures)

Elementary matrix algebra; transposes, inverses (in simple cases), and their properties.

Systems of linear equations, Gauss reduction, elementary matrices. Inverses by the super-matrix method. Linear transformations of  $\mathbb{R}^2$  and other geometrical applications.

#### Determinants, Eigenvalues, Eigenvectors (8 lectures)

Properties of  $2 \times 2$  determinants; extension to  $3 \times 3$  and  $n \times n$  determinants. Calculation of determinants; cofactors. Cramer's Rule for solving sets of linear equations.  $A$  is invertible iff  $\det A \neq 0$ .  $\det AB = \det A \det B$ . Eigenvalues and eigenvectors; application to finding principal axes of a conic.

#### Relations and Functions (2 lectures)

Elementary algebra and properties of relations; functions as special case.

#### Number systems (3 lectures)

Survey of  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ , The Peano axioms, mathematical induction, the well-ordering of  $\mathbb{N}$ . From  $\mathbb{N}$  to  $\mathbb{C}$ : a quick survey.

#### Complex Numbers (4 lectures)

Historical introduction. Complex numbers as real-number-pairs, but construction omitted. Modulus, conjugate, argument, arithmetic operations. Argand diagram, De Moivre's theorem. Trigonometrical identities, roots of unity, roots of an arbitrary complex number,  $e^{i\theta}$ .

#### Polynomials (5 lectures)

Addition and multiplication of polynomials, degree, roots. Division algorithm, remainder theorem, multiple roots; testing for multiple roots. Highest common factor and Euclid's algorithm for integers and polynomials. Irreducible polynomials, factorisation in  $\mathbb{R}[x]$  and  $\mathbb{C}[x]$ .

### Math 102 Calculus

#### First Half Year

(The material in this section was covered in 25 lectures but not in the sequential order as listed.)

#### Sequences

Intuitive ideas and properties of sequences. Formal  $(\forall \epsilon, \exists N)$  definition of the limit of a sequence. Proofs of major properties of sequences.

#### Limits and Continuity

Intuitive ideas and properties of limits and continuity. Formal definition of  $\lim f(x) = L$  in terms of limits of sequences. Derivation of formal properties of  $x \rightarrow a$ ; limits of functions in terms of limits of sequences.

#### Differentiation

Intuitive ideas and properties. Formal definition and derivation of general properties including the chain rule. Intuitive treatment of implicit differentiation. Partial differentiation.

#### Applications and Differentiation

Graphing of functions: first and second derivative tests, points of inflection, and rate-of-change problems.

#### Series

Definition and general properties of series, the geometric series, integral test, p-series, comparison test, limit comparison test, ratio test, alternating series, absolute convergence.

#### Integration

Intuitive ideas and basic properties. Statement and use (but no proof) of the fundamental theorem of calculus. Integration via the method of substitution. Applications: finding areas and volumes. Introduction to double (i.e. iterated) integration through problems involving the finding of volumes of solids with varying cross-sections.

#### Special Functions

Basic properties of and techniques for differentiating and integrating the six basic trigonometric functions and their inverses, the natural logarithm function and the exponential function.

Second Half YearIntegration Theory (6 lectures)

Bounds for sets and functions, construction of the integral via upper and lower sums, area of the ordinate set, integrability of monotonic and continuous functions, basic properties of the integral; anti-derivatives, the fundamental theorem, differentiation of integrals, theorems for integration by substitution and by parts.

Integration Technique and Application (7 lectures)

Standard integrals, the method of partial fractions (including repeated quadratic factors); averages, area between 2 graphs, volumes, volumes of rotation using disks and shells; improper integrals of first and second kind, comparison test and integral test.

Miscellaneous Topics (3 lectures)

The hyperbolic functions and their inverses, the Cauchy mean value theorem, L'Hôpital's rule.

Power Series (4 lectures)

Review of sequences and series, power series, radius of convergence, Taylor polynomials, Taylor series.

Curves (6 lectures)

Parametric plane curves, tangents and normals, arc length, polar coordinates, area and arc length in polar form; conics by the focus/directrix method, standard forms, conics in polar coordinates.

## UNIVERSITY OF CANTERBURY

## Math 111 Algebra

Linear Algebra, Systems of Linear Equations and Matrices (9 lectures)

Solution of systems of linear equations, row operations, reduced echelon form, applications, matrices, invertibility, elementary row operations, link with equations.

Determinants (4 lectures)

Permutation definition, evaluation by row operations, transpose, invertibility.

Vector Geometry (5 lectures)

Vector as ordered triple, sum, scalar multiple, scalar product and geometric applications. Equations of lines and planes, cross product.

Vector Spaces (5 lectures)

Generalisation of  $R^3$  to  $R^n$ . Vector spaces, subspaces, linear independence, bases, dimension.

Linear Transformations (5 lectures)

Linear functions, matrix representation, kernel, solution of homogeneous system. Examples (e.g. rotations, reflections, shears, changes of scale)

Algebraic StructuresIntegers, Congruence and Rationals (10 lectures)

Deduction of properties of the integers from a set of axioms. Induction, division, Euclidean algorithm primes, unique factorisation. Congruence modulo  $n$ , equivalence relations, sum and product of congruence classes. Invertible classes, divisors of zero. Linear diophantine equations. Rational numbers as a field.

Complex Numbers (14 lectures)

Complex numbers as a field. De Moivre's theorem.  $n$ th roots of any complex number.

Polynomials (6 lectures)

Sum, product, scalar multiple of polynomial, degree, zeros, maximum number of roots. Division, factorisation, complex roots.

## Math 112 Calculus

Basic Differentiation (6 lectures)

Real numbers, induction, functions, derivatives, rules for differentiation, differentiation of trigonometric functions, higher derivatives.

Curve Sketching (3 lectures)

Maxima, minima, curve sketching, asymptotes.

Special Functions (5 lectures)

Inverse functions and their derivatives,  $n$ th root functions,  $\sin^{-1}$ ,  $\tan^{-1}$ . Logarithmic, exponential and power functions. Hyperbolic functions.

Basic Integration (6 lectures)

Indefinite integral defined as an antiderivative. Linearity. Substitution, integration by parts, partial fractions, integration of rational functions of sine and cosine.

Limits and Continuity (4 lectures)

Limits of  $f(x)$  as  $x \rightarrow a$  or  $x \rightarrow \infty$ . Algebra of limits. Continuity. Review of the definition of a derivative, evaluation of non-obvious derivatives using the definition. Differentiability implies continuity.

Real-valued Functions of Two Variables (6 lectures)

Geometrical interpretation of graphs. Level curves. Partial derivatives and their geometrical significance. Statement and use of the chain rule. Maxima, minima, saddle points.

Definite Integration (5 lectures)

Area. Approximation by step functions. Basic properties. Fundamental theorem of calculus. Evaluation of definite integrals. Improper integrals.

Mean Value Theorems (3 lectures)

Rolle's Theorem, Cauchy's M.V.T., Lagrange's M.V.T., proof of the theorems used in curve sketching, L'Hôpital's rule.

Taylor's Formula (4 lectures)

Taylor's Theorem, Lagrange's form of the remainder. Expansions of special functions. Applications.

Infinite Series (6 lectures)

Convergence of sequences and series. Geometric series. Comparison test, ratio test, Dirichlet's test. Absolute convergence.

VICTORIA UNIVERSITY

Math 113 Calculus

Differential CalculusFunctions of One Real Variable (17 lectures)

Properties of the standard functions (polynomials, rational, rational power, log, exponential, trigonometric, inverse trigonometric, hyperbolic), standard equations and basic geometrical properties of the ellipse, hyperbola and parabola, (a) their graphs, (b) limit properties, (c) continuity, (d) derivatives. Extremum problems and L'Hôpital's rule.

Functions of Two or More Real Variables (6 lectures)

"Graphs" of some simple functions. Level curves and surfaces. Partial derivatives and some idea of the chain rule. Directional derivative and relative extremum for functions of two variables only.

Integral Calculus (23 lectures)

Riemann sums and definite integrals; the fundamental theorem; integration by substitution; logs, exponentials and hyperbolic fns; integration by parts; int. of rational functions (by partial fractions); volumes of revolution and arc length; Taylor's formula; D.E.'s; polar coordinates; improper integrals.

Math 114 Algebra and Geometry

Vector Geometry (12 lectures)

The algebra of scalar and vector product with geometrical interpretations and consequences. Vector equations of line and plane in Euclidean 3-space with applications to geometrical problems.

Number and Structure (14 lectures)

Some properties of number systems; countability, order, density, completeness, field. The complex numbers; algebraic manipulation of complex numbers with and without use of de Moivre's theorem;  $n$ th roots, geometrical applications. The natural numbers, Peano axioms and induction. The integers and polynomials, divisibility, greatest common divisor, unique prime factorisation. Sequence and series; convergence of a sequence, sequence of partial sums of a series,  $\log_e 2$ , absolute convergence, power series, the ratio test.

Linear Algebra (16 lectures)

Properties of matrices and the algebra of matrices; (symmetry, skew-symmetry, transposes). Elementary matrices and their connection with elementary row/col operations. Inverse matrices - properties of same. Gauss-Jordan method of finding inverses. Elementary properties of determinants. Proof of  $\det AB = \det A \cdot \det B$ . Reduction of a matrix to row echelon form. Definition of rank. Use of row echelon form to solve linear equations. The homogeneous and inhomogeneous case. Type of solution and connection with rank. Use of elementary matrices in proving  $r(A) = n \Leftrightarrow A$  is non singular ( $n \times n$ ). Proof of  $A \text{ ns.} \Leftrightarrow \det A \neq 0$ . Connection between rank and linear dependence of rows/cols. Transformations of the plane ( $\mathbb{R}^2$ ). Special type. Connection between matrices and linear transformations of the plane. Eigenvalues and eigenvectors of  $2 \times 2$  matrices and their connection with the corresponding transformation of the plane.

MASSEY UNIVERSITY

## 60.101 Algebra and Calculus

Algebra and Geometry

Matrices: addition, multiplication by a number, multiplication of matrices. Determinants and inverses. Elementary row operations and the solution of systems of simultaneous linear equations. Plane geometry: straight line, circle, parabola, ellipse and hyperbola. Vectors in a plane. Solid geometry: straight lines, planes and surfaces. Vectors in three-dimensional space.

Calculus

Functions and derivatives, techniques of differentiation. Curve sketching. Maxima and minima. Elementary integration. Trigonometric, exponential and logarithmic functions. Integration by substitution and by parts. Partial fractions. Use of table of integrals. Power series. Number systems. Floating point arithmetic. Errors. Partial derivatives, maxima and minima of functions of two variables.

UNIVERSITY OF WAIKATO

## Math 05.103 Algebra

Draft syllabus only. Order and number of lectures may differ considerably from this.

Logical and Set Theoretical Notation (2½ lectures)

Connectives (no truth-tables); if, only if, necessary, sufficient; quantifiers; direct and indirect proof, counter example, converse, contrapositive; proof by induction, recurrence relations. Brief revision of set notation, Venn diagram, algebra of sets and of subsets of a universal set.

Number Systems' (2½ lectures)

Brief summary of main properties of natural, integer, rational, real and complex numbers. Inequalities, absolute value. Quadratic equations; factorization of polynomials; de Moivre's theorem.

Relations (3 lectures)

Ordered pairs; binary relations from set A to set B as subsets of  $A \times B$ . Domain, range. Composition, inversion. Relations defined by equalities and inequalities; regions bounded by straight lines and simple conics.

Compatibility, equivalence, partial ordering (2½ lectures)

Symmetry, reflexivity, transitivity; compatibility; equivalence, including congruences; partial ordering including inequalities and set-inclusion.



Functions (1½ lectures)

Function from set A into set B, onto set B; composition; one-to-one correspondence; inversion.

Linear Equations (3 lectures)

Solution by elimination, use of parameters. Geometrical interpretation. Determinants of orders 2 and 3 (definitions only, but basic properties introduced by examples and exercises). Uniqueness of solution.

Matrices (4 lectures)

Definition; matrix algebra. Linear equations in matrix form. Inverse matrix (by inverting system of equations).

Finite Sets (3 lectures)

Counting techniques, elementary combinatorics, binomial theorem.

Relations on Finite Sets (4 lectures)

Matrix representation, graphs, trees. Path problems; multiplication of incidence matrices, incidence matrix of composite. Planarity.

Semigroups and Monoids (3 lectures)

Definitions, examples. Isomorphism, homomorphism. Congruence relations on semi-groups.

Groups, Rings, Fields (2 lectures)

Definitions and examples.

Lattices (4 lectures)

Definitions, examples. Distributive lattices. Representation of lattices. Partition lattices.

Boolean Algebras (2 lectures)

Boolean Algebras as lattices; representation of Boolean Algebras; canonical expressions. Calculus of Boolean Algebras. Algebra of subsets revisited.

Logic and Switching (2 lectures)

Truth-tables, propositional calculus. Algebra of switching.

Linear Spaces (5 lectures)

Linear functions from  $C^m$  into  $C^n$ , and their matrices. Linear spaces; bases. Dimension theorem. Linear equations revisited.

Notes

- The syllabus should be regarded as a whole, each section providing prerequisites for the next and later sections. No section should be allowed to become a topic in its own right.
- All definitions should be motivated, and all theory illustrated, by suitable examples and applications drawn from as many different fields as possible.
- In exercises, tests, and examinations, as much emphasis should be given to applications as to theory. Questions should be so designed that students can demonstrate their mastery of basic theory by their ability to apply it.

## Math 05.102 Calculus

Note: This is essentially a methods course. It will attempt, throughout, to motivate concepts, definitions and theorems from applications to other areas of study.

Functions, graphs, operations on functions.

Limits of functions and continuity - concepts and definitions, limit theorems (which may be assumed as axioms).

The Derivative, rules of differentiation, related rates, differentials, implicit differentiation, with applications to numerical analysis, curve sketching and optimization.

Trigonometric functions and identities.

Antidifferentiation, differential equations with separable variables, simple substitutions.

Definite Integral, Riemann sums, numeric integration, Fundamental Theorem of Integral Calculus, with applications to areas, volumes of revolution etc.

Transcendental Functions - logarithmic, exponential, hyperbolic, inverse trigonometric and inverse hyperbolic; exponential growth and decay.

Techniques of Integration - trigonometric substitutions, partial fractions, integration by parts.

Partial Differentiation - introduction, optimization using Lagrange multipliers.

Differential Equations - first order linear, second order with constant coefficients, simple harmonic motion.

UNIVERSITY OF AUCKLAND

Math 26.120 Algebra

Elementary Number Theory (5 weeks)

Division identity; number bases; divisibility; primes; l.c.m., g.c.d.; Euclidean algorithm and the expression of g.c.d. (a,b) in the form  $ma + nb$ ; fundamental theorem of arithmetic; congruences; residues; Fermat's theorem; public-key codes; induction.

Complex Numbers (3-4 weeks)

Algebraic manipulation; geometric representation; polar form; de Moivre's theorem; n-th roots of one and other numbers; the fundamental theorem of algebra and its consequences.

Linear Algebra in 2 and 3 Dimensions

- (i) Vectors (1-2 weeks)  
An introduction to vectors and how they behave leading up to the axioms of linear algebra and easy ideas in differential geometry.
- (ii) Linear equations and how to solve them (1 week)
- (iii) Dependent and independent vectors (1 week)
- (iv) Lines and planes (2 weeks)  
Techniques and ideas for handling the geometry of lines and planes.
- (v) Lengths, distances and angles (1 week)  
Basic notions involving the scalar product and the way that it is used to calculate these things.
- (vi) Vectors in traditional geometry (3-4 weeks)  
A study of many parts of traditional geometry: triangles, theorems of Pappus, Desargues, Morley.
- (vii) Matrices (2 weeks)  
Techniques of dealing with matrices: addition; multiplication, determinants; inverses.
- (viii) Space-time in Newton's and Einstein's Theories (2 weeks)  
This is regarded as an example of a major application of linear algebra. It concentrates on the geometrical foundations which express themselves in kinematics.

Rings and Fields (3 weeks)

This concentrates on the classical examples of rings and fields and goes on to extensions by zeros of polynomials. It ends with the division algorithm and the Euclidean algorithm for polynomials.

Math 26.140 Calculus

Inequalities and functions (1 lecture)

Limits and continuity (6 lectures)

Lim  $\frac{\sin x}{x}$  and related problems (2 lectures)  
 $x \rightarrow 0$

Differentiation, the chain rule, mean-value theorem, maxima and minima (5 lectures)

Differentiation of trigonometric functions (2 lectures)

Cauchy mean-value theorem, L'Hôpital's rule (2 lectures)

Taylor polynomials and Taylor series (the mean-value theorem used to derive the Lagrange form of the remainder) (4 lectures)

Concavity and points of inflection, implicit differentiation (2 lectures)



## FOURTH INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

Berkeley, California, USA : August 10-16, 1980

N.J. (Gus) Gale, President of the New Zealand Association of Mathematics Teachers, was the official New Zealand representative at this congress, and his report follows.

## GENERAL

Attendance at this congress provided me with a unique educational experience. It gave me the chance to extend my knowledge, experience and understanding of what was taking place in other countries in the field of mathematical education that would not otherwise have been possible. I express my sincere appreciation to those who supported my attendance at this important meeting.

ICME 4 was attended by some 2,500 people from over 100 countries. The congress was centred around four invited speakers.

Professor Hans Freudenthal (the Netherlands) *"Major Problems of Mathematical Education"*.

Professor Hermina Sinclair (Switzerland) *"Young Children's Acquisition of Language and Understanding of Mathematics"*.

Hua Loo-Keng (People's Republic of China) *"Applications of Mathematics and Mathematics Education"*.

Seymour Pappert (United States) *"Technology and Mathematics Education"*.

In addition to these plenary sessions there were invited addresses covering such areas as:

Universal Education - What should the mathematics curriculum be in school systems where many students leave at an early age?

Teaching Strategies - Problems involving mixed ability classes in mathematics. How can problem solving in mathematics be taught?

Language and Mathematics - The problems of teaching and of learning mathematics in a second language.

Mathematical Content of the Curriculum - What is the place of geometry in the curriculum? How should the curriculum be affected by the widespread use of hand calculators?

Post-Secondary Mathematics - Should calculus continue to be the unique core of post-secondary mathematics? What can be done to help beginning teachers of mathematics?

Applications - What materials are available worldwide for bringing applications of mathematics into schools?

Mathematics and Industry - Continuing education in industry.

Technology - Microcomputers, the family computer, hand calculators, television.

The Profession of Teaching - What is a professional teacher of mathematics?

Mathematical Competitions - What is taking place in different countries?

Women in Mathematics - How to increase young women's interest in mathematics.

Assessment - National and Internal Assessments of mathematics achievement.

Research in Mathematics Education - Development of mathematical abilities in children.

During the congress some 400 invited speakers spoke on these topics. The quality of these papers varied from excellent to disastrous. The quality was in inverse proportion to the number of delegates who walked out while the talk

was being given. I am pleased to report that no one left while I was presenting my paper and that much interest was created by those who heard of some of the mathematical activities taking place in New Zealand. One worthwhile comment of: "What you are doing in New Zealand is unique", came from Mr. Brian Wilson of the British Council. Several other delegates have requested I send them more information.

In addition, the congress also provided for:

**Project Presentations:** These took the form of discussion groups in special panels and mini-conferences; exhibits of materials; special talks; classroom demonstrations.

**Workshops:** Teachers were able to examine materials and discuss teaching ideas with their colleagues in an informal way.

**Abstracts of Short Communications:** These were displayed in poster format and provided an opportunity for participants to share professional experiences, information and ideas. These were grouped into four general categories:

- \* Teacher Education and Curriculum Planning
- \* School Mathematics Instruction
- \* University Mathematics Education
- \* Miscellaneous: Adult Education, Application, Audio-visual, Bilingual Education, Career Mathematics, Computers, History, Language, Maths Anxiety, Programmes for the Disadvantaged, Radio Teaching, Special Education, Women in Mathematics.

**Working and Study Groups:** Groups continuing from previous congresses continued to meet as groups or mini-conferences.

**Mathematics Teachers Organisations:** Representatives of mathematical associations from forty-six different countries met twice during the congress to discuss mutual problems such as: How to promote membership; how to stimulate professional growth; networks within the association; communication between mathematics teaching organisations throughout the world; student activities.

Editors of Journals met to exchange publications.

Films were also shown on two evenings.

Exhibits of books, computers and other materials were displayed at several locations. This included an impressive "Images in Mathematics" presentation presented by a special team from Great Britain.

School Visitations were also available but time did not allow me the opportunity to join any of these tours.

Because of the wide range of topics available and the large number on at any one time (up to twelve) the choice of which session one would go to became a major decision. I felt that the mini-conferences were inappropriate for the short time available in such a congress as it could not be guaranteed to have the same delegates at all sessions, creating a certain discontinuity in proceedings.

This conflict of areas of interest in choosing which session one should go to I found most frustrating. If it could have been possible for delegates to have had the programme before arriving at Berkeley this would have helped in making decisions.

The other disappointing thing concerning ICME 4 was the non-appearance of congress's Honorary President, George Polya. Unfortunately ill health prevented him from attending and the congress was visibly disappointed that this 93 year old was not able to be present. His great and invaluable influence on mathematics education is reflected in such books as "How to Solve it", "Mathematics and Plausible Reasoning", and "Mathematical Discovery". However, a short but pertinent statement from George Polya was read to the congress at the opening ceremony along with messages from President Carter and Edmund Brown (Governor of California).

During the week before and after the congress I was able to visit schools in north and southern California and discuss matters of mutual interest with school educators and administrators. I found this to be most stimulating and interesting.

I left the United States feeling quietly satisfied with the job that New Zealand teachers are doing, given the limited resources available to them. It is true that schools in the United States and other countries are served with far greater material resources, but the curriculum development in mathematics is very much up with other countries. In fact it is interesting to note that the mathematics curriculum in American high schools is moving towards integrating the different branches of mathematics (e.g. Algebra, Geometry, Trigonometry, Calculus): a process which has always been favoured in this country.

Finally, the question: "Where to from here?" I think this can be summarised from the findings of an extensive survey carried out by the National Science Foundation of America. The project called Priorities in School Mathematics (PRISM) conducted an extensive survey of the opinions of many sectors of society, both lay and professional, and has produced "An Agenda for Action" comprising eight Recommendations for School Mathematics of the 1980s. They are:

1. That problem solving be the focus of school mathematics in the 1980s.
2. That basic skills in mathematics be defined to encompass more than computational facility.
3. That mathematics programmes take full advantage of the power of calculators and computers at all grade levels.
4. That stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics.
5. The success of mathematics programmes and student learning be evaluated by a wider range of measures than conventional testing.
6. That more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population.
7. That mathematics teachers demand of themselves and their colleagues a high level of professionalism.
8. That public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

#### ICME 5

The fifth International Congress will be held in Adelaide, Australia, from 22-29 August, 1984. This is certainly good news for New Zealand mathematics educators and I sincerely hope that a large contingent from New Zealand will be able to attend. There is no question about the immeasurable value that one gains from attending such international meetings. I am sincerely appreciative of the opportunity given to me to attend ICME 4.

*N.J. Gale*

### INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION, ICMI

Report on meeting held at Berkeley University on August 14, 1980

#### BACKGROUND

ICMI was founded following a resolution adopted by the Fourth International Congress of Mathematicians meeting in Rome, in April 1908. A central committee began to "conduct such activities which relate to mathematical or scientific education", to inaugurate "appropriate programmes designed to further a sound development of mathematical education at all levels and to secure public appreciation of its importance". By 1914, twenty-eight countries had joined ICMI and four international congresses had been held: Brussels 1910, Milan 1911, Cambridge 1912, Paris 1914.

The work of ICMI was interrupted by World War I and was not resumed until 1928. World War II again caused an interruption and was not re-established until 1952. ICMI consists of ten members-at-large elected by the General Assembly of the International Mathematical Union (IMU), four ex-officio members, and a host of delegates, each appointed by one of the member nations.

The first International Congress on Mathematical Education (ICME 1) was held in France in 1969, ICME 2 at Exeter, England in 1972, ICME 3 at Karlsruhe, Germany, in 1976, and ICME 4 at Berkeley, California, USA. It has been decided to hold ICME 5 at Adelaide, Australia, 22-29 August, 1984.

#### MEETING OF ICMI DELEGATES, 14 AUGUST, 1980

at Berkeley University, California

Presider: Hassler Whitney, President ICMI.

Bent Christiansen of the Royal Danish School of Educational Studies, Copenhagen, opened proceedings and spoke on the changing role that mathematical educators face today through:

- the broader field of mathematics existing today and its effect on primary, secondary and tertiary education;
- the increased professionalisation of today's mathematics teachers;
- the growth of the international linkage of the mathematical network;

Brief reports were made on the fourteen specialised meetings held under the auspices of ICMI in the last six years. Among them:

- international colloquium on evolving a mathematical attitude in secondary education in Hungary in 1975;
- informatics and mathematics in secondary schools, impacts and relationships in Bulgaria, 1977;
- Southeast Asian Conference on Mathematical Education in Manila, 1978;
- Regional ICMI Seminar in Luxembourg in 1978.
- Seminar on topics in Mathematical Education at Helsinki, Finland, August 1978.

Ubivatu D'Ambrosio (IMECC-UNICAMP, Campinas, São Paulo, Brazil) spoke on the rôle of UNESCO in assisting developing countries within ICMI. Many delegates from these developing countries had their costs of attending ICME 4 covered by grants from UNESCO.

Peter Hilton (Ohio, USA) gave a brief financial report on ICMI but was interrupted by one of the French delegates who rose to a point of order and castigated the organisers of the meeting for not arranging a larger venue and above all for not having interpreting facilities available. It was an embarrassing time for the chairman but was nevertheless an extremely valid point which had the sympathy of the whole meeting. To make his point more valid, the French delegate put his case in his own language (even though he could speak English!) Much was learned from this episode and I am sure that the same mistake will not be made at the next ICMI meeting.

ICME 5: The Australian Delegation then made a formal bid to have ICME 5 held at Adelaide, South Australia, from 22-29 August, 1984. Contrary to the previous item of business, the Australians had prepared their case very thoroughly. They put their case to the meeting with the aid of transparencies in French and Spanish. The Australians had also done some worthwhile preliminary work in selling ICME 5 to delegates with the aid of the Australian Government, Qantas Airlines and Travel & Convention Agencies providing displays, wine etc. Needless to say, the bid to hold ICME 5 in Adelaide, South Australia, was successful even if it appeared to be a fait accompli. The executive committee of ICMI had already approved the bid in a previous meeting. In fact the standing of this general meeting of ICMI was a little hard to comprehend as it became obvious that decisions had already been made by the executive of

ICMI at an earlier meeting. This fact along with the composition of ICMI and ways in which its membership is elected caused some discussion with again no finality being reached except that these matters will be looked at during future meetings. The concern of delegates was certainly expressed most forcibly.

At this point the president, Hassler Whitney, attempted to close the meeting but a strong lobby from a large delegation present prevented this. The delegation had been given an assurance that they would be able to present a petition that had been circulating all week to gain support for the release of Jose Massera being held in detention in Uruguay. The president refused to accept the petition and a very tense situation arose. A compromise was reached by allowing the petitioners to present their case to the full congress at the closing ceremony.

I am sure that the Australian Delegation learned a great deal on what to do and not to do in 1984 and hopefully the meeting in Adelaide will not have the complications that the 1980 meeting foundered on. It is also hoped that an examination will be made of how the executive committee of ICMI is elected.

With ICME 5 now to be held so close to New Zealand it is hoped that a large delegation of New Zealand mathematics educators will be present so as to gain the same worthwhile experience, knowledge and enjoyment that it has been my privilege to receive with ICME 4.

*N.J. Gale*

## GR9 - NINTH INTERNATIONAL CONFERENCE ON GENERAL RELATIVITY AND GRAVITATION

July 14-19, 1980. Friedrich Schiller University, Jena, German Democratic Republic.

About 800 relativists swamped the small Thuringian town of Jena for a week. It coped remarkably well with good accommodation, waiter-served meals (somewhat at the expense of the COMECON pork mole-hill one thought!), free bus travel and waived visa requirements. (Later, as a private tourist in the GDR I encountered a few difficulties but not enough to mar my enjoyment of the country.) A large contingent of Russians were able to come (though not E.S. Fradkin), attracting many big names from the West.

Supergravity continues to hang fire - a fact that worries its adherents, the twistor programme seems to be in a rut and complexification continues its divergence from velocity. So the theoretical front is in turmoil, but exciting developments are occurring in gravitational wave detectors.

Some of the more significant papers presented were:

J.A. Wheeler (Austin). *Einstein's second century.*

In large part being an admission that this eminent physicist still has fundamental worries about the double-slit experiment of quantum mechanics. (Perhaps a lesson for us all?)

Ya. B. Zel'dovich (Moscow). *The theoretical and empirical situation.*

It was noteworthy to have him and Wheeler - respective "fathers" of the Russian and American bombs - at the same venue, and doubly interesting to know of an "uncle" of the Indian bomb in the audience!

V.B. Braginsky (Moscow) and K.S. Thorne (Pasadena). *Present state of experiments on gravitational waves.*

A. Trautman (Warsaw). *The Einstein-Cartan theory.*

A swan-song for his involvement in the theory.

I. Robinson (Dallas). *Complex methods in General Relativity.*

R.A. d'Inverno (Southampton). *Computer methods in General Relativity.*

Algebraic processing is now a thoroughly accepted tool.

H. Stephani (Jena). *Exact solutions of Einstein's field equation.*

He and M.A.H. MacCullum (Cambridge) have just completed a comprehensive survey of presently known solutions.

*W. Brent Wilson*



## SECOND AUSTRALASIAN MATHEMATICS CONVENTION

Sydney 11 - 15 May, 1981

First circulars were distributed to over 400 people in New Zealand, including NZ Mathematical Society members. A second circular will be posted from Sydney to all who returned the questionnaire attached to the first circular. It is not too late to indicate your intention to participate in the convention. However, the later your request for accommodation, the less likely it will be that your first choice of accommodation can be met. *Accommodation deposits and registration fees may be paid in New Zealand currency to The Treasurer, Second Australasian Mathematics Convention, Department of Mathematics and Statistics, Massey University, Palmerston North before 15 March 1981.* It would be appreciated if these could be paid as early as possible.

Programme. At the time of writing (prior to the issue of the second circular) the following invited speakers will be present:

Professor B. Carter (Observatoire d'Astrophysique) - relativity  
 Professor C.W. Curtie (University of Oregon) - representations of finite groups  
 Professor S. Eilenberg (Columbia University) - automata, algebraic topology  
 Professor E. Hewitt (University of Washington) - harmonic analysis  
 Professor B. Mandelbrot (IBM) - communications theory, fractals  
 Professor S. Pappert (MIT) - mathematics education  
 Professor J.T. Stuart (Imperial College, London) - fluid mechanics  
 Professor C.T.C. Wall (University of Liverpool) - manifold theory, topology.

There will be eleven specialist sections for contributed papers, including a section on Mathematics Education. It is possible that an invited speaker on Mathematics Education will deliver a keynote address.

Travel. Convention participants are advised to make travel bookings without delay because airline seats on or near the weekend of 9 - 10 May are being filled rapidly. So far two people have given notice of their willingness to act as group organisers for their region. They are

Ms L.D. Copeland, Okato College, P.O. Box 8, Okato (Taranaki)  
 Dr G.C. Wake, Department of Mathematics, Victoria University of Wellington,  
 Private Bag, Wellington.

A notice from Dr. Wake is printed below. *People in the Wellington region please note.*

The New Zealand Mathematical Society has some money available to assist speakers to travel to the convention (see the announcement below).

Further information is available from the Convention Secretary - Dr. T.M. Gagen, Department of Pure Mathematics, University of Sydney, Sydney, NSW 2006, Australia.

*Dean Halford*

## GROUP FARE - WELLINGTON/SYDNEY

Both the Wellington/Sydney flights in the weekend of 9th and 10th of May 1981 are completely booked out from the point of view of a group of 20 or more. The only possibility Qantas are able to offer is via Christchurch on Sunday 10 May. This would more than extinguish the saving over the Advance Purchase (EPIC) fare. Also the group scheme incurs the inflexibility of at least 20 travelling together both ways. Accordingly, it is suggested that, *for the Wellington group, we use individual Advance Purchase fares (presently \$296 return - an increase expected before May!)*. I am more than happy to coordinate activities as far as individuals are concerned, but *advise that the appropriate flights are almost full and early booking is strongly recommended.* Some people may like to investigate the possibility of travelling on an organised package tour group - these are very cheap and available through various travel agents.

*G.C. Wake, 4 November 1980*

## APPLICATIONS FOR TRAVEL ASSISTANCE TO THE SECOND AUSTRALASIAN MATHEMATICS CONVENTION

The New Zealand Mathematical Society has a fund of about \$2000 which is to be used to assist New Zealand speakers to travel to the Second Australasian Mathematics Convention, to be held in Sydney 11-15 May, 1981. In addition, a smaller amount is available to assist speakers from the South Pacific region (excluding New Zealand and Australia) to travel to the Convention.

Special application forms were distributed with the Society's Newsletter Number 18, but more are available from the Secretary if required.

Applications are invited from anyone intending to present a paper. Applicants do not have to be members of the New Zealand Mathematical Society, but the special application form must be used. It is likely that most grants-in-aid will not exceed \$150.

*Applications close with the Secretary, NZNS, Department of Mathematics and Statistics, Massey University, Palmerston North, New Zealand on 30 January, 1981. All applicants will be notified of the outcome as soon as possible after the closing date.*

PLEASE SEND IN YOUR APPLICATION AS EARLY AS POSSIBLE

## Problems

*Readers are invited to send problems for this section. Some indication should be given of how a problem has arisen and whether a complete solution is known and attribution of sources should be provided for problems that are not original. Attempts at solutions should be sent to the setter or to the Editor.*

### Problem 5 (Resisted flight time)

Is the time of flight for a projectile longer or shorter if air resistance is taken into account?

*Comments:* This problem was asked in a Physics 100-level finals exam at Victoria University in 1979. Although in most real cases the problem is elementary (the time of flight is shorter if air resistance is taken into account), it is not entirely clear what the answer is in general. Taking the case of vertical motion with initial speed  $U$  and air resistance equal to  $kf(v)$ ,  $v$  a typical speed, the time of flight is:

$$T = \int_0^U \frac{dv}{g + kf(v)} + \int_0^V \frac{dv}{g - kf(v)},$$

where  $g$  is the acceleration due to gravity and  $V$  is the return velocity given by (height up = height down)

$$\int_0^V \frac{v dv}{g - kf(v)} = \int_0^U \frac{v dv}{g + kf(v)}.$$

For  $k$  small, clearly  $T(k) \leq \frac{2U}{g}$  as  $T'(0) = -\frac{2}{g^2 U} \int_0^U v f(v) dv < 0$ .

Also, for  $f(v) = v^n$ , we easily obtain:

(i) when  $n = 1, 2$   $T \leq \frac{2U}{g}$ , for all  $k$

(ii) when  $n = 0$ ,  $T \leq \frac{2U}{g}$  for  $\frac{k}{g} \leq 0.7$  (approx) and  $T \geq \frac{2U}{g}$  for  $\frac{k}{g} \geq 0.7$ .

A note on Problem 3, December 1979.

The problem concerns sequences generated by

$$x_{n+1} = (3x_n + 1) \text{ div}^* 2$$

for any positive integer  $x_0$ ; where  $\text{div}^*$  denotes indefinitely repeated exact integer division. W.B. Wilson noted that all sequences seem to converge to one, though this is yet unproved.

Informally, it seems fair to regard such sequences as pseudo-random processes,  $x_{n+1} = x_n Y_n$ , where  $Y_n$  has (approximate) distribution

value	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{3}{8}$	$\dots$
probability	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\dots$

and Wilson noticed that as  $E[Y_n] = 1$ , the process should be stationary. However, this is an inaccurate observation, (even informally), as the process is not an additively homogeneous one, so that  $E[Y_n]$  is irrelevant. Such processes with (pseudo-)independent identically distributed multiplicative increments, can be easily rewritten in the usual addition form merely by putting

$$Z_n = \log Y_n$$

$$\text{then } Z_{n+1} = Z_n + \log Y_n \quad (\text{the usual random walk form}),$$

$$\text{and now } E[\log Y_n] = \log \frac{3}{4} < 0,$$

so the process is on average decreasing. It is not therefore surprising that all sequences move fairly rapidly downwards, though with some moderately long oscillations. It is an interesting coincidence that 1 is the only fixed point or loop (this is not true for most similar processes, for instance

$$(3n-1) \text{ div}^* 2 \quad \text{or} \quad (3n+5) \text{ div}^* 2).$$

In summary, the operation  $\text{div}^* 2$  reduces a positive integer by an "average" factor of 2; (as  $3n+1$  is automatically even this gives an average factor of 4 for this particular process); so that for

$$x_{n+1} = f(x_n) \text{ div}^* 2$$

to be (pseudo-) stationary it is sufficient to have

$$f(n) \doteq 2n.$$

This is a fairly tight result as is seen by the two following examples, which addicts might like to investigate past the  $x_0 = 100$  which I tried.

$x_{n+1} = (2x_n - [\sqrt{x_n}]) \text{ div}^* 2$  which has all sequences apparently converging to 1, or a loop involving 51, (though some sequences are very long);

$x_{n+1} = (2x_n + [\sqrt{x_n}]) \text{ div}^* 2$  in which there appear to be several diverging sequences(!) even for  $x_0 < 100$ .

*W.F.C. Taylor, University of Canterbury*

\* \* \* \* \*

My only pleasure in Church used to be calculating the date of Easter. You divide by 19, excluding fractions. It was such a relief to be able to do that. One was never allowed to exclude fractions in arithmetic at other times.

Bertrand Russell as quoted in  
*Personal Record 1920-1972* by Gerald Brennan

## Conferences

\*\*\* 1981 \*\*\*

- January 2 - 8  
(Banff, Canada) *Winter Research Institute on Geometric Quantization*  
Details from Mrs. Pat Dalgetty, Secretary, Geometric Quantization Conference, Department of Mathematics and Statistics, The University of Calgary, Calgary, Alberta, Canada T2N 1N4.
- January 5 - 8  
(Barbados) *Third Caribbean Conference on Combinatorics and Computing*  
Details from C.C. Cadogan, Department of Mathematics, University of West Indies, P.O. Box 64, Bridgetown, Barbados, West Indies.
- January 7 - 15  
*International Conference on Algebraic Geometry*  
Details from Tomás Sánchez, Facultad de Ciencias, Sección de Matemáticas, Prado de la Magdalena, Valladolid, Spain.
- January 10 - 11  
(Claremont, California) *Differential Equation and Applications to Ecology, Epidemics and Population Problems*  
Details from Stavros N. Busenberg, Department of Mathematics, Harvey Mudd College, Claremont, California 91711, U.S.A.
- January 12 - 16  
(Claremont, California) *NSF Regional Conference on Global Topological Methods in Applied Mathematics*  
Details from Kenneth L. Cooke, Mathematics Department, Pomona College, Claremont, California 91711, U.S.A.
- January 12 -  
February 6  
(Hobart) *Australian Mathematical Society 21st Summer Research Institute*  
Details from R. Lidl, Department of Pure Mathematics, University of Tasmania, GPO Box 252C, Hobart, Tasmania 7001, Australia.
- February 8 - 12  
(Kuwait) *Conference on Algebra and Geometry*  
Details from M.A. Al-Bassam, Department of Mathematics, Kuwait University, Kuwait, State of Kuwait.
- February 25 - 28  
(Houston, Texas) *Nonlinear Problems in Science*  
Details from John C. Polking, Department of Mathematics, Rice University, Box 1892, Houston, Texas 77001, U.S.A.
- March 2 - 5  
(Baton Rouge, Louisiana) *Twelfth Southeastern Conference on Combinatorics, Graph Theory and Computing*  
Details from K.B. Reid, Department of Mathematics, Louisiana State University, Baton Rouge, Louisiana 70803, U.S.A.
- March 2 - 6  
(Los Alamos, New Mexico) *Nonlinear Problems: Present and Future*  
Details from Organizing Committee, Centre for Nonlinear Studies, Los Alamos Scientific Laboratory, University of California, P.O. Box 1663, MS 457, Los Alamos, New Mexico 87545, U.S.A.
- March 12 - 13  
(Pittsburgh, Pennsylvania) *Computer Science and Statistics: the Thirteenth Symposium on the Interface*  
Details from William F. Eddy, Department of Statistics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213, U.S.A.
- March 16 - 20  
(Haifa) *International Conference on Convexity and Graph Theory*  
Details from Joseph Zaks, Department of Mathematics, University of Haifa, Haifa 31999, Israel.
- March 25 - 27  
(Baltimore, Maryland) *Conference on Information Sciences and Systems*  
Details from Wilson J. Rugh, Electrical Engineering Department, The John Hopkins University, Baltimore, Maryland 21218, U.S.A.
- March 26 - 28  
(Birmingham, Alabama) *International Conference on Spectral Theory of Differential Operators*  
Details from Ian Knowles, Department of Mathematics, University of Alabama in Birmingham, Birmingham, Alabama 35294, U.S.A.
- April 21 - 24  
(Kuala Lumpur) *Second Southeast Asian Conference on Mathematical Education*  
Details from C.K. Lim, Department of Mathematics, University of Malaya, Kuala Lumpur, Malaysia.
- April 23 - 26  
(Plattsburgh, New York) *Recent Advances in Non-Commutative Ring Theory: A George H. Hudson Symposium*  
Details from P. Fleury, Chairman, G.H. Hudson Symposium, Department of Mathematics, State University of New York at Plattsburgh, Plattsburgh, New York 12901, U.S.A.

- April 30 - May 1  
(Pittsburgh,  
Pennsylvania) *Twelfth Annual Pittsburgh Conference on Modelling and Simulation*  
Details from William G. Vogt, Modelling and Simulation Conference,  
348 Benedum Engineering Hall, University of Pittsburgh, Pittsburgh,  
Pennsylvania 15261, U.S.A.
- May 11 - 13  
(Milwaukee,  
Wisconsin) *Thirteenth ACM Symposium on Theory of Computing*  
Details from Walter A. Burkhard, Publicity Chairman, SIGACT - 81  
Symposium, Department of Electrical Engineering and Computer Sciences,  
University of California, San Diego, La Jolla, California 92093, U.S.A.
- May 11 - 15  
(Sydney) *Second Australasian Mathematics Convention*  
Details from T.M. Gagen, Department of Pure Mathematics, University of  
Sydney, Sydney, New South Wales 2006, Australia.
- May 16 - 23  
(Kozubnik,  
Poland) *International Conference on Functional-Differential Systems and  
Related Topics*  
Details from D. Przeworska-Rolewicz, Mathematical Institute, Polish  
Academy of Sciences, Sniadeckich 8, P.O. Box 137, 00-950 Warszawa,  
Poland.
- May 21 - 22  
(Washington, D.C.) *Third Symposium on Mathematical Programming with Data Perturbations*  
Details from Anthony V. Fiacco, Department of Operations Research,  
School of Engineering and Applied Science, The George Washington  
University, Washington, D.C. 20052, U.S.A.
- June 9 - July 3  
(Cape Town) *Symposium on Categorical Algebra and Topology*  
Details from K.A. Hardie, Department of Mathematics, University of  
Cape Town, Rondebosch 7700, South Africa.
- June 22 - 27  
(Tübingen) *International Symposium on Stochastics and Analysis*  
Details from H. Heyer, Mathematisches Institut der Universität  
Tübingen, Auf der Morgenstelle 10, 74 Tübingen, West Germany.
- June 23 - 26  
(Dundee) *Biennial Conference on Numerical Analysis*  
Details from G.A. Watson, Department of Mathematics, University of  
Dundee, Dundee DD# 4HN, Scotland.
- June 28 - July 5  
(Weimar) *Ninth International Congress on the Application of Mathematics in  
Engineering*  
Details from H. Matske, President of the IX. IKM, Karl-Marx-Platz 2,  
53 Weimar DDR, East Germany.
- June 29 - July 10  
(London, Ontario) *Current Trends in Algebraic Topology*  
Details from V.P. Snaith, Department of Mathematics, The University of  
Western Ontario, London, Ontario, N6A 5B9, Canada.
- June 30 - July 2  
(Bethlehem,  
Pennsylvania) *Fourth IMACS International Symposium on Computer Methods for Partial  
Differential Equations*  
Details from R.S. Stepleman, Computing Technology and Services Division,  
Exxon Research and Engineering Company, P.O. Box 51, Linden, New Jersey  
07036, U.S.A.
- July 13 - 24  
(Cambridge) *NATO Advanced Research Institute on Nonlinear Optimization*  
Details from M.J.D. Powell, DAMTP, Silver Street, Cambridge, CB3 9EW,  
England.
- July 19 - 25  
(Cambridge) *Summer Meeting in Category Theory*  
Details from P.T. Johnstone, University of Cambridge, Department of  
Pure Mathematics and Mathematical Statistics, 16 Mill Lane, Cambridge  
CB2 1SB, England.
- July 20 - 24  
(Swansea) *Eighth British Combinatorial Conference*  
Details from A.D. Keedwell, Honorary Secretary, British Combinatorial  
Committee, Department of Mathematics, University of Surrey, Guildford,  
Surrey GU2 5XH, England.
- August 3 - 7  
(Rio de Janeiro) *International Seminar on Functional Analysis, Holomorphy and  
Approximation Theory*  
Details from Guido I. Zapata, Instituto de Matemática, Universidade  
Federal do Rio de Janeiro, Caixa Postal 1835, 21910 Rio de Janeiro,  
Brazil.

- July 6 - 11  
(Budapest) *Sixth Hungarian Colloquium on Combinatorics*  
Details from E. Györi, J. Bolyai Mathematical Society, H-1368 Budapest, Pf.240, Hungary.
- August 5 - 7  
(Snowbird, Utah) *1981 ACM Symposium on Symbolic and Algebraic Computation*  
Details from B.F. Caviners, Department of Mathematical Sciences, Rensselaer Polytechnic Institute, Troy, New York 12181, U.S.A.
- August 11 - 21  
(Berlin) *Sixth International Conference on Mathematical Physics*  
Details from R. Seiler, Institut für Theoretische Physik, Freie Universität Berlin, Arnimallee 3, D-1000 Berlin 33, Federal Republic of Germany.
- August 23 - 28  
(Brisbane) *Ninth Australian Conference on Combinatorial Mathematics*  
Details from S. McDonald, Mathematics Department, University of Queensland, St. Lucia, Queensland, Australia.
- August 23 - 28  
(Montréal) *Tenth Conference on Stochastic Processes and their Applications*  
Details from A. Joffe, Centre de recherche de mathématiques appliquées, Université de Montréal, Case postale 6128, Montréal, Québec, Canada H3C 1J7.
- August 30 -  
September 6  
(Kiev) *Ninth International Conference on Nonlinear Oscillations*  
Details from Organising Committee, Institute of Mathematics, Repin Str. 3, 252004, Kiev-4, U.S.S.R.
- September 8 - 10  
(Austin, Texas) *International Symposium on Semi-infinite Programming and Applications*  
Details from James Vick, Mathematics Department, University of Texas, Austin, Texas 78712, U.S.A.
- September 21 - 26  
(Metz) *Journées Arithmétiques*  
Details from Georges Rhin, Département de Mathématiques, Université de Metz, Ile du Saulcy, 87000 Metz, France.
- \*\*\* 1982 \*\*\*
- August 11 - 19  
(Warsaw) *International Congress of Mathematicians*  
Details from Czeslaw Olech, Institute of Mathematics, Polish Academy of Sciences, Sniadeckich 8, P.O. Box 137, 00-950 Warszawa, Poland.

M.R.C.

## COMBINATORIAL MATHEMATICS

The Ninth Australian Conference on Combinatorial Mathematics will be held at the University of Queensland, Brisbane, Queensland, 23-28 August, 1981. All interested persons are cordially invited to attend. Contributed papers are welcome in all areas of combinatorics, pure and applied. Invited speakers are being arranged. One or two series of instructional lectures are being planned. Accommodation on campus will be available. The third annual general meeting of the Combinatorial Mathematics Society of Australasia will be held at the Conference.

Those who are not members of C.M.S.A. but are interested in attending the Conference please write to: Dr. S. Williams, Director C.M.S.A., Department of Mathematics, University of Queensland, St. Lucia, Queensland 4067, Australia.

The Combinatorial Mathematics Society of Australasia was formed in 1978 to promote combinatorial mathematics: the investigation, construction, enumeration, and application of discrete configurations. It disseminates information about combinatorics and combinatoricists through its newsletter *Combinatorics*, and conducts an annual conference, the proceedings of which are published. There are currently eight-six members from all over the world.

Any interested person is invited to join C.M.S.A. Annual subscription for 1981 is \$A4 for those in full-time employment and \$A2 otherwise. Members receive the newsletter and a reduction in the annual conference registration fee.

Please address all enquiries, giving your full name and address, to Dr. S. Williams at the above address.

# Secretarial

## CALL FOR NOMINATIONS

The seventh annual general meeting of the Society will be held in Sydney in May 1981 during the Second Australasian Mathematics Convention.

Nominations for the positions of Incoming Vice-president, 2 members of Council and Auditor must reach the Secretary, NZMS, Department of Mathematics and Statistics, Massey University, Palmerston North by 13th March 1981. Nominations must be countersigned by the nominee and should include brief biographical detail suitable for publication in the April Newsletter.

Present officers of the Society, together with their retirement dates, are listed below.

### OFFICERS OF THE SOCIETY, JUNE 1980 - MAY 1981

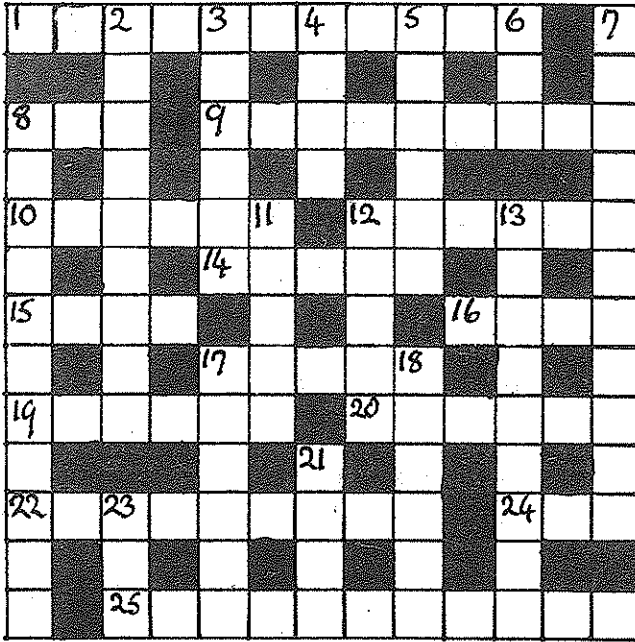
President:	Dr W.D. Halford (1982)	Maths. Dept., Massey University, Palmerston North.
Incoming Vice-President:	Dr D.B. Gauld (1983)	Maths. Dept., University of Auckland, P.B. Auckland.
Immediate Past President:	Mr J.C. Turner (1981)	Maths. Dept., University of Waikato, P.B. Hamilton.
Secretary:	Dr G.M. Thornley (1981)	Maths. Dept., Massey University, Palmerston North.
Treasurer:	Mr H.S. Roberts (1981)	Appl. Maths. Div., Dept of Scientific & Industrial Res., Box 1335, Wellington.
Councillors:	Dr J.H. Ansell (1983)	Maths. Dept., Victoria University, P.B. Wellington.
	Dr M.R. Carter (1983)	Maths. Dept., Massey University, Palmerston North.
	Mr D.C. Harvie (1981)	Maths. Dept., Victoria University, P.B. Wellington.
	Mr R.S. Long (1982)	Maths. Dept., University of Canterbury, P.B. Christchurch.
	Dr G. Olive (1982)	Maths. Dept., Otago University, Box 56, Dunedin.
Editor:	Dr W.B. Wilson	Maths. Dept., University of Canterbury, P.B. Christchurch.
N.Z.A.M.T. (Alternates)	Mr N.J. Gale	Papanui High School, Box 5-220, Christchurch.
	Mr B.R. Stokes	Hamilton Teachers' College, P.B. Hamilton.
Auditor:	Mr A.R. Clark	Accounting Dept., Victoria University, P.B. Wellington.
The Society's official address is:		The New Zealand Mathematical Society Inc., c/o The Royal Society of New Zealand, Box 12-249, Wellington.

However, normal correspondence should normally be sent direct to the Society's Secretary, at the address given in the list of officers.

# Crossword

N<sup>o</sup> 2

by Matt Varnish



## CROSSWORD N<sup>o</sup> 1 SOLUTION

### Across:

1. Hessian, 5. Octet, 8. Ear,  
9. Swallow, 10. Bleep, 11. Feet,  
13. Out, 14. Run, 16. Isopod,  
18. Fermat, 21. Set, 22. Psi,  
23. Lapp, 26. Eight, 27. Lemmata,  
29. Hod, 30. Nines, 31. Summers.

### Down:

1. Host, 2. Shape, 3. Ill, 4. Newton,  
5. Orbit, 6. Theorem, 7. Topknots,  
12. Two, 15. E-IN-STEIN, 17. Octagon,  
19. Ell, 20. Fields, 22. Paths,  
24. Plane, 25. Mass, 28. Mum.

## SYMBOLICALLY SPEAKING (it will give you the bird)

All answers are words of current english usage. One is a proper noun.

### Across:

1. +. (1,5,5)  
8. Be Nero, eh? (3)  
9.  $C \div 10 = ER$ . (9)  
10. XX. (6)  
12. {VO}? (6)  
14. Omit N. (5)  
15. 12th St., Tiger. (4)  
16. This. (4)  
17. About 51 100 left over. (5)  
19. ——. (6)  
20. OO. (6)  
22. (ZNOX)<sup>i</sup>. (4,5)  
24. N. (3)  
25.  $V \geq$ . (4,3,4)

### Down:

2. LSZ, NP. (4-5)  
3. X<sup>C</sup>. (6)  
4. Ĉ. (4)  
5. CC'. (6)  
6. ... p q r s t ... (3)  
7. I. (5,6)  
8. XIII. (6,5)  
11. S. (5)  
12.  $\Xi$  An(up)d. (5)  
13. (:)<sup>A</sup>. (9)  
17. Note; Re + N. (6)  
18. Co. (6)  
21. F = T. (4)  
23. ... klmopq ... (3)

\* \* \* \* \*

The Newsletter is the official organ of the New Zealand Mathematical Society (Incorporated). It is produced by Ann Tindall, Beverley Haberfield, Graham Wood, Ian Coope and Brent Wilson of the University of Canterbury Mathematics Department and printed at the University Printery. Contributions are solicited from all and sundry (without distinction) for subsequent Newsletters, the next dead-line being mid-March, 1981. Merry Christmas.