Number 79     August 2000

# NEWSLETTER

OF THE

**NEW ZEALAND MATHEMATICAL SOCIETY (INC.)**

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*Contents*



## PUBLISHER'S NOTICE

This newsletter is the official organ of the New Zealand Mathematical Society Inc. This issue was assembled and printed at Massey University. The official address of the Society is: The New Zealand Mathematical Society,
c/- The Royal Society of New Zealand,
P.O. Box 598, Wellington, New Zealand. However, correspondence should normally be sent to the Secretary: Dr Charles Semple, Secretary, NZ Mathematical Society,
Department of Mathematics and Statistics,
University of Canterbury,
Private Bag 4800, Christchurch.

## NZMS Council and Officers

## Web Sites

The homepage of the New Zealand Mathematical Society with URL address: http://www.math.waikato.ac.nz/NZMS/NZMS.html (Webmaster: stephenj@math.waikato.ac.nz)
The newsletter is available at: http://IFS.massey.ac.nz/mathnews/NZMSnews.html
Editorial enquiries and items for submission to this journal should be submitted as text or LATEX files to r.mclachlan@massey.ac.nz.

## EDITORIAL

### Hello

Welcome to the Newsletter, my second as editor after taking over from the steady hand of Mike Hendy, and 79th overall. We must be doing something right: the Australian Mathematical Society has decided to start a newsletter, and to model it on ours. If they come up with a logo of the letters AMS in the shape of a kangaroo, your editor intends to sue. In his address *The First 25 Years of the New Zealand Mathematical Society* (printed in Newsletter 76), Garry Tee has commented on the history and role of the newsletter, and I see no need for any major changes. I suspect that the *Local News* will remain the most popular item; it also serves as a valuable record of who did what when. However, there is some scope for publishing more mathematical or general interest features, and I urge everyone to send in anything they'd like to share. It was once famously remarked (by Professor Jim Campbell, I learn from Garry) that the only occasion when mathematicians at the New Zealand universities learned of each other's work was when they met each other on leave at Oxbridge. Surely the Newsletter can close the gap just a bit.
What goes in an Editorial? I might have reported on what's happening and going to happen in the Society; but luckily Graeme Wake's prompt contributions of his President's Column has relieved me of that burden. Failing that, there seems to be a formula for these things in which one starts with an observation or anecdote, develops it a bit and then pontificates a while on some pressing issue. Let's see how I go…
Remember that crack about Oxbridge? Last month I was walking down the street on my way to the Cambridge train station when who should I meet coming the other way but Rua Murray! Thirty years later and it's still the same story! (He didn't tell me about his latest research, but I did learn that I'd been spotted by Mary Silber (U. Chicago) at 6am the day before, arriving at Heathrow in my 'KAM tori' T-shirt—it seems my life is turning into a minor subplot in a David Lodge novel.) Rua, fresh from Ph.D. and postdoc, resurfaced in New Zealand at the 1999 Canterbury Colloquium, and immediately badgered me about how we had to get together, get all the New Zealand mathematicians together to meet and work together, make things happen. Shamefacedly I realized that five years ago it was me holding forth and espousing this principle. Five years and what's happened?
My own department, now not even called a department in its own right, has been reduced from 16 to 11.4 academic staff. We teach just as many students, but university papers have been resized, so our EFTS numbers have fallen. They may fall further, judging by demographics and shifting demand. We now face the loss of up to 2 further positions in the infamous 'repositioning', and the outlook for statistics is even worse (although the contracting out of Business Statistics does seem to have been averted as a result of massive protest). At 36 I am still the youngest staff

member. Spiralling out from there we have the student loan problem, low academic standards, low pass rates, the task of creating a highly educated, literate, and numerate public (not to mention a knowledge economy) keen to stay in New Zealand and pay off their loans; behind all this moves the stately, imperturbable barge of demographics. As the number of workers supporting each retired person falls gracefully from 4 to 2, the demand in North American and Europe for young, skilled workers is going to be unquenchable. The Tertiary Education Advisory Commission (http://www.teac.govt.nz), now sitting, will consider some of these matters, but it's a knotty mess and no mistake. Faced with this situation, is it any wonder that we keep up our overseas links and retreat to those Oxbridge common rooms from time to time?

In Britain, the outlook is altogether different. True, they are worried about a brain drain of academics to the US, to which the government has rushed in with £50m to stem the flow; but it seems that a combination of competition (introduced under the Tories) and more money (Labour) is producing great results, with lots of mobility, lots of foreign staff coming in, young people getting promoted quickly, and also—perhaps due to the high salaries in banks and software—lots of students. The recent spending review included an extra £1b annually for science research, and 5.4% real per annum growth in all education spending, for three years. (Hint to the NZ government: note the word *real*. 2.3% dollar growth is not an increase at all.) Like the unreasonable effectiveness of mathematics, there seems to be some kind of unreasonable effectiveness of the UK economy at the moment.

Well, I have no more insight than you or the next person what to do about this. To take just one issue, the one raised by Rua: there are, at least, a few new young mathematicians now in New Zealand, and a whole slew more coming through, enticed back by Marsden and FORST postdocs. We have to find a way of keeping them.

*Robert McLachlan*

## PRESIDENT'S COLUMN

This is the issue of the newsletter which acts as a prelude to the Annual Meetings scheduled to coincide with the NZ Mathematics Colloquium 27–29th November. We hope you will be there. Clearly the vigour of our discipline and our Society needs your support.

Colleagues in Palmerston North especially are experiencing threats on their jobs as repositioning bites. Looking at the repositioning exercise in Massey, I thought of the very successful exercise in the United States a few years ago in which the AMS played a key role in rescuing the Department of Mathematics at Rochester from demolition. We should be active in promoting the underpinning role our subject plays and support our Massey colleagues.

The Australian Mathematical Society meetings have passed. We formalised the agreement for an NZMS observer at their Council meeting and we were represented in Brisbane this year by Professor Gaven Martin (University of Auckland – thanks Gaven). Gaven reports that the issues focussed on issues of concern here **. . . membership recruitment and mathematical promotion**. From their minutes I notice that they have now appointed a Vice-President (Annual Conferences)—Walter Bloom. Their schedule of meetings is:

> Thirty-seventh Applied Mathematics Conference, ANZIAM 2001, to be held at the All Seasons Barossa Valley Resort from 3rd to 7th February, 2001 with Professor E.O. Tuck as Director.
>
> The Forty-fifth annual meeting of the Australian Mathematical Society, AustMS 2001, to be held at the Australian National University from 22nd to 26th September 2001, with Professor A.G.R. McIntosh as Director.
>
> The Forty-sixth annual meeting of the Australian Mathematical Society to be held at the University of Newcastle in September 2002.

In 2003 the Australian Mathematical Society winter meeting and ANZIAM will be in association with the International Congress of Industrial and Applied Mathematics (ICIAM) in Sydney 7–11 July.

Of immediate concern to us is to whether or not New Zealand schedules the 2003 NZ Mathematics Colloquium also in association with ICIAM 2003. A different option is to take up Walter Bloom's suggestion of a joint meeting in Newcastle in September 2002, OR BOTH?

What do you think? These options need to be resolved at the forthcoming November meetings. I had earlier signalled the former as a definite option.

We have recently circulated an interesting report on the Marsden Fund 1995–9 around the Council for comment. This was the report prepared by the MIS subject convenor for the RSNZ Committee in March. I would like further discussion at the November meetings. In the meantime, this report is available from our Secretary, Charles Semple

C.Semple@math.canterbury.ac.nz

Comments are invited.

The NZMS Lecturer 2000, Professor John Guckenheimer, was welcomed by all main centres, plus Kapiti Island (!!) in May/June. Thank you to all the local organisers. He wrote an article on his experiences which is reprinted elsewhere in this Newsletter.

Mathematically speaking, there are at least two other regular meetings coming up in the next couple of months, both here in Christchurch. Firstly, the NZ Statistical Association one day meeting on 1st September and then a week after the NZ Mathematics Colloquium, the Australasian Conference on Combinatorics.

There are a number of Councillors retiring this year. We are also badly in need of a Treasurer as Dr Mick Roberts is retiring from this position soon. Please consider putting your nominations to Dr Semple before November. NZMS

needs you! Note also the membership form in this newsletter. Distribute it to colleagues and students.
See you in Hamilton,

*Graeme Wake*
*President*

## LOCAL NEWS

## UNIVERSITY OF AUCKLAND

### Department of Computer Science

Associate-Professor John Hosking is now the Head of Department.
The Centre for Discrete Mathematics and Theoretical Computer Science was founded in 1995 by Douglas Bridges, Cris Calude, Marston Conder, Bob Doran, Jeremy Gibbons and Peter Gibbons.
The 5th anniversary of the CDMTCS was celebrated by a Workshop, held at the University of Auckland Department of Computer Science, on May 26. Jeremy Gibbons is now at the University of Oxford, but the other five founders attended the Workshop. The following lectures were presented:

Professor Marston Conder, “Graph symmetries, 2-groups, Sierpinski’s gasket and the Gray codes".

Dr Michael Atkinson, “Abstract data types viewed as abstract machines".

Professor Clark Thomborson, “How combinatorial graph theory can improve software security".

Professor Reinhard Klette, “Multigrid convergence and image analysis".

Dr Georgy Gimel’farb, “Random fields as image models: discrete vs continuous paradigms".

Garry J. Tee, “History of algebraic computation".

Dr Steve Reeves, “Program development, refinement and Z".

Dr Michael J. Dinneen, “Recent work on graph minors".

Professor Dan Archdeacon, Dr Paul Bonnington and Professor Josef Siran, “How to eliminate crossings in graphs by adding handles".

Dr Bakh Khoussainov, “Extracting algebraic information from finite automata".

Professor Rod Downey, “Reals, randomness and reducibilities".

Professor John C. Butcher, “A special group with applications".

Luminita Simona Dediu, “Apartness spaces – a framework for constructive topology".

Professor Douglas S. Bridges, “More constructive ideas on apartness spaces".

Margartet Ng, “Regularity preserving metrics".

Chi-kou Shu, “Computing approximations of a Chaitin’s Omega number".

Professor Cristian S. Calude, Dr Elena Calude and Dr Peter Kay, “Liars, demons and fractals".

**Seminars**

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|---|---|
| **Professor Karl Svozil** | (Technische Universität Wien), “Some recent developments in the theory of quantum information". |
| **Chad Bamrick** | (Research Systems, Inc), “Data analysis and visualization software". |
| **Michael Sanders,** | “Evolving locomotion controllers for virtual creatures". |
| **Professor Mike Atkinson** | (University of St Andrews/University of Otago), “The combinatorics of shunting data around networks". |
| **Dr Bob Fink** | (Lawrence Berkeley National Laboratory), “A next generation Internet protocol". |
| **Dr Andre Nies** | (University of Chicago), “Reducibilities on computably enumerable (C.E.) reals". |
| **Associate Professor John Hosking,** | “An application of a pattern language for evolving frameworks". |
| **Professor Bob Doran,** | “Computer architecture and the ACE Computers". |

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| **Sasha Rubin,** | “Finite automata and relational structures". |
| **Professor Rick Thomas** | (University of Leicester), “Automatic semigroups and formal languages". |
| **Zhou Peng and Zili Deng,** | “Graph Algorithms". |
| **Professor Sukumar Gosh** | (University of Iowa), “Self-stabilizing distributed systems". |

**Department of Mathematics**
On the evening of Tuesday July 11, at a ceremony at the University of Auckland to farewell the 2000 NZ team to the 41st International Mathematical Olympiad, a special presentation of a medal was made to Arkadii Slinko. The citation which accompanied the medal came in a letter (in Latvian) to Marston Conder:
“Dear Professor Conder,
This year the 50th anniversary of the Latvian Mathematical Olympiad was celebrated. In this connection the Mathematical Society of Latvia and The University of Latvia have established a special medal. It is awarded to outstanding personalities of Olympiad movement in Latvia and abroad.
I'm glad to inform you that this medal is awarded to the professor of your university, Mr Arkadii Slinko, for his outstanding activities in Olympiad movement through many years and especially for the support he has provided to mathematical olympiads in Latvia."
Tamsin Meaney has completed her Ph.D. with her thesis on “Mathematics classroom development in indigenous communities".
John Guckenheimer, Professor of Mathematics and Theoretical and Applied Mechanics at Cornell University, the New Zealand Mathematical Society Lecturer for 2000, gave three lectures here.
Dr Zorana Lazarevic, of the University of Alaska at Fairbanks, is back visiting the Department, this time for about four months. She will contribute to topology seminars and will teach Mathematics for Business and Technology 2.
Dr Tim Marshall is visiting here, en route from Al-Akhawayn University (Morocco) to the Mathematics Institute in Prague.
Professor Donald James, of Pennsylvania State University, is visiting here for six months.

**Seminars**

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|---|---|
| **Dr David McIntyre,** | “Hanf's theorem and Gaifman's theorem". |
| **Dr Chris Good** | (Birmingham University), “The axiom of choice in topology" (6 lectures). |
| **Rosheen Gray,** | “Mathematics with the graphic calculator – professional development for Junior Secondary Mathematics teachers". |
| **Professor John C. Butcher,** | “Heun and the Runge-Kutta method". |
| **Karyn Woodruffe** | (Pakuranga College), “Open source software: an alternative for education". |
| **Professor Peter M.W. Gill,** | (University of Nottingham), “Construction and use of Stewart atoms" (joint seminar with Department of Chemistry). |
| **Peter Hughes** | (ACE), “A major New Zealand initiative in teaching mathematics to year 1 to 6 children". |
| **Professor Peter Nyikos** | (University of South Carolina), “Hereditary normality and metrizability of manifolds" (2 lectures), “Hilbert's first problem and the foundations of mathematics", “New techniques in set-theoretic topology" (2 lectures), “Small uncountable cardinals and applications", “Topological properties of trees", “Hereditary properties in locally compact locally connected spaces", and “Various axioms and their uses". |
| **Dr Kay Irwin** | (School of Education), “Family income, mathematical concepts and language". |
| **Professor Zuowei Shen** | (National University of Singapore), “Cardinal interpolation". |

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| **Professor John Guckenheimer** | (Cornell University), NZMS Lecturer, “Thinking about thinking", “Computing periodic orbits of vector fields", and “Canards in periodic orbits". |
| **Professor Mark Wilson** | (University of Montana), “Asymptotics of multivariate sequences". |
| **Sanka Liyanage,** | “A framework for informal assessment questions of secondary School". |
| **Dr Ye Yoon Hong & Dr Mike Thomas,** | “Super-Calculators and mathematics examinations". |
| **Tamsin Meaney,** | “An ethnographic case study of a community-negotiated curriculum development project". |
| **Dr Arkady Leiderman** | (Ben-Gurion University of the Negev), “Kolmogorov’s superposition theorem and its applications". |
| **Professor Gaven Martin,** | “100 years of the Beltrami equation". |
| **Professor Laurent Jay** | (University of Iowa), “Solution of nonlinear systems for implicit Runge-Kutta methods by inexact simplified Newton iterations". |
| **David Barton & Stuart Laird** | (Engineering Science), “A discussion about the Form 7 calculus textbook “Delta Mathematics"”. |

**Department of Statistics**

Associate-Professor David Scott is now Head of Department.

Brian McArdle has been promoted to Associate-Professor, and both Renate Meyer and Maxine Pfannkuch have been promoted to Senior Lecturer.

Dr Marti Anderson has been awarded a University of Auckland PostDoctoral Fellowship, for her project on “The importance of habitat structure on the ecology of reef fishes, combining multivariable field experiments with statistical modelling".

The statistical software toolbox “R" was released on February 29, and it has already been used extensively by scientists around the world. Robert Gentleman and Ross Ihaka began writing R in 1992, Paul Murrell later joined them to design graphics aspects of the system, and 12 academics from around the world have joined the core team developing R. The 15 developers corresponded by e-mail for some years, before they all met at Vienna in 1999. An active international community of academics using R had grown, even before the general release on February 29. R has grown to a few hundred thousand lines of code, with the master copy kept at the University of Wisconsin. It is designed to be an extensible toolbox, and contributions have been provided from over 100 universities. But the core team of 15 developers alone have authority for making corrections, additions and other changes to the system.

**Seminars**

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|---|---|
| **Associate Professor Andy Philpott** | (Engineering Science), “Yacht performance prediction using computer simulation". |
| **Dr Geoff Nicholls** | (Mathematics), “Spontaneous magnetisation in the plane". |
| **Dr Ilze Ziedins,** | “Virtual partitioning – dynamic priorities for unbuffered resources". |
| **Dr Golbon Zakeri** | (Engineering Science), “Web-based stochastic optimization on meta-computing platforms". |
| **Dr Geoffrey Pritchard,** | “Must-run auctions in electricity markets". |
| **Dr Ross Taplin** | (Murdoch University), “Pseudo-modelling of outliers for estimation of regression and time series models". |

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| **Dr Duncan Murdoch** | (University of Western Ontario), “Perfect sampling: not just for Markov chains?". |
| **Dr Michael C. Minotte** | (Utah State University), “High-order histosplines: superior binned density estimation". |
| **Professor Don L. McLeish** | (University of Waterloo), “The geometry of the high, low, open, close: smoothing financial time series, estimating volatility and correlation". |
| **Professor Jerry Lawless** | (University of Waterloo), “Analysis of duration times in clinical and observational studies". |
| **Professor John Deely** | (Purdue University), “Robust Bayesian selection of the best supplier". |
| **Dr Cynthia Struthers** | (University of Waterloo), “Sequential design for generalized linear models". |
| **Dr Roger Sugden** | (Goldsmiths College, London), “Cochran’s rule and Edgeworth expansions for simple random sampling". |
| **Dr Randy Sitter** | (Simon Fraser University), “Design issues in fractional factorial split-plot experiments". |
| **Dr Gareth James** | (University of Southern California), “Functional principal component models for sparsely sampled data". |
| **Dr Marti Anderson,** | “A comparison of permutation methods for linear models". |
| **Bo Cai,** | “Lagrange interpolation adaptive rejection sampling". |
| **Dr Mark Berliner** | (Ohio State University), “Hierarchical Bayesian space-time models: geophysical applications". |
| **Professor Mark Wilson** | (University of Montana), “Asymptotics of multivariate generating functions". |

*Garry J. Tee*

## UNIVERSITY OF CANTERBURY

## Department of Mathematics and Statistics

Dr Charles Semple has been awarded (and accepted) a NZ science and technology post-doctoral fellowship. This three year fellowship (one of 24 awarded by the Foundation for Research, Science, and Technology) is based in the Mathematics and Statistics Department of The University of Canterbury. The title of the project is “Geometry of the space of X–trees and its applications to phylogenetic analysis.” Dr Semple is currently working with Associate Professor Mike Steel.

Dr Irene Hudson has had the pleasure of accepting a continuing honorary appointment as Adjunct Fellow, Institute of Land and Food Resources—Forestry, The University of Melbourne. The appointment is until the end of 2002.

The department would like to congratulate Drs Luminita Simona Dediu and Britta Basse on completing their Doctorates. Dr Basse is now a University of Canterbury post-doctoral fellow, and is working on modelling cell growth with Professor Graeme Wake.

Professor Wake has recently returned from Sultan Qaboos University in Oman, where he acted as an advisor to that University on the future direction of Mathematics at that University. In conjunction with his position at Canterbury as Dean of post-graduate research, Professor Wake has attended a conference on post-graduate research in Adelaide. Professor Wake has also been an invited speaker at the National University of Singapore.

**Seminars**

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|---|---|
| **Dr David Wall** | “Implicit Volterra history equations, their numerical solution and properties of dynamical systems defined through Volterra history equations.” |
| **Professor Malcolm Faddy** | “Extended binomial modelling and analysis of count data.” |
| **Dr Rachel Fewster** | (University of Auckland) “Models for species distribution and dispersal.” |
| **Professor John Guckenheimer** | (Cornell University) “Computing periodic orbits of vector fields.” |

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| **Professor Graeme Wake** | "How tough is your steak?" |
| **Dr Frank Lad** | "Assessing the value of a second opinion: the role and structure of exchangeability." |
| **Dr Bruce van Brunt** | (Massey University) "On a singular Sturm-Liouville problem involving an advanced functional differential equation." |
| **Dr Aaron T Gulliver** | (University of Victoria, Canada) "Optimal linear rate 1/2 codes." |
| **Clemency Williams** | (Brown University, Rhode Island) "Metrics, Mythology, Magic: a glimpse into the history of ancient Indian Mathematics." |
| **Professor Douglas Bridges** | "Apartness spaces as a foundation for constructive topology." |

*Chris Price*

## INDUSTRIAL RESEARCH LIMITED

### Applied Mathematics Team

Shaun Hendy has joined the team as a permanent staff member, after completing his NZ Science and Technology Post-Doc at the end of June. Brent Walker was awarded a Bright Future Scholarship and will be leaving us in September to begin his PhD studies at the Cavendish Laboratory at Cambridge. Brent has been working with Shaun Hendy on condensed matter modelling since November 1999. We wish him all the best for his studies.

Steven White and Graham Weir went to the World Geothermal Congress in Japan at the end of May. Steve gave three talks entitled "Rock alteration above a diorite intrusion", "Modelling corrosion and scaling in a flowing geothermal well" and "Modelling water-rock interaction in high-temperature granites". Graham gave a talk entitled "A mathematical model coupling heat and mass flow and extension in the Taupo Volcanic Zone". Warwick Kissling, Graham, Shaun and Brent attended the Wellington-Manawatu Applied Maths Day in June at Massey.

Finally, Steve has had his joint proposal with the University of Utah, entitled "Sequestration of $CO_2$ in geologic formations", funded by the U.S. Department of Energy. This project may involve travel to Utah during the northern hemisphere ski season (a.k.a. winter) although Steve is unsure as to whether his Whakapapa season ticket will be accepted in Utah.

*Shaun Hendy*

## MASSEY UNIVERSITY

### Institute of Fundamental Sciences

#### Mathematics

*"To be or not to be that is still the question….*".

The only comment that can be given is that we are still being repositioned. More details shall not be given as this may be sensitive to some staff. Many staff are furious and very bitter and are feeling betrayed about the mismanagement of funds at Massey.

Tammy Smith and Robert McKibbin attended the World Geothermal Congress (May 28–June 10, 2000). The WGC is held once every five years and this meeting was the first one to be held in Asia. It took place on May 28–June 10, 2000 in Japan. The venues were: first half—Beppu City, Kyushu, second half—Morioka City, Tohoku. The theme was "Sustaining Geothermal Energy into the 21st Century". In Morioka Tammy presented their joint paper "An Investigation of Boiling Processes in Hydrothermal Eruptions".

Robert McLachlan went to the United Kingdom on conference leave in July. He was plenary invited speaker at an LMS symposium on "Geometric Integration", in fact the first meeting solely devoted to this subject. Robert gave three lectures on splitting methods. More than 80 people converged on Durham for the 11 days of the symposium and a good time was had by all, although apparently a seaside trip to Whitby can't compete with the NZ Maths Research Institute's locations.

Mike Hendy and Barbara Holland are currently somewhere in Scandinavia. They have been invited to do some research with Kati Huber and Vince Moultan in Sundsval, Sweden.

Last month Benny Chor was farewelled over lunch which was held at Wharerata. We hope that Benny has enjoyed his time with us and that he will come back one day.

Mark Johnston will be taking up the position of Lecturer in Operations Research in the Mathematics Department at the University of Essex, Colchester in December. Congratulations! We wish Mark all the best for the future.

Chris Palliser (currently a postdoc) will be leaving Massey at the end of the month to join the Dairying Research Corporation in Hamilton as a Computer Modelling Scientist. All the best Chris!

Nicolas Allsop has successfully defended his thesis titled "The Quotient between Length and Multiplicity". Congratulations Nick! Nick commenced his PhD under the supervision of Wolfgang Vogel and Kee Teo. After Wolfgang's death, Dr. Lê Tuân Hoa became a supervisor. Nick was invited by Professor Jurgen Stuckrad (who also became Nick's supervisor) to do research for his thesis with him in Germany. Nicholas is now training to become an actuary.

Anton Raviraj Selvaratnam recently gained his MSc with his thesis titled "Geometrical Interpretations of Bäcklund Transformations and Certain Types of Partial Differential Equations". Congratulations Anton! Anton is now Head of

Department of Mathematics at Waihau College, Tuatapere ($46^o$ south).
The new foundation paper in Mathematics within the Foundations Studies Programme for overseas students started this Semester. From Margaret Walshaw: We have five students only but apparently that is a sufficient number to make the programme viable. This Semester they concentrate on English and Mathematics, and next Semester they take up either science or business subjects, in place of the Mathematics, and retain the English focus.
The Third Annual Wellington-Manawatu Applied Mathematics Day was held in the Russell Room at Wharerata (our Staff Club) on Thursday 15 June 2000. (For some of us a day off exam marking.) It was pleasing to see that 40+ people turned up, from as far north as Albany (Adrian) and as far south as Christchurch (Graeme Wake), with Victoria University, Industrial Research Ltd, AgResearch, Landcare and Massey University sandwiched in between. The day was a success, talks were interesting, weather was fine and moreover it is nice to catch up with your friends and colleagues. The speakers were:

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| **Professor John Guckenheimer** | (Cornell University), "Dynamical Systems and Neural Systems". |
| **Professor Paul Callaghan** | (Massey University – Physics), "Dispersive Flow". |
| **Alan Coulson** | (Industrial Research Limited), "Wireless Local Area Networks". |
| **Peter Donelan** | (Victoria University of Wellington), "The Geometry of Robot Control". |
| **Yong Hong** | (Victoria University of Wellington), "Porous Media". |
| **Mark Johnston** | (Landcare), "Applied Maths Research at Landcare". |
| **Ken Louie** | (AgResearch), "Modelling Clover-Ryegrass Growth". |
| **Tammy Smith** | (Massey University – Mathematics), "Hydrothermal Eruptions". |
| **Professor Graeme Wake** | (Canterbury), "Modelling of Cell-Growth". |

In April Robert McLachlan launched the "Mathematics Graduate Seminar Series", a series of talks by and for mathematics graduate students. It was inaugurated by showing the celebrated BBC documentary "Fermat's Last Theorem". The participants so far have been:

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| **Matthew Perlmutter** | (Institute of Fundamental Sciences – Mathematics), "Introduction to Symplectic and Poisson Manifolds with a View Toward Conformal Dynamics". |
| **Donna Giltrap** | (Institute of Fundamental Sciences – Physics), "Gasification of Biomass". |
| **Richard Love** | (Institute of Food, Nutrition and Human Health), "Models of Dough Sheeting". |
| **Li Xun** | (School of Global Studies),"Modelling Population Dynamics with Fossil Pollen Data". |
| **Tammy Smith** | (Institute of Fundamental Sciences – Mathematics), "Towards a Hydrothermal Eruption Flow Model". |
| **Margaret Walshaw** | (Department of Technology, Science and Mathematics Education),"Gender Research and the Postmodern". |
| **Jonathan Marshall** | (Institute of Fundamental Sciences – Mathematics), "Analyticity of Solutions to Functional Differential Equations". |
| **David Pidgeon** | (Institute of Fundamental Sciences – Mathematics), "An Introduction to Jet Spaces and Differential Equations". |
| **Padmanathan Kathirgamanathan** | (Institute of Fundamental Sciences – Mathematics), "Estimation of source of a pollution in the river". |
| **Patrick Rynhart** | (Institute of Fundamental Sciences – Mathematics), "Mathematical Modelling of Granulation". |
| **Padma Senerath** | (Institute of Fundamental Sciences – Mathematics), "An Introduction to Riemannian Space and its basic facts". |
| **Anthony Poole** | (Institute of Molecular BioSciences), "Gene Finding: How can we go about finding RNA in whole genomes?" |

**Seminars**

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|---|---|
| **Professor Benny Chor,** | (Technion, Haifa, Israel) (on sabbatical leave at Institute of Fundamental Sciences, Massey University), "Private Information Retrieval". |
| **Finlay Thompson** | (Victoria University of Wellington), "Quaternionic Frobenius Manifolds and Lorentzian Metrics". |
| **Stephen Joe** | (University of Waikato), "Assessing rank-1 lattice rules using average discrepencies". |

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| **Professor John Guckenheimer** | (Cornell University, USA), NZMS Lecturer 2000, "Thinking about Thinking" (Public Lecture). |
| **John Hudson,** | "General position in theory and practice". |
| **Mike Fellows** | (Victoria University of Wellington and University of Victoria, Victoria, Canada), "Coordinatized Kernels and Catalytic Branching: New Methods for Designing Efficient Algorithms for Small Parameter Ranges of Hard Problems". |
| **Barry Blundell** | (Department of Computer Science, University of Waikato, "Volumetric Visualisation: The Past, Present and Future". |
| **Dean Halford,** | "Some features of General Relativity Theory". |
| **Dr Igor Boglaev,** | "Domain Decomposition for Advection-Diffusion Problems". |
| **Peter Kelly,** | "Dropout in first year extramural classes – is it a maths problem or a problem with maths?" |

*Marijcke Vlieg-Hulstman*

**Institute of Information and Mathematical Sciences**

**Mathematics**

**Seminar**

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|---|---|
| **Frederick Heung Yeung Lam** | (MSc Student), "Ramanujan, Elliptic Functions and Sums of Squares". |

*Adrian Swift*

**Institute of Information Sciences and Technology**

**Statistics**

Two or three years have passed since these notes described the major reorganisaton of the Massey science faculties into a Science College. That accouunt finished with the hint that sometime in the future there would be an account of the benefits. Read on.

The statistics group has partially averted a threat that about half of them be repositioned to somewhere out of Massey, the number having now been reduced to three. For the time being the Vice-Chancellor has decreed that business statistics be taught by statisticians. Apart from this, student numbers have fallen, particularly in the year following Massey's reorganisation into Colleges, hence the need to reposition three of us.

Despite the battles at home some have organised short term overseas repositioning.

The 15th Australian Statistics Conference provided a welcome break, with several attending and giving talks, including two of our PhD students. Earlier in the year Chin Diew Lai and Geoff Jones found themselves alone on the Wollongong Campus on Australia Day, but stayed on long enough to work with John Rayner. Chin Diew also visited Bordeux, France for the Second International Conference on Mathematical Methods on Reliability. Mark Bebbington has just returned from visiting collaborators and attending conferences in USA and Canada. Steve Haslett is about to leave for the UK for a month exploring small area estimation at the ONS and Southampton.

We still receive visitors. Dr Yuthana Siriwathananukul from Prince of Songka University, Thailand spent three months working with Graham Wood on animal nutrition, and Dr Qiao Chungui is arriving soon to spend a year working on genotype by environment interactions.

*Greg Arnold*
*Geoff Jones*

## UNIVERSITY OF OTAGO

## Department of Mathematics and Statistics

In the last several months the Department has weathered three resignations, which, as this is being written, are just beginning to take effect. First to leave was Dr David Tan, an applied mathematician, and next comes Professor Bryan Manly and Dr Caryn Thompson. We are currently advertising for their replacements, but it is a stressful time as we cope with heavier teaching loads, the loss of their research output, and, most importantly, their cheery faces around the department. Hopefully the new staff will have cheery faces too!

Richard Barker attended the International Workshop on Wildlife Population Assessment in the University of Queensland from 29 June–8 July.

John Clark attended the Ring Theory section of the 25th Ohio State-Denison University Algebra Conference in Columbus, Ohio, from 18–21 May.

Derek Holton attended Logos 6, one of a series of meetings organised by the Mathematics Education Unit at the Auckland University, from 18–21 April. This particular meeting centred around graduate degrees and research in mathematics education in New Zealand. As a result of the discussions, people interested in mathematics education are now in touch and are sharing information related to their graduate courses. Hopefully too, we'll see some joint research come out of this meeting.

A report on the AERA meeting, New Orleans, 27–28 April by Derek Holton: This was my first AERA meeting and an interesting and stimulating event it was. I'm not sure what the final number of participants was but it was in excess of 2500. So finding people and places was not easy and neither was tracking down what talks you wanted to hear from the mass of talks available.

Apart from visiting the swamps of Louisiana, the highlight of the conference for me was attending the talk by Howard Gardner. He is the "inventor" of multiple intelligences, which roughly means that you can be intelligent in maths, dancing and a lot of other things. Howard's interesting plenary session was marred by the sounding of fire alarms. Most of the faithful sat put but I sneaked out (in fear of my life!) only to be told by an official-looking person that it was a false alarm. So I sneaked back in. The alarm continued for another 20 minutes! During this time Howard continued as if nothing was happening, except for an odd joke aimed at the noise with which he was competing. I found it difficult to concentrate but Howard discovered a new multiple intelligence: being able to give a plenary session with accompanying fire alarms.

Gerrard Liddell ran a display for Science Festival 2000. The CyberMath session used interactive software to introduce maths to the public. New software by John Shanks covering maths from school to first year university proved popular and captivated children. Other interactive software showed the applications of maths from permutation group basis and Rubik's cube, to robots in engineering.

Bryan Manly is just in the process of packing up and moving overseas after 27 years at the University of Otago. Earlier in the year he used up some holiday leave at the Max Planck Institute for Limnology in Ploen, giving a course for research students, and collaborating with the staff.

After leaving Otago, Bryan is going to Norway to give a workshop on resource selection by animals, to the Joint Statistical Meetings in the US, then up to the annual meeting of the Alaskan Section of the ASA, before starting work in his new capacity as a statistical consultant with WEST Inc. in Wyoming.

He anticipates making many return trips to New Zealand to visit friends, his two daughters, who still live in Dunedin, and his five grandchildren.

A report on 7th Western Black Bear Workshop, Coos Bay, Oregon by Junior Research Fellow Darryl MacKenzie: In early May I travelled to Coos Bay, Oregon, to attend the 7th Western Black Bear Workshop to present a paper on a method for estimating population size, "Negative binomial models for counts of unique individuals". The workshop was attended primarily by biologists, geneticists and field workers from the western United States, Canada and Alaska who work with or research black and grizzly bears. Being the lone statistician out of over 100 participants (and a Kiwi to boot) made me a very small minority. The workshop started on Tuesday afternoon with a session "Social Aspects—Bears and people" which included an interesting talk about a court case between the Arizona Game and Fish Department and the family of a teenage girl who was mauled in her sleep by a black bear at a camping ground, and another presentation by the Oregon Police Department detailing how they broke a local black bear poaching ring. On Tuesday evening we piled into some bright yellow school buses and headed to Sunset Bay for an icebreaker session where we sampled some of the local delicacies and beverages. Over the next 2 days, 4 more sessions were held: "Research Methods and Techniques", "Grizzly Bears—Management and Conservation", "Management of bear-human conflicts", and "Ecology—Bears and their environment", with a field trip on the final day, a jet boat tour up the Rouge River. My personal highlight was seeing a black blur during the jet boat ride which, I was later informed, was a black bear. However I found it very educational listening to the many practical difficulties speakers faced with their data collection (something those of us who sit in front of a computer often forget) and at the same time the lack of a good study design with little forethought of how they intended to analyse their data. To many readers this latter point may be nothing new and one that can only be addressed by the breaking down of the interdisciplinary barriers that still appear to be firmly in place through education and effective communication. I was somewhat surprised and saddened that many participants approached me after my talk commenting that it was one of the first papers they had heard presented by a statistician that they could understand and follow.

Our Head of Department, Vernon Squire has finally surfaced from his office with the onset of the inter-semester break only to dash off to two conferences within a couple of weeks of one another. The first meeting, the 10th International Offshore and Polar Engineering Conference, was held in Seattle, USA, from 27 May–3 June. The second was the International Glaciological Society International Symposium on Sea Ice and its Interactions with the Ocean, Atmosphere and Biosphere, held in Fairbanks, Alaska from 19–23 June. Vernon presented different papers at each conference and was on their Editorial Boards. He came back to New Zealand between the end of one and the start of the second, which was rather silly in hindsight but was necessary because of exams and some rather heavy budget-oriented administration.

We welcomed Greg Trounson to the Department as Computer Programmer on 12 April.

**Seminars**

**Craig Ansley** (Frank Russell Company N.Z. Limited), "Mathematics and finance, marriage or affaire".

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|---|---|
| **Professor John Guckenheimer** | (Cornell University), “Thinking about thinking". Professor John Guckenheimer (Cornell University), “Computing periodic orbits of vector fields”. |
| **Darryl MacKenzie** | “Assessing the goodness-of-fit of mark-recapture models—PhD progress report”. |
| **Stefanka Chukova** | “Proportional rate models in environmental studies. Applications in cost analysis”. |

*Lenette Grant*

## UNIVERSITY OF WAIKATO

### Department of Mathematics

In April Alfred Sneyd attended a conference in Japan on electromagnetic processing of materials. He is currently on study leave until the end of the year. Most of this time is being spent in France.

There were a number of other travellers in the department. Rua Murray spent nearly a month in the UK and Germany where he presented two conference papers as well as give seminars at Strathclyde University, the University of Surrey, and the University of Paderborn. Ian Craig has just returned from a similar trip to the UK and France. He visited Grenoble (where he met up with Alfred), University College London, and the University of Manchester. Ernie Kalnins attended a mathematical physics workshop held on the Gold Coast. He came back for a week and is currently away for two weeks attending a conference in Dubna, Russia, on group-theoretic methods in physics. His post-doc, Jonathan Kress, is also attending this conference, but left much earlier to spend six weeks visiting the University of Sydney as well as attending a conference in Rome. Those of us left behind had to endure the usual grind.

We have had several visitors over the past few months. Anant Vyawahare of Nagpur, India, visited us for six weeks under the sponsorship of Rotary International. Giovanni Santoboni from the University of Maryland visited Rua for two weeks in May. They collaborated together on dynamical systems. Also visiting was James Lyness from Argonne National Laboratory. He and Stephen Joe continued their work on the structure of lattice rules.

Preparations for the Colloquium to be held here in November are well in hand. Don’t forget to register for this event; a notice about the Colloquium is elsewhere in this issue.

The cellphone transmitters have now been installed on the roof of our building and are set to be activated very soon. The radiation levels in the rooms on our floor have been measured and measurements will be done again when the transmitters are fully functioning.

### Seminars

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|---|---|
| **W. Moors,** | “Construction of pathological Lipschitz functions”. |
| **A. Sneyd,** | “Waves in liquid metal and in water”. |
| **A. Vyawahare** | (Nagpur University), “Vedic Mathematics”. |
| **G. Santoboni** | (University of Maryland), “Advection of active particles in open chaotic flows”. |
| **E. Kalnins,** | “Group representation theory and Bessel functions”. |
| **M. Reddy** | (University of the South Pacific), “Some new results in the structure of lattice rules”. |
| **S. Joe,** | “Average discrepancies for rank-$1$ lattice rules”. |
| **R. Murray,** | “What is the ergodic theory of dynamical systems?”. |

***Stephen Joe***

### Department of Statistics

Bill Bolstad was on sabbatical for the first half of 2000. While on leave, he attended ISBA2000 in Crete, NORSTAT, in Grimstad, Norway and IBC2000 at Berkeley, San Francisco. For the second half of the year, Lyn Hunt will be on leave. She will be working at the University of Queensland with Professor Kaye Basford, furthering her research into clustering techniques.

June and July saw a large contingent of department members head overseas to various conferences. Murray Jorgensen and Lyn Hunt attended CSNA2000 in Montreal, Canada, where they presented papers on clustering. Murray’s paper was on the work he has been doing into the clustering of internet traffic data. Nye John, Judi McWhirter, Murray Jorgensen and Lyn Hunt attended ASC15 in Adelaide, Australia. Nye and Judi also took part in the Statistics Education Workshop and Murray and Lyn participated in the Data Mining workshop. Honorary lecturer Harold Henderson also attended IBC2000. In April, James Curran attended the spring meeting of the UK Forensic Science Society and the California Association of Criminalists, in Napa, California. He was enabled to do this

because of his award from the Joint Presidents for the best young researcher in Forensic Science.
The department is currently hosting David Johnson who is on a return visit from Loughborough University. While he is here, David is working with Nye John and David Whitaker on a book for teaching first year management statistics.
In July, the department was visited by Dr Bruce Weir and Dr Christopher J. Basten from North Carolina State University. They presented a very successful Two-Day Workshop on Continuous and Discrete Trait Mapping on 11-12 July 2000, which was attended by about sixty participants.

**Seminars**

| | |
|---|---|
| **Dr Marti Anderson** | (Department of Statistics, University of Auckland), "A comparison of permutation methods for linear models". |
| **Dr Robert Gentleman** | (Department of Statistics, University of Auckland), "Computational algorithms for censored data". |
| **Dr John Buckleton** | (Institute of Environmental Science and Research Ltd, Auckland), "Statistics in forensic science". |
| **Mr Donal Krouse** | (Industrial Research Ltd, Lower Hutt), "Statistical process control and improvement for short-run production". |
| **Mr Bruce Miller** | (Productivity Improvement Consultant, Auckland), "Life as an industrial statistician". |
| **Dr Mark Rizzardi** | (Humboldt State University, California), "Statistical methods for ordinal-valued flower phenology data". |
| **Dr Alain C. Vandal** | (Clinical Trials Research Unit, University of Auckland), "A covariate-based measure of source dependence for epidemiological capture-recapture modelling, with an application to the Auckland Leg Ulcer Study". |
| **Dr Randy Sitter** | (Simon Fraser University, Canada), "Design issues in fractional factorial split-plot experiments". |
| **Dr Ian Westbrooke** | (Department of Conservation, Christchurch), "Using trellis graphics to display and analyse multi-dimensional data". |

*Judi McWhirter*

## VICTORIA UNIVERSITY OF WELLINGTON

**School of Mathematical and Computing Sciences**

We are scrambling here to tackle the deficit, with various timelines, working groups and structures being set up. There is considerable consultation of staff, which gives me heart. Otherwise it is business as usual.
John Harper, Young Hong, Leigh Roberts and Peter Donelan attended the 3rd Wellington/Manawatu Applied Maths day at Massey—Young and Peter gave talks and everyone enjoyed the day.
Peter Donelan is presenting a paper at Ball 2000, a symposium being held in Cambridge UK to honour the 100th anniversary of the publication of Robert Ball's "Treatise on the Theory of Screws". He is also visiting David Chillingworth at Southampton University.
John Harper is overseas in July and August for the IUTAM Symposium on Free Surface Flows in Birmingham, UK and the Isaac Newton Institute for Mathematical Sciences workshop on Free Boundary Problems in Industry in Cambridge, UK.
Rod Downey has a visitor: Walker White from Cornell. Walker has just finished his PhD on computable model theory with Richard Shore. He is here for two months supported by a Marsden Grant. Rod's postdoc Denis Hirschfeldt recently spoke at the Ershov meeting in Novosibirsk, and will soon be heading to Paris for the Logic Colloquium. The connections between mathematics and dancing are well documented, but to add to this lore Rod recently taught at the Hamilton winter school in scottish country dancing.
James Oxley from Louisiana State University, and Charles Semple from the University of Canterbury, visited Geoff Whittle for two weeks in June.
Vladimir Pestov and Finlay Thompson both spoke at the Australian National Colloquium on Geometric Analysis held at ANU in June.
Mark McGuinness went to two sea ice conferences at the University of Alaska, Fairbanks, during two weeks in June, and presented a poster on the penetration of solar radiation into sea ice. There was a surprising number of New Zealanders there. It was mid-summer, warm, and the sun barely went down at night. Mark will visit Kerry Landman at the University of Melbourne in August.

**Seminars**

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|---|---|
| **Dr. Finlay Thompson,** | "Quaternionic gerbes". |

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|---|---|
| **Dr. Zbigniew Piotrowski** | (Youngstown State University), "Separate and joint continuity". |
| **Dr. Roger Sugden** | (Goldsmith's College, London), "Cochran's rule and edgeworth expansions for simple random sampling". |
| **Dr. Peter Donelan,** | "Darboux motions and their singularities". |
| **Dr. Corrin Lakeland** | (Otago), "Robust statistical parsing". |
| **Dr. Peter J. Smith,** | "Random matrices in communication systems". |
| **Fiona Walls,** | "The effects of teaching practices on children's learning of mathematics: longitudinal case study research". |
| **Dr. Christopher Atkin,** | "Some remarks on hyperbolic secants". |
| **Dr. Denis Hirschfeldt,** | "Structure theorems and nonstructure theory in computable structure theory". |
| **Dr. Rick Schoenberg** | (UCLA), "Short-term exciting, long-term correcting models for earthquake catalogs". |
| **Dr. Hilary Ockendon** | (Oxford University), "Spinning and weaving: mathematical problems from the textile industry". |
| **Dr. Peter Winbourne** | (British Council visitor from the University of the South Bank, UK), "Constructing narratives about learners' identities: how might such narratives inform the teaching of mathematics?". |
| **Dr. Graham Weir** | (Industrial Research Ltd), "Orientation of failure planes in stress space". |
| **Professor Andre Nies** | (University of Chicago), "Model theoretic properties of degree structures". |
| **Dr. Mark McGuinness,** | "Sun on sea ice". |
| **Dr. Paul Martin,** | "XML—Whizzy new web technology". |
| **Dr. John Hine with Professor Brian Corbitt** | (the JADE Professor of E-Commerce, VUW); Jenny Mortimer (CIO of Telstra Saturn); a forum on "The future of the internet". |
| **Dr. Ed Mares,** | "An introduction to the logic of belief revision". |
| **Dr Jian Yang,** | "Semiparametric maximum likelihood estimation of ARMA models". |
| **Professor John Guckenheimer** | (Cornell University), "Computing periodic orbits of vector fields". |
| **Dr. Andy Cockburn** | (Canterbury), "Understanding and supporting web navigation". |
| **Dr. Arkady Leiderman** | (Ben-Gurion University of the Negev, Israel), "Kolmogorov's superposition theorem and its applications". |
| **Dr. Charles Semple** | (University of Canterbury), "A characterization for a set of characters to define a phylogenetic $X$-tree". |
| **COMP Honours Seminars** | Simon Brandon: "Planning the loading of pallets onto trucks", and Kirk Jackson: "Understanding frameworks through visualisation". |

*Mark McGuinness*

## NZMS Lecturer 2000

*Professor John Guckenheimer toured New Zealand during May/June 2000 as NZMS Lecturer 2000. He spoke in all university centres, giving general and seminar talks in the general area of dynamical systems and their application. He was a keynote speaker also at the Wellington/Manawatu Applied Mathematics day in Massey University on 15th June.*
*He hails from the Mathematics Department of Cornell University, New York. He provided this commentary on the eve of his return to the United States. We thank him for this and his many contributions while he was with us.*

*Graeme Wake*

As NZMS Lecturer, I visited seven mathematics departments and groups during May and June, 2000. Here I reflect briefly upon my experiences as I prepare to return to the United States. Based in Christchurch during my two months in New Zealand, I thoroughly enjoyed my trip to Otago and tour of the North Island. The mathematical discussions were interesting for me. As a bonus, I was able to sample the splendid outdoors of the country and enjoy the wonderful kiwi hospitality.

The New Zealand mathematics community is very small, perhaps the size of two large departments in the United States. The mathematics communities in Boston, New York or Los Angeles are each much larger than the whole country. This small size and the long distances between New Zealand and North America brought me back to an earlier period in my career when I was a member of a department with thirteen faculty. The University of California at Santa Cruz was founded in the 1960's and was still very young when I arrived as a young faculty member in 1973. Not being a "founder", I was a newcomer throughout my twelve years on the faculty there. UCSC was about fifty miles from Stanford and about seventy-five miles from Berkeley, but it still felt isolated and remote. I learned there what it was like to do mathematics research with a sense of being out of the mainstream. My attitude was a state of mind due to having been fortunate to have spent my student and postdoc years in major mathematical centers. In the best of circumstances, doing mathematics research requires great perseverance. Continuing to pursue research vigorously in an isolated environment needs even greater fortitude. The isolation of New Zealand is far greater than that I experienced in Santa Cruz. I admire those of you who continue to do research here in New Zealand. There are few colleagues with whom to share your efforts, few research students to ask probing questions and few opportunities for interactions to help shape our sense of which mathematical problems are important and why. Mathematics research in these circumstances is a labour of love, and my hats are off to those of you who continue to do it.

As the New Zealand economy struggles with fluctuating world markets for wool, lamb and dairy products, the secondary impact of a poor economy on universities is evident. The prevailing mood is that resources for higher education and scientific research are meagre, a situation that is unlikely to improve in the near term. Wherever I go, two frequent topics of mathematical conversation are always jobs and research support. Both are clearly vital to the profession and to its renewal. The job market for academic positions in the US goes through cyclical swings that are sensitive to the economy. Since I completed my Ph.D. at the end of the "golden age" of the 1960's, concern about declines in research funding have been continuous throughout my career. Both jobs and research funding are in short supply now in New Zealand. That is discouraging, but I believe that many of the issues affecting mathematics are largely independent of economic factors. They have more to do with the role of mathematics in education and science.

Throughout the world, mathematicians are highly dependent upon teaching as the primary financial support for the profession. Mathematics remains a large part of education in primary and secondary schools. Training qualified staff and writing sound curricula provides demand for maintaining strong academic mathematics programs in universities. However, mathematics departments with research aspirations cannot thrive on teaching prospective schoolteachers. Paradoxically, computers seem to have decreased the demand for mathematical skills. The way in which mathematics is used in modern "high tech" enterprises is different from the past. A lot of what we trained individuals to do is now done automatically with much less human effort. Despite past predictions about the need for more mathematical training of a technologically proficient workforce, it appears that the discipline of mathematics may not be the primary provider of such training. If science, engineering and business oriented disciplines decide that they are dissatisfied with the courses taught by mathematicians, they will increasingly decide to teach the requisite mathematics themselves. In Europe, the United States and New Zealand, reshaping university mathematics so that it remains attuned to the needs of students in a changing world is a big challenge. I fear that our response to this challenge has been rather feeble. This weakness, along with the perception that other disciplines lead to better jobs, may be reflected in declining mathematics enrolments. With its small size and good balance between pure and applied mathematics (and statistics) within its mathematics departments, New Zealand could easily demonstrate international leadership in curriculum reform for university mathematics. We need to look beyond our perennial sense that today's students are less capable than those of yesterday and focus upon how to most effectively educate students for the world that they will face tomorrow. I believe strongly in the necessity of keeping mathematics at all levels responsive to the forces that change our world.

On the research side, mathematics remains "medium tech", needing less equipment and technical personnel than other scientific professions. One benefit of this status is that it is possible for individuals to practice mathematics without laboratories, technicians or supplies. The only equipment we use regularly are computers, and even inexpensive personal computers are more powerful than "supercomputers" of a decade ago. Nonetheless, I think that the stimulation of working with colleagues is critical to most individuals in maintaining their enthusiasm for doing research. As long as each department maintains breadth adequate for its educational responsibilities, I believe that increased concentration in selected areas might increase enthusiasm for research and lead to greater distinctiveness of New Zealand's mathematics in international settings.

I am impressed by New Zealand's export of mathematical talent, with the best students encouraged to go overseas. While this diminishes the stimulation coming from research students in the local research environment, I think that it promotes international communication and serves to lessen the impact of geographical isolation. I am a firm believer in a free a market as possible for academic manpower. The United States, as a large importer of mathematical talent, has demonstrated the importance of creating an attractive working environment and being open to hiring in an international framework. You can take pride in the numbers of highly talented students from New Zealand who study abroad and succeed in developing strong research careers. Having few fellowships or assistantships for research students in New Zealand makes it difficult to keep students here, but with the small number of faculty openings domestically, it is wise to encourage students to seek overseas opportunities as well as domestic ones. All of us work as part of an international community, one that has become more tight knit with faster, cheaper communications and transportation systems. On balance, it seems to me that sending students aborad for advanced training is healthy for everyone, as long as sufficient attention is given to ensuring "critical mass" in research groups here.

On a personal level, one aspect of New Zealand life that I found difficult was the lack of heat in buildings. You are a stoic, hardy lot when it comes to the weather!

## THE CRAWLER

In the words of the immortal "computer error messages in haiku" spam,

The web site you seek
Can not be located but
Countless more exist.

Exactly this happened to me the other day. (OK, today, while I was supposed to be writing the editorial.) Here's what I found instead. First, if you're feeling confident, take a look at http://www.claymath.org. The Clay Mathematics Institute, formally launched at MIT in May (for the opening ceremony, see http://www-tech.mit.edu/V119/N26/26cmi.26n.html), was founded by businessman Landon T. Clay to "further the beauty, power and universality of mathematical thought." All right! Their scientific panel consists of Alain Connes, Arthur Jaffe, Ed Witten, and Andrew Wiles, who are no slouches. They sponsor a range of activities, but by far the most spectacular are their US$1m prizes offered for solving any of seven fundamental mathematical problems—P vs. NP, the Hodge, Poincaré, and Birch–Swinnerton-Dyer conjectures (which reminds me of hearing Sir Peter Swinnerton-Dyer described as "two-thirds of the Birch–Swinnterton-Dyer conjecture embodied in just one man"), the Riemann hypothesis, and problems related to the Yang-Mills and Navier-Stokes equations. Don't be shy, have a go.

The mention of Alain Connes, who seems to be heavily involved in the Institute, made me wonder what he was working on, so I trolled his recent papers in MathSciNet. (You can spend a happy hour or two searching for the phrase 'Featured Review'.) There I found the paper MR 99h:81137 by Alain Connes and Dirk Kreimer, *Hopf algebras, renormalization and noncommutative geometry*, which concerns (inter alia ...) functions from vector fields to diffeomorphisms of the form $f \mapsto \phi_f(x) = x + hc_1 f + h^2 c_2 f' f + \ldots$, where $h$ is a small parameter and the terms in the series are the so-called elementary differentials of $f$. Sound familiar? These are known in numerical analysis as $B$-series, named after John Butcher. It was noticed by Christian Brouder (*Runge-Kutta methods and renormalization*, preprint, http://xxx.lanl.gov/abs/hep-th/9904014) that many of the combinatorial identities established by Connes and Kreimer in their study of renormalization were already proved by John Butcher in 1972 in a study of the algebraic properties of Runge-Kutta methods. John has studied the group formed by Runge-Kutta methods, but now Connes and Kreimer have shown they they (or, equivalently, the rooted trees) also form a Hopf algebra. It seems that this algebra allows one to tell which terms in, say, a Feynman diagram or a divergent series should be grouped together to allow the series to converge. Well-deserved flights of angels must have been playing around John's head when he heard about this one...Brilliant! It's the Revenge of Numerical Analysis.

Back to web crawling, it's World Mathematics Year at the moment (http://wmy2000.math.jussieu.fr), and groups in Montreal, Paris, London, and elsewhere have had the idea of promoting maths by putting posters in subway trains. The posters look great and can be seen at http://omega.CRM.UMontreal.CA/math2000/affiches/ and http://www.newton.cam.ac.uk/wmy2kposters/index.html. (The Newton Institute were originally giving away copies of these, but seem to have run out—let me know if you succeed in getting any hard copies of the posters.) While looking for the Newton Institute posters I came across a nice essay at http://www.cms.cam.ac.uk/news6/ by Tom Körner, next year's Forder Lecturer, who has a nice twist on an old saw:

> *It is said that the Amazon rain forests are a major resource for humanity because they contain so many different plants some of which will have presently unknown but vital uses in the future. Pure mathematics represents a similar resource full of species—some beautiful, some grotesque, all useless at present (or they would already be applied mathematics) but a few which will have remarkable uses in the future.*

*Robert McLachlan*, R.McLachlan@massey.ac.nz

## OBITUARY

### Emeritus Professor T.R.F. Nonweiler
### BSc(Manc), PhD(Belf), CEng, FRAeS, FIMA, MIPENZ
### 8.2.1925 — 17.12.1999

Terence Reginald Forbes Nonweiler was Professor of Applied Mathematics at Victoria University of Wellington from 1975 to 1991. Born in 1925 in Wandsworth, London, his interest and ability in both mathematics and drama developed early and stayed with him all his life.

After his 1944 Honours degree in Mathematics from Manchester, Terence went to the Royal Aircraft Establishment, Farnborough, where he began working on high-speed flight. In 1951 he moved to the Cranfield College of

Aeronautics, where he discovered that he liked lecturing, which drew on some of the skills he had developed in the theatre.
Terence's "waverider" design for a supersonic wing in 1958 won him the Royal Aeronautical Society Gold Medal, and many years later led to his invitation (in 1990) to address the First International Waverider Symposium in Washington. He moved to Queen's University, Belfast in 1958, and in 1961 went to the University of Glasgow, where he was Professor of Aerodynamics and Fluid Mechanics and, in 1971, became Dean of Engineering.
Terence's final move was in 1975, to the Victoria University of Wellington Chair of Applied Mathematics. His research here was mainly in numerical analysis. His 1984 textbook "Computational Mathematics" is distinctly original and contains much useful information not easily found elsewhere.
Terence had enormous energy: besides his academic career he had another in the theatre, both as actor and director, sometimes amateur, sometimes professional. (His performance as King Lear opposite his son Barry, then a Victoria University of Wellington English lecturer, as the Fool, was particularly memorable.) He was also an urbane, charming colleague; this defused more than one fraught meeting.
After retiring, he continued his scientific work until his death. Recently he was researching the aerodynamical safety of small cars in strong winds.
Our sympathy goes to all his family.

*J.F. Harper*
*Professor of Applied Mathematics*
*Victoria University of Wellington*

## NEW COLLEAGUES

**Dr Lutz Grosz**

**Associate Professor James Sneyd**

Dr **Lutz Grosz** arrived March 2000 to take up a position of Lecturer in Mathematics at Massey University, Albany. As reported in the April Newsletter, he completed his Diploma of Mathematics at the University of Hannover/Germany in 1988 and then joined the Research Group for Numerical Methods on High Performance Computers at the University of Karlsruhe/Germany, working on scalable software for the solution of non-linear PDEs. After 7 years at Karlsruhe, Lutz took up a position in the School of Mathematical Sciences at ANU, to contribute to the scientific software project with Fujitsu/Japan. He finished his PhD in Mathematics in 1997 at the University of Karlsruhe. Recent research interests lie in the area of preconditioned, iterative linear equation solvers. Lutz has already has made a significant impact on the Mathematics group as he has been instrumental in setting up a small Beowulf cluster of 4 Pentium III processors to improve the Institute's computational facilities. The success of this venture encourages us to try to obtain funding for expansion of the cluster. On a personal front, he and Andrea have settled in well and are looking to buy a piece of dirt in the country from their base on the Whangaparaoa. His yellow BMW motorcycle is an object of admiration in the car park!

**James Sneyd** took up his position as Associate Professor, Mathematics at Massey University, Albany in January 2000. As reported in the April Newsletter, after BSc studies at Otago University, he moved to New York University where he did a PhD under Charles Peskin. He then spent a post-doctoral year at Oxford, followed by a period as assistant professor at the University of California, Los Angeles. In 1994 he returned to NZ and spent three years at Canterbury University, helping to set up the Biomathematics Research Centre. In 1997 he returned to the United States, and spent the next three years at the University of Michigan, where he still holds an appointment as Professor. In 1998 Sneyd won the American Association of Publishers prize for the best mathematics book of 1998 ("Mathematical Physiology", J. Keener and J. Sneyd, Springer-Verlag). He has been an invited speaker at numerous international conferences, including four Gordon Conferences, the 1995 ICIAM meeting, and the 1998 SIAM meeting. In 2000 he is an invited speaker at the New Zealand Mathematics Colloquium, while in 2001 he is a plenary speaker at the SIAM Snowbird conference on Dynamical Systems. He is also organising the American Mathematical Society short course on Mathematical Biology for their annual meeting in January 2001.
His research interests are in mathematical biology and physiology, particularly the application of differential

equation theory, nonlinear dynamics, and bifurcation theory to the study of self-organisation and nonlinear waves.

## BOOK REVIEWS

## SPRINGER-VERLAG PUBLICATIONS

Information has been received about the following publications. Anyone interested in reviewing any of these books should contact

David Alcorn
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**Ambrosio L**, Calculus of variations and partial differential equations: topics on geometrical evolution problems and degree theory. 347pp.
**Bachman G**, Fourier and wavelet analysis. (Universitext) 510pp.
**Balakrishnan R**, A textbook on graph theory. (Universitext) 280pp.
**Balser W**, Formal power series and linear systems of meromorphic ordinary differential equations. (Universitext) 299pp.
**Berggren L**, Pi: a source book. 736pp.
**Bridson M**, Metric spaces of non-positive curvature. (Grundlehren der mathematischen Wissenschaften, 319) 643pp.
**Callahan JJ**, The geometry of spacetime. (Undergraduate Texts in Mathematics) 320pp.
**Cohen H**, Advanced topics in computational number theory. (Graduate Texts in Mathematics, 193.) 570pp.
**Cohn P**, Introduction to ring theory. (Springer Undergraduate Mathematics Series) 229pp.
**Dafermos C**, Hyperbolic conservation laws in continuum physics. (Grundlehren der mathematischen Wissenschaften, 325) 443pp.
**Demazure M**, Bifurcations and catastrophes: geometry of solutions to nonlinear problems. (Universitext) 301 pp.
**Duistermaat JJ**, Lie groups. (Universitext) 334pp.
**Dullerud GE**, A course in robust control theory. (Texts in Applied Mathematics, 36) 417pp.
**Eisenbud D**, The geometry of schemes. (Graduate Texts in Mathematics, 197) 305pp.
**Engl K-J**, One-parameter semigroups for linear evolution equations. (Graduate Texts in Mathematics, 194) 586pp.
**Evans G**, Analytic methods for partial differential equations. (Springer Undergraduate Mathematics Series) 299pp.
**Evans G**, Numerical methods for partial differential equations. (Springer Undergraduate Mathematics Series) 290pp.
**Exner GR,** Inside calculus. (Undergraduate Texts in Mathematics) 211pp.
**Hoppensteadt FC**, Analysis and simulation of chaotic systems. (2nd ed) (Applied Mathematical Sciences, 94) 315pp.
**Hurley DJ**, Geometry, spinors and applications. (Springer Praxis Books) 369pp.
**Jacod J**, Probability essentials. (Universitext) 250pp.
**Kevorkian J**, Partial differential equations. (2nd ed) (Texts in Applied Mathematics, 35) 645pp.
**Mardesic S**, Strong shape and homology. (Springer Monographs in Mathematics) 489pp.
**Murota K,** Matrices and matroids for systems analysis. (Algorithms and Combinatorics, 20) 483pp.
**Nathanson MB**, Elementary methods in number theory. (Graduate Texts in Mathematics, 195) 513pp.
**Neukirch J**, Cohomology of number fields. (Grundlehren der mathematischen Wissenschaften, 323) 699pp.
**Onishchik AL**, Lie groups and Lie algebras II: discrete subgroups of Lie algebras and cohomologies of Lie groups and Lie algebras. (Encyclopaedia of Mathematical Sciences, 21) 223pp.
**Radjavi H**, Simultaneous triangularization. (Universitext) 318pp.
Rynne BP, Linear functional analysis. (Springer Undergraduate Mathematics Series) 273pp.
**Saito M**, Gröbner deformations of hypergeometric differential equations. (Algorithms and Computation in Mathematics, 6) 254pp.
Schiff JL, The Laplace transform: theory and applications. (Undergraduate Texts in Mathematics) 233pp.
**Seroul R**, Programming for mathematicians. (Universitext) 429pp.
**Struwe M**, Variational methods: applications to nonlinear partial differential equations and Hamiltonian systems. (3rd ed) (Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge, 34) 274pp.
**Thorisson H**, Coupling, stationarity, and regeneration. (Probability and its Applications) 353pp.
**Tymoczko T**, Sweet reason: a field guide to modern logic. (Textbooks in the Mathematical Sciences) 670pp.
**Üstünel AS**, Transformation of measure on Wiener space. (Springer Monographs in Mathematics) 296pp.
**Varga RS**, Matrix iterative analysis. (2nd ed) (Springer Series in Computational Mathematics, 27) 358pp.
**Wagon S**, Mathematica in action. (2nd ed) 592pp.
**Zelikin MI**, Control theory and optimization I: homogeneous spaces and the Riccati equation in the calculus of variations. (Encyclopaedia of Mathematical Sciences, 86) 283pp.

**A History of Algorithms: From the Pebble to the Microchip**
by Jean-Luc Chabert *et alia*, (translated from French by Chris Weeks)

Springer, Berlin, 1999, 9+524 pages, DM 98.00. ISBN 3-540-63369-3.

The mathematical documents which survive from the ancient civilisations of Mesopotamia, Egypt, China and India consist largely of worked examples of calculations, demonstrating rules for performing various operations, such as arithmetic operations and solving linear and quadratic equations. Some ancient Greek philosophers developed mathematics as a logical structure, with proofs of theorems and constructions. In particular, Euclid treated greatest common divisors in his **Elements** (*c*.-300), stating a set of rules to apply to a pair of commensurable quantities, and he proved that those rules construct the greatest common divisor in a finite number of steps. (A Chinese text, which uses equivalent rules to reduce fractions to lowest terms, might pre-date Euclid.)

Abū Ja'Far Muhammad ibn Mūsā al-Khwārizmī (*c*779–*c*850) was born at Khwārizmīa (now in Uzbekistan), and he became a scholar at the House of Wisdom in Baghdad, founded by the Caliph al-Mamun. Al-Khwārizmī wrote clear and systematic texts on arithmetic, algebra, astronomy, chronology, scientific instruments *et cetera*, which have had immense influence on the development of science. Some of his works were translated into Latin in the 12th century, with his name corrupted into Algorizmi or Algorithmi; and consequently arithmetic based on the Indian decimal system became known as algorithm, in Latin, English and other European languages. By the 17th century, European mathematicians were using that word for any set of definite rules in mathematics. But by 1930 the word algorithm had practically ceased to be used, except for the Euclidean algorithm for greatest common divisor.

During the 1930s, several mathematicians attempted to formalize the intuitive concept of a well-defined computation, and very diverse definitions were proposed by Kurt Gödel, Alonzo Church, Stephen Kleene, Alan Turing and Emil Post. They agreed that, for a computation to be regarded as well-defined, it had to involve a finite number of steps. The fact that those diverse definitions of a well-defined computation proved to be logically equivalent persuaded most mathematicians to accept that common formalization, with the Turing Machine formulation being regarded as the simplest and clearest version. Major advances were made by Church and Turing, who proved that some proposed types of computation (including Hilbert's proposal for a computation which would decide, for every mathematical statement, whether it was true or false; and Turing's suggestion of a computation which would decide, for every computation, whether it would halt after a finite number of steps) could not be done by any well-defined computation.

During the late 1940s, the several groups of pioneers of computers realized that, in order to get a computer to perform any specific task, the operation of the computer must be controlled by minutely detailed unambiguous instructions for the successive steps. A word was needed for such a set of instructions, and so the obsolete word algorithm was revived for that purpose. A computer program is (or should be) an implementation of an algorithm, written in a language which the computer can accept; and so algorithms are now a very important subject. Problems such as Hilbert's Decidability Problem and Turing's Halting Problem are now described as "algorithmically unsolvable".

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This sourcebook on the History of Mathematics, written by a team of 8 French mathematicians (with Jean-Luc Chabert as editor), focusses on the development of algorithms. There are 15 chapters, on Algorithms for Arithmetic Operations, Magic Squares, Methods of False Position, Euclid's Algorithm, From Measuring the Circle to Calculating $\pi$, Newton's Methods, Solving Equations by Successive Approximations, Algorithms in Arithmetic, Solving Systems of Linear Equations, Tables and Interpolation, Approximate Quadratures, Approximate Solutions of Differential Equations, Approximation of Functions, Acceleration of Convergence, and Towards the Concept of Algorithm; plus biographical Appendix and indices.

In the Introduction, the Editor acknowledges that some of the topics treated do not match the modern concept of an algorithm—e.g. some iterative processes, which do not converge exactly in any finite number of steps. There are about 200 passages (up to several pages long) from original texts, some in the original English or from published English translations, but mostly translated from the French edition (1994). Many of those French texts had been translated from Babylonian, Egyptian, Sanskrit, Greek, Latin, Chinese, Arabic, Provençal, Italian, Japanese, German or Russian; and some of those German texts are translations from Babylonian or Arabic, *et cetera*.

Isaac Newton (1642-1727) developed a very powerful method for solving an equation of the form $f(x) = 0$, where $f$ is a differentiable real–valued function of a real variable (pp. 170–174). Choose a suitable initial estimate $x_0$ of some root $\alpha$ of that equation, and then repeat the step: $x_i = x_{i-1} - f'(x_{i-1})/f(x_{i-1})$ for $i = 1, 2, 3, \ldots$ until $x_i$ is sufficiently close to the desired root $\alpha$. Newton did not attempt to settle the questions of choosing suitable initial estimates to give convergence, and of how many steps are required to give a result which is sufficiently close to a root. Newton's method was simplified and generalized by Joseph Raphson in 1690, and hence the method is usually called the Newton–Raphson algorithm.

"The question of convergence for Newton's method appears to have been discussed properly for the first time in 1768 by Mourraille in his **Traité de la Résolution des Equations en Général** [London & Marseille, 1768]. Jean–Raymond Mourraille, a mathematician and astronomer, was mayor of Marseille from 1791 to 1793. It is curious to note that his work has passed almost unknown, since it contains many novel ideas. In particular, contrary to Newton and Raphson, it emphasizes the geometrical aspect—it is here that the name of the 'tangent method' makes sense—and it is the geometrical representation that is used to explain the behaviour of the iterative sequence produced by the Newton algorithm" (p.178). How curious it is, that such a significant advance in numerical analysis should be linked with the Revolutionary anthem *Marseillaise*!

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The interesting information provided in this book is blemished by many defects in the text and in its translation.
The Editor begins the Introduction with the remarkable declaration that "Algorithms have been around since the beginning of time"—a robustly Platonist view. The famous allegorical woodcut depicting the triumph of the algorists over the abacists (from Gregor Reisch, **Margarita Philosophica**, 2nd edition, 1503) has been clumsily re-drawn (p.3), and mis-dated as 1504. The references in the various Bibliographies are not consistent in style: some give references to collected editions and others do not, some write Œuvres and others write Oeuvres. In Chapter 3, many of the references are mis-numbered in the text.
The Egyptian symbol for 1/7 has been printed illegibly (p.7).
The invention of binary arithmetic is attributed to Gottfried Leibniz, and part of his 1703 paper on binary arithmetic is presented (pp. 40-43). But Leibniz knew (no later than 1674) of John Napier's book Rabdologiæ (Edinburgh, 1617), which includes very clear algorithms for binary arithmetic, as far as square roots.
Brian Randell's book **The Origins of Digital Computers** (1st edition, Springer Verlag, Berlin, 1973) is cited (mis-spelling his name as Randall) for the assertion that "During the 1930s, electronic circuitry designed to carry out arithmetic operations were developed in the USA, France and Germany, and these all used the binary number system" (p.44). In fact, Randell showed that such electronic circuitry was operating in England from 1932, partially operating in the USA by 1939, and a design for such circuitry was developed in Germany by 1939 but was rejected by the German government. Also, "For numerical information, the numbers are represented in binary form" (p.44) – but some computers have used decimal numbers (including IBM650 and IBM1620), and some others have represented numbers in other bases, including $-2$ and $3$.
The statement that in 1424 al-Kāshī's calculation of $\pi$ was "equivalent to an accuracy of 16 places of decimals" (p.145) is somewhat misleading, since he calculated $2\pi$ as a decimal fraction, with all 16 decimal places correct.
Machin's important formula $\pi/4 = 4 \arctan(1/5) - \arctan(1/239)$ is misprinted twice, omitting the factor of 4 (pp. 161 & 163). The statement that, for calculating $\pi$, "In 1719, M. de Lagny [13] calculated 127 places of decimals starting with the Euler formula: $\pi/4 = \arctan(1/2) + \arctan(1/3)$" (pp 161 & 162) is rather misleading, since Euler was then 12 years old.
In the Bibliography to Chapter 5 a website address is incorrect (p.167).
At the start of Chapter 6 on Newton's Methods, a vignette of a young man is presumably intended to portray Isaac Newton as a young man (p.169) — but the first known portrait of Newton was painted by Sir Godfrey Kneller in 1689, when Newton was aged 46.
The Babylonian examples of square roots (pp. 200–201) do not include the most remarkable example: the clay tablet YBC 7289 is inscribed with a square and numbers giving $\sqrt{2}$ correct to 3 sexagesimal places as 1;24,51,10 .
Daniel Bernoulli's use of series, including "both integer series and trigonometric series" (p.223) presumably refers to "power series and trigonometric series".
The discussion of nested multiplication for polynomial evaluation states that "Traces of this method of evaluating a polynomial using this approach, are already to be found in a text by Newton (see Section 5.1), however, a systematic method for finding, not only the constant term, but also all the other coefficients of a transformed polynomial $Q(x) = P(x+u)$ did not properly appear before the beginning of the 19th century" (p.231). Newton is not mentioned in §5.1; but in §6.1 Newton used nested multiplication to convert a quartic polynomial in $y$ to a quartic polynomial in $p$, where $y = p + 1$ (p.174).
The discussion of Fermat's Little Theorem, that if $a$ is not a multiple of prime $p$ then $p$ divides $a^{p-1} - 1$, repeats the misunderstanding by L. E. Dickson, who wrongly claimed that Fermat's Little Theorem "appeared to be known in the particular case of $a = 2$ to the Chinese in about 500 B.C. who knew that $2^p - 2$ was divisible by prime $p$" (p.252).
An important paper (1885) on rate of convergence of iterative methods for solving linear equations is attributed to Alexander Ivanovich Nekrasov (pp. 306–309) — but the biographical Appendix (p.503) correctly gives his dates as 1883–1957! In fact, that 1885 paper was written by Pavel Alekseyevich Nekrasov (1858-1924).
The claim (p.327) that "The first table of sines was drawn up in a work by al-Khwārizmī" (9th century) is incorrect, since in 499 Āryabhata started his book Āryabhatiya with a short table of sines.
The first table of logarithms, published by John Napier in 1614, was <u>not</u> "obtained by successive extraction of square roots" (p.329). Napier's friend Henry Briggs based his tables of decimal logarithms (published 1617 and 1624) on successive extraction of square roots.
In the 1790s, Gaspard de Prony carefully organized teams of human computers to produce huge mathematical tables of unprecedented accuracy. It is not merely "possible that the 'industrial' approach adopted by Prony had an influence on Babbage" (p.343), since Charles Babbage repeatedly extolled Prony's organizational approach as the inspiration for his Difference Engines.
Alan Turing is said (p.468) to have attended in 1935 "a course on the foundations of mathematics given in Cambridge by Von Neumann" — but it was M. H. A. Newman who gave that course.
The biographical entry on Abu l-Fath 'Umar ibn Ibrahim Al–Khayyām (1048–1131) states that "We have no definitive information about the life of 'Umar ibn Al–Khayyām" (p.483). But a considerable amount of information about him (including the actual days of his birth and death), from reliable sources, is presented by John Andrew Boyle, 'Omar Khayyam: astronomer, mathematician and poet', *Bulletin of the John Rylands Library*, v.52 No.1 (Autumn 1969), 30–45.
The table published in 1620 by Joost Bürgi was not a table of logarithms (p.488), but a table of anti–logarithms.

Arthur Cayley (1821-1895) is said (p.488) to have “only published one work”(!); but in addition to about 1000 papers he did publish only one book.
Simon Stevin (1548–1620) did not use the decimal point for the first time (p.510) in his pamphlet *La Disme* (1585). Rather, his account of decimal fractions was hampered by his exceedingly cumbersome notation (pp. 38–40).

The translation reads awkwardly, omitting some words and mis–spelling many others, e.g. “Massassuchetts” for “Massachusetts” (p.46), and “H. W. Tumbull” for “H. W. Turnbull” (p.334). There are some serious errors in the translation, including “worthy” for “unworthy" (p.52), and “infinite” for “infinitesimal” (!) (p.401).
There are many errors in dates, including the following: “second century before Christ” for “second millenium before Christ” (p.11), “13th-14th century” for “14th-15th century” (p.52), “14th century” for “15th century” (pp. 52, 64, 503 & 506), “12th century” for “13th century” (p.84), “seventh century” for “fifth century” (p.97), “1713” for “1913” (p.137), “eighth century” for “ninth century” (p.175), “the 1920s” for “the 1910s” (p.188), “6th century” for “7th century” (p.272), “1859” for “1959” (p.370), and “1499” for “499” (p.504). Qian Baocong’s edition of the Ten Mathematical Manuals is dated as 1963 (pp. 93 & 95) and as 1964 (p.111). John Wallis’s important book Arithmetica Infinitorum (Oxford, 1655) is mis-dated as 1656 on pp. 127, 138, 156 & 167; but it is correctly dated as 1655 on pp. 197 & 514. Āryabhata is said to have been born *c*476 and to have written his book in 510 (p.484)—but Āryabhata explained that he wrote that book at the age of 23 and he gave his horoscope, which establishes 476 as the year of his birth and 499 as the year of his book. Charles Babbage is said to have been born at Teignmouth in 1792, but he was born in South London on 1791 December 26 (p.484). Thomas Harriot’s dates are given as Oxford *c*1560—London 1621, but 1560 was the year of his birth (p.496).
How did this book get accepted for publication?

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**An Introduction to the Mathematical Theory of Inverse Problems**
by Andreas Kirsch, (Applied Mathematical Sciences, 120)
Springer-Verlag, 1996, 282pp, DM 119.00 ISBN 0-387-94530-X.

Chapter Headings:

1. Introduction and Basic Concepts
2. Regularization Theory for Equations of the First Kind
3. Regularization by Discretization
4. Inverse Eigenvalue Problems
5. An Inverse Scattering Problem
A. Basic Facts from Functional Analysis
B. Proofs of the Results of Section 2.7

This monograph presents some mathematical properties of a few regularization methods for linear ill-posed inverse problems (chapters 2 and 3) followed by some results concerning the direct problems for recovering spatially-varying parameters from eigenvalue data or far-field scattering data (chapters 4 and 5). Chapter 1 gives a brief introduction to inverse problems and the notion of ill-posedness drawing on largely mathematical examples such as Cauchy’s problem and numerical differentiation.
I think that a reader’s reaction to this book can largely be determined by their reaction to a statement made in the opening sentences of section 2.1, titled ”A General Regularization Theory”. In this section the author outlines the general mathematical framework that he will use, and defines a regularization strategy for solving linear inverse problems of the type $Kx = y$. He begins “For simplicity, we assume throughout…that the compact operator $K$ is one-to-one. This is not a serious restriction since we can always replace the domain…by the orthogonal complement of the kernel of $K$.” I wonder what most New Zealand mathematicians would make of that restriction. As a mathematician who researches methods for solving inverse problems, and actually tries to solve real inverse problems using measured data, I think that this restriction is serious, and undesirable. It is probably not too strong to say that for the past twenty five years, leading research into solving inverse problems has largely been about determining values for the essentially unmeasured components in the solution. So the author has thrown the baby out with the bath water and it seems that this book is not about methods for solving inverse problems as we know them today. The generalized inverse of Moore and Penrose, and Tikhonov’s pioneering generalization to regularizing the inverse, both set unmeasured values to zero. For some time it has been understood that this is not a universally good idea and that choosing the specific value zero for unmeasured solution components is often silly. For example, when your CD player has a data error, it is preferable for it to interpolate between the adjacent values to generate a pleasant (though incorrect) sound than it is to set the value to zero and subject you to an annoying click. Remarkable work into quantifying listener preference has lead to excellent interpolation schemes that are not linear functions of the data and hence cannot be structured as a projection onto a subspace. X-ray crystallography is another example, and probably the inverse problem that has used more CPU cycles than any other scientific problem. In that case the magnitude of the Fourier transform of electron density is measured, with the phases being unknown. Simply

claiming that the phases could be set to zero, or that any linear function of the solution could be set to zero, is banal. Or try suggesting this restriction to a Radio Astronomer who measures the 2-dimensional Fourier transform of the sky-brightness on a finite number of concentric half circles. The very first attempts (in the 60s) at imaging in Radio Astronomy did what this book suggests and set the unmeasured values to zero. But that scheme was quickly rejected when it was found that large chunks of the sky were predicted to have negative brightness. Needless to say, none of the lovely pictures that you see of super-nova remnants use the author's assumption.

That is not to say that regularization methods with zero default are not useful techniques. They certainly have their place, and I use them frequently for dealing with simple inverse problems where basically any technique would work. After all they are quick to compute and the standard quadratic regularizers can be completely understood in terms of the singular-value decomposition. And we can work around the author's restriction by considering the domain of $K$ to be all of image space with some directions corresponding to very small singular values; Whether a datum is unmeasured or $10^{-100}$ of the noise level is irrelevant for recovering images.

The author considers noise on data $y$ so measured data $y^\delta$ satisfies $\|y^\delta - Kx\| \leq \delta$. A *regularization strategy* is defined as a family of linear bounded operators which map $y^\delta$ to the solution $x^{\alpha,\delta}$ with $R_\alpha K \to I$ as $\alpha \searrow 0$. The strategy $\alpha(\delta)$ is called *admissable* if $\alpha(\delta) \to 0$ and $\sup \|x^{\alpha,\delta} - x\| \to 0$ as the noise level $\delta \to 0$. The main thrust thereafter is determining the power $p$ in the asymptotic relation

$$\|x^{\alpha,\delta} - x\| \propto \delta^p$$

for small $\delta$, preferring strategies with larger order of convergence $p$. Since $\delta$ is not under user control, this criterion is not actually of value. Any inversion scheme needs to perform for a finite noise level—the noise level of the measured data—and its performance for other data with other noise levels is usually not relevant. About twenty years ago the literature abounded with examples poking fun at these types of criteria for choosing regularization strategies, by presenting scenarios where the regularization is clearly ridiculous for the measured data though performs well for data that was not actually measured. Fortunately that kind of polemic no longer appears, instead the community has taken to heart that performance for the actual data measured is the only criterion to aim for. So in this basic direction the author also fails to represent contemporary theory of inverse problems.

Two admissable regularization strategies are considered in some detail, being the well-known Tikhonov method and Landweber iteration. I had not previously heard of Landweber iteration and I now know why. Landweber iteration is equivalent to a steepest-descent minimization of $\|Kx - y\|$ which is terminated after a finite number of steps to achieve regularization. In a modern context this algorithm is disgusting. For ill-conditioned problems the steepest-descent algorithm is extremely inefficient and besides $\|Kx - y\|$ is a bad thing to aspire to minimize. Landweber can be excused for suggesting this algorithm in 1951 when computation came at a premium and computing anything was the goal. But now the issue is about computing something optimal and including this algorithm is like including a section on watering your horse in a contemporary book on road transport.

I have now read to page 50, or so, and am wondering if there is anything of value in this book. I scanned the remaining chapters and unfortunately can report that the book continues in the same vain and so gave up trying to describe it.

In many ways this book is defined more by what it omits than what it contains, and it omits all of the advances in solving inverse problems from the last thirty years. I asked a (non-mathematician) colleague who works in inverse problems to give me a second opinion. He returned the book a day later and summarized his view by say that it looked as though a pure-mathematician (his words) had attended a first lecture on inverse problems and then developed a lengthy formal theory. So he also viewed this book as ignorant of the field and pointlessly formal.

Perhaps naively, I used to believe that every volume in the Springer series in Applied Mathematics represented something near excellence in the field, and that the series therefore built to a kind of state-of-art. Now I see that the series can also contain books like this one that is just trite nonsense.

I'd like to end this review on a positive note, and can do so by pointing out that there are good books on regularization and inverse problems available. I heartily recommend the excellent monograph *Rank-Deficient and Discrete Ill-Posed Problems: Numerical Aspects of Linear Inversion* by Per Christian Hansen rather than the woeful chapters 1-4 of this book. Chapters 5-6 are easily usurped by *An Introduction to Inverse Scattering and Inverse Spectral Problems* by Chadan, Colton, Päivärinta and Rundell. Both these books are from SIAM.

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**Topological Vector Spaces**

by H.H. Schaefer, with M.R. Wolff, (2nd ed), (Graduate Texts in Mathematics, 3)
Springer-Verlag, Berlin e.a., 1999, 375pp, DM 119.00. ISBN 0-387-98726-6.

So deep-seated and indelibly bright an imprint Schaefer's fine monograph has left inside my hippocampus nearly two decades ago, that when I learned about the second edition coming out and resolved to write this review, I knew I could have accomplished the task even before getting a copy of the book at my disposal. The book in question—or, rather, the 1971 Russian translation of the 1966 original—served as the subject matter for one in a series of exams that I, then a PhD student at the Department of Geometry and Topology of Moscow State University, had to pass back in 1981. After having carefully and slowly read the book (or, to be completely honest, the first four chapters of it, out of then five), I then spent a week or so jotting down all the major definitions, results, and their proofs strictly out of my head, without opening the book a single time. Finally, I went through the impressive bundle of paper sheets produced in such a manner yet again, this time with the book at hand, checking the accuracy of my own rendering of the theory. I wish I had enough determination and stamina to apply this method more widely and on more occasions.

Now let us get down to business. A *topological vector space* (TVS) is simply a vector space (typically, over the ground field of scalars $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$) equipped with a Hausdorff topology making the vector space operations continuous. This concept, along with major operations and basic concepts (such as metrizability and boundedness) forms the subject of Chapter I. As a matter of fact, the above concept is exceedingly general, as only those TVS in which convex open sets form a base for the topology are currently known to allow for a deep and elegant theory. Such topological vector spaces are called *locally convex spaces* (LCS). They are introduced and studied in Chapter II, as well as some of their major and most immediate subclasses (such as bornological and barrelled spaces) and operations preserving local convexity (of both projective and inductive kind).

Historically the first examples of the kind were provided by Banach spaces, that is, complete normed vector spaces. The most common among them are the so-called classical Banach spaces, including the spaces $l_p$ of $p$-summable sequences of scalars, the spaces $L_p(X, \mu)$ of $p$-summable functions on a measure space $X$, the space $l_\infty$ of all bounded sequences equipped with the supremum norm, and so forth. This development is primarily linked with the name of Stefan Banach. Norbert Wiener has laid a claim of having stated the concept of a normed space independently from Banach, and was always bitter about not having received recognition on this account. In any case, isolating a concept, however useful, is one thing, but building around it a consistent and rich theory is quite another, and it is Banach's 1932 book *Théorie des opérations linéaires* that is universally recognized as the first course of abstract functional analysis in its modern form. The foundations for the theory are provided by such results as the Hahn-Banach Theorem (asserting, roughly speaking, that bounded linear functionals are plentiful on every normed — more generally, locally convex — space), Banach-Steinhaus Theorem (establishing the equivalence of uniform boundedness and pointwise boundedness for families of operators between two normed spaces), and Banach's homomorphism theorems, including the Closed Graph Theorem (a linear operator between two Banach spaces is bounded if and only if its graph is closed). These results and much more, in each case extended to as general a subclass of locally convex spaces as possible, can be found in Chapter III, titled *Linear Mappings*.

The setting of normed spaces, while extremely broad and important (for instance, it includes all Hilbert spaces, which serve as the mathematical foundation for quantum theory), is still not broad enough in that many vector spaces equipped with 'natural' topology are non-normable. Such are, for instance, spaces of $C^\infty$-functions equipped with the topology of uniform convergence on compacta with all derivatives. To accommodate such objects, the concept of a locally convex space was introduced by John von Neumann. Many major tools, concepts, and results from theory of normed spaces admit more or less straightforward extensions to this, much more general, setting. Most notably, the Hahn-Banach theorem is valid, and leads to the profound and very efficient *duality theory,* where properties of and structures related to a locally convex space $E$ are mirrored by properties of and structures related to the dual space $E'$ formed by all continuous linear functionals on $E$. A particularly important part in the duality theory is played by the *weak topology* on a LCS $E$, that is, the coarsest topology on $E$ which preserves the continuity of each continuous linear functional $f\colon E \to \mathbb{K}$. Duality theory and its applications form the subject of Chapter IV of the book of the review, which also treats tensor products of various types (theory of which is largely due to Grothendieck's effort).

It is natural to ask, is theory of locally convex spaces just limited to picking up known results and concepts from theory of normed spaces and then trying to extend them to the widest possible setting? The answer is 'no,' as it became clear in the fifties, when the 21-year-old Alexander Grothendieck put forward the concept of a *nuclear space*. This is not uncommonly seen as the most important development ever since the introduction of the concept of a normed space. To understand the place of nuclear spaces in the theory, notice that every finite-dimensional vector space can be made into a topological vector space in a unique fashion, and its topology is of course normable. Thus, normed spaces can be considered as generalizations of finite-dimensional spaces. Nuclear spaces capture another property of finite-dimensional vector spaces, going in the 'orthogonal direction' to that assumed by normed spaces, and have no analogues whatsoever in theory of normed spaces. As with every deep concept, nuclear spaces can be defined in a multitude of equivalent ways. Here is one: a LCS $E$ is nuclear if and only if every continuous linear map $f$ of $E$ into each Banach space $F$ is *nuclear,* that is, can be represented in the form

$$f(x) = \sum_{n=1}^{\infty} \lambda_n f_n(x) y_n,$$

where $(\lambda_n)$ is an absolutely summable sequence of scalars, $(f_n)$ is an equicontinuous sequence of linear functionals on $E$, and $(y_n)$ is a bounded sequence in $F$.

It is easy to see that indeed every finite-dimensional vector space is nuclear. Infinite-dimensional examples of nuclear spaces are many, including most of the known spaces of functions and spaces of distributions. Nuclear spaces display a remarkable resilience, surviving under a much broader variety of basic operations than, for instance, normed spaces (subspaces, quotients, products, inverse limits, inductive limits, projective tensor products, locally convex direct sums...) Every nuclear space is *multihilbertian,* that is, can be obtained from a suitable family of Hilbert spaces through taking the infinite product and proceeding to a locally convex subspace. It is not particularly difficult to observe that every bounded subset of a nuclear space is precompact; at the same time, the unit ball of a normed space $E$ is precompact if and only if $\dim E < \infty$. Thus, the intersection of the class of normed spaces and that of nuclear spaces consists exactly of all finite-dimensional vector spaces. Nuclear spaces are treated in Chapters III and IV.

Chapter V of the book, called *Order Structures,* deals with ordered topological vector spaces, that is, those TVS equipped with a partial order in a way consistent with both algebraic operations and topology. The principal source of examples of ordered TVS is provided by function spaces of various kinds — indeed, every vector space made up of (real or complex valued) functions on some set $X$ contains in the most natural fashion a *positive cone,* $P$, which is simply the collection of all functions $f$ from our space with the property $f(x) \geq 0$ for all $x$. In such a context, one can talk of *positive functionals* $\phi$ as those linear functionals preserving the order: $\phi(f) \geq 0$ whenever $f \in P$. Axiomatizing these observations leads to the notions like *topological vector lattices*. In particular, the well-known *Stone-Weierstrass theorem* can be reformulated in this context in a very elegant and useful form: if $F$ is a vector sublattice of the space $C(X)$ of all continuous real-valued functions on a compact space $X$, such that $F$ contains the constantly-one function and separates points in $X$, then $F$ is uniformly dense in $C(X)$.

The second edition also contains a newly-written Chapter VI, presenting the basics of theory of $C^*$- and $W^*$-algebras. A $C^*$-algebra $A$ is simply a complex Banach space equipped with a multiplication making it into an associative (unital or not) algebra, and carrying an involution $*$ (an anti-linear map from $A$ to itself) satisfying the properties $(xy)^* = y^*x^*$, $\|xy\| \leq \|x\|\,\|y\|$ and $\|x^*x\| = \|x\|^2$ for all $x, y \in A$. Commutative $C^*$-algebras are exactly algebras of all continuous complex-valued functions on compact spaces (the Gel'fand-Naimark theorem). The structure of noncommutative $C^*$-algebras is significantly more complex and far from being well understood. Since studying commutative $C^*$-algebras is essentially the same as studying compact topological spaces, noncommutative $C^*$-algebras are often thought of as 'noncommutative,' or 'quantum,' analogues of compact spaces. The master example of a non-commutative $C^*$-algebra is $L()$, the $C^*$-algebra formed by all bounded linear operators on a Hilbert space . Remarkably, every $C^*$-algebra is isomorphic with a $C^*$-subalgebra of one of the form $L()$.

Fewer in number, and thus somewhat more amenable to classification, are *von Neumann algebras,* also known as $W^*$*-algebras*. A $C^*$-algebra $A$ with unit is called a von Neumann algebra if $A$, thought of as a mere normed space, is the dual to a suitable normed space: $A = A_*'$. The space $A_*$ is in such case uniquely determined by $A$ up to an isometric isomorphism, and called the *predual* of $A$. In the commutative case, every $W^*$-algebra is isomorphic to the algebra of all essentially bounded measurable functions on a measure space. This is why general (not necessarily commutative) von Neumann algebras can be thought of as objects of *noncommutative measure theory*. This area has witnessed some of the most remarkable advances in modern mathematics, including the work of Alain Connes and others on classification of factors (so are called von Neumann algebras whose centre is one-dimensional).

What relates $C^*$-algebras with the subject of the book, is the existence of a canonical partial order on every $C^*$-algebra $A$, commutative or not: an element $x \in A$, by definition, belongs to the positive cone if $x = y^*y$ for some $y \in A$. This order and many related concepts are of paramount importance for the theory of $C^*$-algebras. In the commutative case, the order thus defined coincides with the usual partial order between functions.

A large number of exercises is scattered throughout the book, ranging from straightforward to very hard. The book is concluded with an Appendix on the spectral theory of positive operators, whose contents are not easily found elsewhere in the literature.

By now, the book has firmly established itself both as a superb introduction to the subject and as a very common source of reference. It is becoming evident that the book itself will only become irrelevant and pale into insignificance when (and if!) the entire subject of topological vector spaces does. An attractive feature of the book is that it is essentially self-contained, and thus perfectly suitable for senior students having a basic training in the area of elementary functional analysis and set-theoretic topology. My view — let even possibly biased for sentimental reasons — is that the book under review would make for a very practical and useful addition to every mathematician's personal office collection.

*Vladimir Pestov*
*Victoria University of Wellington*

**Parameterized Complexity**

by Downey, R. G. and Fellows, M. R., (Monographs in Computer Science)
Springer-Verlag, New York, 1999, xvi+533 pp. US$59.95. ISBN 0-387-94883-X.

Informally speaking, computational complexity asks how the number of steps needed to produce the output of an algorithm grows as a function of the size of the input. In order to classify an algorithm, one compares the asymptotic rate of growth (as the size of input goes to infinity) with a number of reference rates: linear (good), polynomial (still feasible), exponential (intractable), etc. From the early 1970s when Cook and Levin showed that boolean cnf formula satisfiability was *NP*-complete, thousands of problems in computing were shown to be *NP*-complete or *NP*-hard, problems where computational complexity seems to grow exponentially with size. Parameterized complexity adds a whole new dimension to computational complexity and extends the range of feasibility into areas previously regarded as intractable. A simple example of this is the NP-complete problem of vertex cover: given input consisting of a graph on $n$ vertices and a positive number $k$, whether there is a set of $k$ vertices such that every edge of the input graph is incident with at least one of these vertices. This problem requires time linear in $n$ but exponential in the parameter $k$; so it is solvable in a very good time when the parameter is bounded. Such problems are called fixed-parameter tractable (FPT).

Downey and Fellows are the two main architects of this area and their book is the first to appear on parameterized complexity. Below is a very brief summary of the book. The book has three distinct parts. Chapter 1 is an idiosyncratic introduction into the subject of the book, which is written using some interesting metaphors in a scholarly style, while also managing to be quite entertaining. Here is discussed the geographic place of parameterised complexity on the scientific globe, its history and general background, motivations and various methodologies with their particular points. It also presents a bird's eye view of the book. The following seven chapters constitute the first part of the book "Parameterized Tractability". Chapter 2 introduces the fundamental definitions of fixed-parameter tractability motivated by three concrete examples from theoretical computer science: vertex cover, graph genus and graph linking number. This chapter also offers an alternative view of being FPT through an analog of the classical notion of *advice*. Chapter 5 is a follow-up of the advice view introduced here. Chapter 3 describes two ad hoc methods. Several applications of the method of bounded search trees are given. For example, it is shown that this technique can solve the vertex cover using an algorithm running in time of $O(2^k n)$ for each $k$, where $n$ is the size of the input and $k$ is a parameter. Moreover, the flexibility of the technique allows for a serious improvement of the algorithmic solution with the running time $O(kn + (4/3)^k k^2)$. This means that the NP-complete problem vertex cover (the classical version where $k$ is not fixed) has a good practical algorithm so long as the parameter $k$ is no more than about 70. Then follows a description of the method of reduction to a problem kernel and how it can be combined with the method of search trees to give feasible parameterized solutions to some $NP$-hard problems. Chapter 4 reviews some very interesting connections to classical complexity. Applications are given such as a new result that the vapnik-chervonenkis dimension is unlikely to have a fully polynomial-time approximation scheme. Chapters 6, 7 and 8 present the most difficult and deepest material in the first part of the book. These chapters contain an excellent exposition of very general and powerful methodologies for demonstrating FPT based on bounded treewidth and pathwidth combinatorics (Chapter 6) and the results of Robertson and Seymour on graph minors and well-quasi-orderings (Chapter 7). A lot of background material is included. Chapter 6 starts with an introduction to automata theory. Where else one can read about the Myhill-Nerode theorem for tree automata?! In Chapter 7 the authors give a nice account of the beautiful ideas behind the proof of the infamous Graph Minor Theorem. It is very readable despite the fact that reverse mathematics threatens us with the formidable logical complexity of this theorem ($\Pi^1_1$ comprehension). I would also like to mention a very clever and elegant FPT approximation (linear-time) algorithm for pathwidth (p. 186).

In the second part, the authors turn to the study of parameterized intractability. They develop a new framework to define parameterized complexity classes of problems that are believed to be fixed-parameter intractable: $W[1], W[2], \ldots, W[P]$, etc. It is shown that these classes form a so-called $\mathbf{W}$-hierarchy $FPT \subseteq W[1] \subseteq W[2] \subseteq \ldots \subseteq W[P]$, and it is conjectured that these inclusions are proper. They prove fundamental $W[1]$-completeness and hardness results on clique, independent set, vapnik-chervonenkis dimension, short turing machine satisfiability and many other well known computational problems from logic, formal language theory and molecular biology, which provide us with the evidence that these problems are unlikely to be FPT.

The third part of the book considers structural aspects of parameterized complexity. Amongst other interesting material it initiates a study of the structure under parameterized reducibilities, a structure of daunting beauty and complexity. The area is wide open at the moment and raises a cornucopia of challenging problems. One section is an excellent survey of computability techniques (unsurpassed in literature in my opinion), which is helpful as it gets particularly involved here.

The book ends with an impressive appendix. The first part of the appendix is a problem compendium of over one hundred and fifty problems which are known to be either FPT or complete/hard for various levels of the $\mathbf{W}$ -hierarchy. The list is a reasonably complete and has references to more. The second part of the appendix is the "Research Horizons" section which contains an interesting overview of the subject, lists some particularly interesting open problems and indicates research directions. An extensive bibliography contains almost 500 references. The only criticism of this book I can make is in regards to certain aspects of referencing. There are only 4 symbols in the list of notations in the index. Moreover, some concepts in the book are missing from the index. For example, there is no reference to Coles' Theorem and Coles' Graph (p. 164). Several typos also crawled in. For example, there is a reference to K. and M. Cattell while only Kevin Cattel should be referenced. A big and fundamental book like this needs a more comprehensive index.

Personally I like this book very much: it is written in a superb style, it is very readable, it provides an excellent introduction to the area, it contains masses of new and interesting information from different areas of mathematics and computing, with many bright new and innovative ideas (at least for this reviewer). The depth and variation of techniques are testimony to the maturity of the current work in parameterized complexity. It could also be used for both undergraduate and postgraduate theory of computation courses, as well as courses on design and analysis of

algorithms. There is a large number of exercises of various difficulty. The numerous bibliographical notes provide an interesting historical account of the subject.
A natural prerequisite would be the classic book by Garey and Johnson *Computers and Intractability: A Guide to Theory of $\mathcal{NP}$-completeness*. Actually, the reviewer considers this book as "the" sequel to Garey and Johnson. For those interested in the design and analysis of algorithms I can highly recommend this book. It will surely be a must for any scientist who deals with issues of computational intractability.

*Asat Arslanov*
*School of Computer Science and Software Engineering*
*Monash University*

CENTREFOLD

## Professor Michael D Hendy

Mike Hendy is a leader in mathematical biology in New Zealand, and his work in this field has attracted considerable attention internationally. In recognition of this, he is currently on leave from Massey University, having taken up a prestigious visiting position, the Mercator Professorship, at the University of Greifswald in northern Germany. This visiting position is supporting the newly established undergraduate degree programme in BioMathematics at Greifswald, the first in Germany giving equal weight to mathematical and biological studies.
Mike was born in Dannevirke where he attended the local high school and was Dux Literarum in 1962. The following year he became a student at Victoria University of Wellington, taking a Mathematics major for his BSc degree when Professor Jim Campbell was Head of the Department. In 1967 Mike graduated BSc(Hons) from Victoria.
Upon completing his degree Mike was appointed to the Ministry of Works staff in Wellington, where for nine months he was engaged in hydrological research. Later in 1967 he took up a Junior Lectureship in the Department of Mathematics at Massey University, joining the five other mathematicians and one computer scientist then housed in a pleasantly situated but rodent-ridden building on the Palmerston North campus (the only Massey campus in those days!).
Having been thoroughly introduced to extramural as well as internal teaching, Mike decided that he needed "time out" to do a PhD, so in 1969 he and his wife Beth proceeded to the University of New England at Armidale, Australia, where Mike studied with Dr E M Horadam. His thesis dealt with the finite evaluation of the class number of quadratic number fields. Mike and Beth's Australian experience was very productive—they went with no children and returned with one!
Mike returned to the Department of Mathematics at Massey University as a Junior Lecturer in 1973 and during the next 20 years he progressed through the ranks of Lecturer, Senior Lecturer and Associate Professor. His research continued to be focused for a few years in the field of algebraic number theory and included supervision of his first PhD student Neville Jeans.
Around the middle of the 1970s, Mike turned his talents towards mathematical biology. Then began a collaborative development, initially with Massey colleagues Les Foulds and David Penny, involving phylogenetic trees in the theory of evolution. This watershed in Mike's research career was based on his expertise in using discrete mathematics, particularly combinatorics and graph theory, to analyse the theoretical models. Mike largely pioneered the application of Hadamard transforms to the reconstruction of evolutionary trees from DNA sequence data, a problem renowned for its difficulty. The originality and resulting success achieved in this work has brought Mike many international plaudits including an associate editorship of the journal Molecular Biology and Evolution. Massey University recognised his leadership in this field by appointing him to a Personal Chair in Mathematical Biology in 1993.
Mike relates: "At high school I was encouraged into mathematics by teacher Robin Paterson who introduced me to the fascinating Fermat's Last Theorem and the Four Colour Problem. These guided me into number theory and graph theory for graduate study. Because of timetable clashes I had to choose between mathematics and biology in

Form 5, a hard choice, so it is with great satisfaction that I now have a personal chair that encompasses both disciplines in the title.

“I was led into evolutionary theory after attending a debate in 1973 at Massey between Dr Duane Ghish (Center for Creation Science, California) and our local geochemist Dr (now Professor) Robert Brooks. After hearing the creationist, Beth challenged me to apply some mathematics to the issue. Fortunately, David Penny was already working with sequence data for evolutionary questions and was more than willing to guide a mathematician into this fascinating area. Later we were able to refute one of Ghish’s major theses, Popper’s claim that evolution was untestable and therefore not a scientific theory."

The development of this Massey-spearheaded research on evolution is an exemplary story of interdisciplinary collaboration, led by Professor Mike Hendy on the mathematical side and by Professor David Penny on the biological side. It has spawned several conferences, workshops and seminars in New Zealand, in exotic or historic places like Kaikoura, Akaroa and the old Dannevirke Hospital. It has produced several PhD students; those for whom Mike was chief supervisor are Mike Steel (Canterbury) and Mike Charleston (Oxford) (what has the name “Mike" got to do with trees?). Mike Steel was awarded the Royal Society of New Zealand’s Hamilton Memorial Prize (1995), and Mike Charleston has been granted a (UK) Royal Society University Research Fellowship. Current PhD students are Barbara Holland and Paul Gardner. Mike was also second supervisor to some PhD students in biology including Peter Waddell (Tokyo) and Liz Watson (Stockholm). Many international visitors have come to Massey because of the work being done in this area of mathematical biology.

These achievements attest to the scholarship of Mike Hendy, who has been an invited speaker at several international conferences, and has been in constant demand as a reviewer or referee for many scientific journals. Currently he has 81 research publications to his credit, ranging over number theory, mathematical biology and graph theory. A textbook Geometry and Linear Algebra [Dunmore Press, 1986], co-authored with Gillian Thornley, is now in its second edition.

As a teacher, Mike has contributed widely to the programme at Massey University, being well-rated and well-liked by internal and extramural students at all levels. In addition to his doctoral students, Mike has supervised postdoctoral fellows, masterate students and summer research students. He has also been very actively involved in administration, serving on various departmental, faculty and University committees, with long service in particular to the Doctoral Research Committee. He was Acting Dean of the Faculty of Information and Mathematical Sciences for eleven months.

Mike is an advocate for mathematics within the wider community. He has given talks to secondary school students, judged at science fairs, and been a mentor for prospective Mathematics Olympiad participants. Several NZ Mathematics Colloquia have benefited from Mike’s organisational skills, where again he endeavoured to reach out mathematically to as wide an audience as could be mustered.

Professionally, Mike is a Fellow of the Institute of Combinatorics and its Applications, and is a member of the following bodies: the NZ Mathematical Society, the NZ Association of Scientists, the Society of Molecular Biology and Evolution, and the Combinatorial Mathematics Society of Australasia. He has been an Assistant Chief Examiner of the NZ University Bursaries and Entrance Scholarship Mathematics with Calculus examinations.

The New Zealand Mathematical Society has benefited considerably from Mike’s enthusiasm. He served two terms on the Council (from 1993 to 1998). As Editor of its Newsletter from 1994 to 1999 he set high standards for the publication and wrote an insightful, often challenging editorial column. His earlier co-editorship of the Problems Section of the Newsletter also set challenges, of a more technical kind. It is entirely fitting that these Centrefold pages be devoted to honouring one who has contributed much to the Society.

Mike continues to bring challenges to himself and others, whether they be in the professional arena or in personal endeavours. For a number of years he ran between home and work each day (a return distance of about 16 kilometres) and took part regularly in harrier and road running events in the Manawatu. Mike also wrote computer software to analyse the results of these events. Nowadays his physical activity has slackened a little, but Mike appears to have more than compensated by increasing his academic efforts with outstanding success. His attitude and abilities have inspired others and will continue to do so.

*Dean Halford*

Centrefolds Index

## CONFERENCES

### 2000

September 10-13 (Melbourne) **The 4th Biennial International Conference of the Engineering Mathematics and Applications Conference**
email: `emac2000@rmit.edu.au`
homepage: `http://www.ma.rmit.edu.au/emac2000/emac2000.html`

October 23-27 (Manila, Philippines) **The Third Asian Mathematical Conference**
email: `amc2k@math01.cs.upd.edu.ph`
homepage: `http://math01.cs.upd.edu.ph/AMC2000/`

November 26-29 (University of Waikato) **2000 New Zealand Mathematics Colloquium**
Contact Ian Hawthorn
homepage: http://www.math.waikato.ac.nz/Coll2000/

December 4-8 (Christchurch) **25th Australasian Combinatorial Mathematics and Combinatorial Computing Conference**
Contact either Charles Semple or Mike Steel
email: c.semple@math.canterbury.ac.nz, m.steel@math.canterbury.ac.nz
homepage: http://www.math.canterbury.ac.nz/accmcc.shtml

December 10-12 (Bond University, Gold Coast, Queensland) **First Australian Conference on Mathematics and Art**
Contact Michelle Brown
e-mail: michelle.brown@bond.edu.au

## 2001

**February 3-7 (Barossa Valley, South Australia) ANZIAM 2001: The 37th Applied Mathematics Conference**
email: anziam2001@maths.adelaide.edu.au
homepage: http://www.maths.adelaide.edu.au/anziam2001

July 1-5 (Kruger National Park, South Africa) **Warthog Delta'01: Third Southern Hemisphere Symposium on Undergraduate Mathematics Teaching**
email: samern@scientia.up.ac.za
homepage: http://science.up.ac.za/delta01

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## ANZIAM 2001
## The 37th Applied Mathematics Conference

ANZIAM 2001 will take place from 3rd February to 7th February, 2001 at the All Seasons Barossa Valley Resort, situated in Australia's premier wine growing region.
The ANZIAM conference is an established annual gathering of applied mathematicians, scientists, engineers and students. It provides an interactive and traditionally informal forum for the presentation and discussion of research on applied and industrial problems arising in many different scientific fields.
The conference is being organized by the South Australian branch of ANZIAM. More information regarding the conference and the venue can be found on our web site

http://www.maths.adelaide.edu.au/anziam2001

A formal call for papers will be made in October. Papers are invited in all areas of Applied Mathematics. If you intend to submit a paper please email us at

anziam2001@maths.adelaide.edu.au

or register your interest by filling in the form at our web site. This will allow us to reserve a place for you in the program.
Registration will be available via the web later in the year and a second announcement will be made once this is in place.

Invited Speakers:

Professor R Elliott, Universities of Adelaide and Alberta (*Financial Mathematics*)
Professor D Katz, Duke University (*Biomedical Engineering*)
Professor L K Forbes, University of Tasmania (*Medical Technology*)
Professor G Nemhauser, Georgia Tech (*Operations Research*)
Dr H Sidhu, Michel Medalist 1999, UNSW, Australian Defence Force Academy (*Combustion*)
Dr G Stewart, University of Iowa (*Numerical Analysis*)
Dr A Tordesillas, Michell Medalist 2000, University of Melbourne (*Solid Mechanics*)

and one more to be announced.

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**Related Events**
The Mathematics-in-Industry Study Group, MISG 2001, will be run from January 29 to February 2 at the City East campus of the University of South Australia, Adelaide. For details contact Assoc. Prof. P. Howlett at

phil.howlett@unisa.edu.au or visit their web site http://www.misg.unisa.edu.au/

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**NEW ZEALAND MATHEMATICS COLLOQUIUM 2000**
**The University of Waikato**
**26–29 November, 2000**

The Department of Mathematics at the University of Waikato invite you to attend and participate in this year's NZ Mathematics Colloquium, to be held at the University of Waikato in Hamilton.
Registration and a welcoming reception will be held on Sunday evening (November 26). The scientific programme will run Monday 27 to Wednesday 29. Comprehensive conference information (including an on-line registration form) can be found at:

http://www.math.waikato.ac.nz/Coll2000/

**INVITED SPEAKERS**

- John Cleary (Waikato)
- Nat Friedman (SUNY Albany) TO BE CONFIRMED
- Peter Jackson (Auckland)
- Vaughan Jones (Berkeley)
- Ernie Kalnins (Waikato)
- James Sneyd (Massey at Albany) ANZIAM INVITED SPEAKER
- Alf van der Poorten (Macquarie) NZMS SPEAKER

**ADDITIONAL ATTRACTIONS**

- Special discussion forum: "The state of mathematics in New Zealand" –Panel discussion chaired by Robert Mclachlan
- NZMS Annual General Meeting
- ANZIAM New Zealand branch Annual General Meeting
- Mathematical video, poster and book displays
- Excursions to Waitomo Caves and Mt Aroha

**REGISTRATION**
Registrations will be processed via the Colloquium webpage:

http://www.math.waikato.ac.nz/Coll2000/reg.html

If you do not have WWW access, we can send a paper registration form if you write to us at the address below.

**CALL FOR PAPERS**
Contributed papers are invited for inclusion in the programme of talks. Please indicate your interest in presenting a talk on the registration form (online or paper version). Your abstract may be submitted using the online form at

http://at.yorku.ca/cgi-bin/amca/submit/caek-01

If you do not have WWW access, then send a paper (or email) version of the abstract to the conference organizers at the address below. You should submit your abstract by October 1, 2000 to guarantee inclusion in the programme.

**ACCOMMODATION, MEALS AND CONFERENCE DINNER**

We have reserved a block of rooms at College Hall (on-campus student accommodation). Bed and breakfast costs $47 per night. Other meals can be purchased on the campus, or in local restaurants. The Conference Dinner will be held at The Station (an on-campus restaurant), and will cost $35.

**FEES**

- Full registration $120 ($100 if paid by October 1st)
- Student registration $60 ($50 if paid by October 1st)
- One or two days $40 per day

**NOTICE TO STUDENTS**

The organizers are particularly keen to encourage participation from graduate and post-graduate students. To better

facilitate this, the NZMS generously provides some financial support for students. If you wish to be considered for this financial support, please indicate this on the registration form. We may contact you to ask for additional details of the costs you expect to incur.
The NZMS also awards the annual Aitken prize for the best student talk at the Colloquium. Please indicate on your registration form if you would like yourself (and your talk) to be considered for this award.
Further details appear elsewhere in this issue.

**ADDRESS**

Please contact the Colloquium organizers for any additional information:

| | |
|---|---|
| Secretary, NZMC 2000 | Web: `http://www.math.waikato.ac.nz/Coll2000/` |
| Department of Mathematics | Email: `nzmc2000@math.waikato.ac.nz` |
| University of Waikato | Tel: (07) 8384713 |
| Private Bag 3105 | Fax: (07) 8384666 |
| Hamilton | |
| New Zealand | |

# NOTICES

## AITKEN PRIZE (NZMS STUDENT PRIZE)

The New Zealand Mathematical Society offers a prize, known as the Aitken prize, for the best contributed talk by a student at the annual New Zealand Mathematics Colloquium.
Named in honour of the New Zealand born mathematician Alexander Craig Aitken, this prize will be offered for the sixth time at the 2000 Colloquium to be held at the University of Waikato during the week 27-29 November 2000.
The prize will consist of a cheque for NZ$250, accompanied by a certificate. Entrants for the prize must be enrolled (or have been enrolled) for a degree in Mathematics at a university or other tertiary institution in New Zealand in the year of the award.
During the Colloquium, they should give a talk on a topic in any branch of the mathematical sciences.
A judging panel will be appointed by the New Zealand Mathematical Society Council, and make recommendations to the New Zealand Mathematical Society President and Vice-President for the prize. Normally the prize will be awarded to one person, but in exceptional circumstances the prize may be shared, or no prize may be awarded.
Entrants should clearly indicate their willingness to be considered for the prize when they register their intention to contribute a talk at the Colloquium.

### NOTICE OF ANNUAL GENERAL MEETING

The Annual General Meeting of the New Zealand Mathematical Society will be held during the 2000 New Zealand Mathematics Colloquium at the University of Waikato on Monday 27 November 2000. The exact time and place of the AGM are currently being arranged.
Items for the Agenda should be forwarded by Monday 16 October 2000 to the New Zealand Mathematical Society Secretary, Dr Charles Semple, Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch (fax number: (03) 364 2587, email address: c.semple@math.canterbury.ac.nz).

### CALL FOR NOMINATIONS FOR NEW ZEALAND MATHEMATICAL SOCIETY COUNCIL POSITIONS

As the terms of office of the Immediate Past President (Rob Goldblatt) and two Council members (Mick Roberts and Dennis McCaughan) come to an end in 2000, nominations are called for the resulting vacancies on the New Zealand Mathematical Society Council:

(i) Incoming Vice-President.
(ii) Council members (two), including Treasurer.

The term of office of the Incoming Vice-President is one year, after which that person is expected to become President for a two-year period, and then Immediate Past President for a further year.
The term of office of a Council member is three years. Council members may hold office for two (but no more than two) consecutive terms.
Nominations should be put forward by two proposers. The nominee and the two proposers should be current Ordinary or Honorary members of the New Zealand Mathematical Society. The nominations, including the nominee's consent, should be forwarded by Monday 30 October 2000 to the New Zealand Mathematical Society Secretary, Dr Charles Semple, Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch (fax number: (03) 364 2587, email address: c.semple@math.canterbury.ac.nz). If nominations are sent by email, the two proposers and the nominee should each send separate email messages to the Secretary.

### NZMS RESEARCH AWARD

This annual award was instituted in 1990 to foster mathematical research in New Zealand and to recognise excellence in research carried out by New Zealand mathematicians.
The award for 2000 will be announced at the 2000 Mathematics Colloquium in Hamilton in late November. Other recipients to date have been John Butcher and Rob Goldblatt (1991), Rod Downey and Vernon Squire (1992), Marston Conder (1993), Gaven Martin (1994), Vladimir Pestov and Neil Watson (1995), Mavina Vamanamurthy and Geoff Whittle (1996), Peter Lorimer (1997), Jianbei An (1998), and Mike Steel (1998).

**Call for nominations 2000/2001**

Applications and nominations are invited for the NZMS Research Award for 2001. This award will be based on mathematical research published in books or recognised journals within the last five calendar years: 1996-2000. Candidates must have been residents of New Zealand for the last three years.
Nominations and applications should include the following:

1. Name and affiliation of candidate.
2. Statement of general area of research.
3. Names of two persons willing to act as referees.
4. A list of books and/or research articles published within the last five calendar years: 1996-2000.
5. Two copies of each of the five most significant publications selected from the list above.
6. A clear statement of how much of any joint work is due to the candidate.

A judging panel of three persons shall be appointed by the NZMS Council in advance of the receipt of nominations. The judges may call for reports from the nominated referees and/or obtain whatever additional referee reports they feel necessary. The judges may recommend one or more persons for the award, or that no award be made. No person shall receive the award more than once. The award consists of a certificate including an appropriate citation of the awardee's work, and will be presented (if at all possible) around the time of the AGM of the Society in 2001.
All nominations (which no longer need to include the written consent of the candidate) and applications should be sent by 31 March 2001 to the NZMS President, Graeme Wake, at the following address:

Professor Graeme Wake
Department of Mathematics and Statistics
University of Canterbury
Private Bag 4800
Christchurch, New Zealand

Please consider nominating any of your colleagues whose recent research contributions you feel deserve recognition!

## MARSDEN FUND

By early September, the results of the year 2000 funding round should be available on the Royal Society's web site `http://www.rsnz.govt.nz/funding/marsden_fund`. This year 61 preliminary applications were received, compared to 46 in 1999 of which six were funded. In 1995–1999, 53 out of 435 applications in the Mathematical and Information Sciences were funded, at an average value of $69,000 p.a.

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Details of foreign membership dues of Category I, II in 2001 are available from Charles Semple, Department of Mathematics and Statistics, University of Canterbury, Private Bag 4800, Christchurch (fax number: (03) 364 2587, email address: c.semple@math.canterbury.ac.nz).

# MATHEMATICAL MINIATURE 12

# Pascal’s triangle, Padé approximations and an application

The following triangular array is formed by adding adjacent cells in a row, to give the cell between them on the next row. This is just as for Pascal’s triangle, even though the cells are vectors of polynomials rather than integers.

$$\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\1-z\end{bmatrix}\qquad\begin{bmatrix}1+z\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\1-z+\frac{1}{2}z^2\end{bmatrix}\qquad\begin{bmatrix}2+z\\2-z\end{bmatrix}\qquad\begin{bmatrix}1+z+\frac{1}{2}z^2\\1\end{bmatrix}$$

$$\begin{bmatrix}1\\1-z+\frac{1}{2}z^2-\frac{1}{6}z^3\end{bmatrix}\qquad\begin{bmatrix}3+z\\3-2z+\frac{1}{2}z^2\end{bmatrix}\qquad\begin{bmatrix}3+2z+\frac{1}{2}z^2\\3-z\end{bmatrix}\qquad\begin{bmatrix}1+z+\frac{1}{2}z^2+\frac{1}{6}z\\1\end{bmatrix}$$

The vectors on row number $k = 0, 1, 2, \ldots$ represent approximations to $\exp(z)$ of order $k$ in the sense that the rational function formed from the two components, say $N(z)$ and $D(z)$, of any cell on this row satisfies

$$\frac{N(z)}{D(z)} = \exp(z) + Cz^{k+1} + O(z^{k+2}).$$

The values of $C$ corresponding to row number $k$ have the same magnitude but alternate in sign. This is why adding an adjacent pair to form an entry in the next row of the triangle increases the order by 1.

Many relationships exist between triples of entries in this table and we will explore one simple example. The relationship is between the central three entries on rows $2n-4$, $2n-2$ and $2n$. Denote the entry in the centre of row $2n$ by

$$V_n(z) = \begin{bmatrix}N_n(z)\\D_n(z)\end{bmatrix}$$

and we have

$$V_n(z) = \alpha_n V_{n-1}(z) + \beta_n z^2 V_{n-2}(z), \qquad n = 2, 3, \ldots, \tag{1}$$

where $\alpha_n = 2(2n-1)/n$, $\beta_n = 1/n(n-1)$. The right-hand side represents an approximation to $\exp(z)$ with order at least $2n-2$, irrespective of the values of $\alpha_n \neq 0$ and $\beta_n$. One can easily verify that the values actually used, give the correct $z^0$ and $z^n$ terms in $V_n(z)$ and a proof of (1) can be built on these observations.

Given a function $f$, assumed to be analytic in a neighbourhood of zero with $f(0) \neq 0$, there may exist for particular non-negative integers $l$ and $m$ a pair of polynomials $N$ of degree $l$, and $D$ of degree $m$, such that [illegible]. In this case the rational function $N/D$ is known as an $(l, m)$-Padé approximation to $f$. For some functions the complete Padé table exists and, as we see by rotating the Pascal’s triangle we have introduced into tabular form, $\exp$ is one of these functions. The tabular arrangement in this case, where a few more entries have been squeezed in, is

| | $l=0$ | $l=1$ | $l=2$ | $l=3$ |
|---|---|---|---|---|
| $m=0$ | $\frac{1}{1}$ | $\frac{1+z}{1}$ | $\frac{1+z+\frac{1}{2}z^2}{1}$ | $\frac{1+z+\frac{1}{2}z^2+\frac{1}{6}z^3}{1}$ |
| $m=1$ | $\frac{1}{1-z}$ | $\frac{2+z}{2-z}$ | $\frac{3+2z+\frac{1}{2}z^2}{3-z}$ | $\frac{4+3z+z^2+\frac{1}{6}z^3}{4-z}$ |
| $m=2$ | $\frac{1}{1-z+\frac{1}{2}z^2}$ | $\frac{3+z}{3-2z+\frac{1}{2}z^2}$ | $\frac{6+3z+\frac{1}{2}z^2}{6-3z+\frac{1}{2}z^2}$ | $\frac{10+6z+\frac{3}{2}z^2+\frac{1}{6}z^3}{10-4z+\frac{1}{2}z^2}$ |
| $m=3$ | $\frac{1}{1-z+\frac{1}{2}z^2-\frac{1}{6}z^3}$ | $\frac{4+z}{4-3z+z^2-\frac{1}{6}z^3}$ | $\frac{10+4z+\frac{1}{2}z^2}{10-6z+\frac{3}{2}z^2-\frac{1}{6}z^3}$ | $\frac{20+10z+2z^2+\frac{1}{6}z^3}{20-10z+2z^2-\frac{1}{6}z^3}$ |

Now the application. There exists an ordinary differential equation counterpart to Gauss-Legendre quadrature in which, for each time step of length $\Delta t$, the solution is advanced using an implicit Runge-Kutta method, containing $n$ stages evaluated at the zeros of the Legendre polynomial $P_n$, adapted to the interval $[t, t + \Delta t]$. The error generated in a step is equal to $O(\Delta t^{2n+1})$. To be acceptable for the solution of the type of “stiff" problems arising in the discretisation of time-dependent partial differential equations, the numerical method must be stable for the solution of $y' = \lambda y$ whenever $\lambda$ is in the left half-complex-plane. Write $z = \lambda \Delta t$ and this means that the value of the $(n, n)$ Padé approximation to $\exp(z)$ must lie in the unit disc, whenever $z$ is in the left half-plane. Using (1) it follows that this approximation can be written in continued fraction form as

$$R(z) = 1 + \frac{1}{-\frac{1}{2} + \frac{1}{z}+} \; \frac{1}{\frac{12}{z}+} \; \frac{1}{\frac{5}{z}+} \; \cdots \; \frac{1}{\frac{a_n}{z}},$$

where $a_n = 4(2n-1)$ for $n$ even, and $a_n = 2n - 1$ for $n$ odd. The proof that $|R(z)| < 1$ for $\mathrm{Re}\, z < 0$, is left as an exercise, but hinges on the facts that the left half-plane is closed under addition, multiplication by a positive real and by the mapping $z \mapsto z^{-1}$.

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