PROFILE

Rick Beatson

Rick Beatson began his studies in Mathematics at University of Canterbury in 1970. Prior to confirming his enrolment on the final year of honours, he thought to keep his options open and attended a lecture in an engineering department. Luckily for the mathematics community in NZ, it was reputed to be the worst and most boring lecture he had sat in (apparently the lecturer was famous around campus for this), thus cementing his decision to continue studying the subject that we all love. He finished honours in 1973, in an impressive cohort of 33 students, and was awarded the Cook Memorial Prize (for best student in that year's class). He continued his studies in mathematics, graduating with an MSc in 1975 and a PhD in 1978. The topic was approximation theory and the supervisor was Allan McInnes. After graduation, he took a Postdoctoral fellowship in the University of Otago before making the jump across the Pacific to the wild lands of America. He was an Instructor at University of Texas, Austin from 1978 to 1981 and then he held a tenure-track position at the University of Connecticut from 1981 to 1985, with a stint at DSIR–Lincoln in the middle. He returned to NZ in 1985 as a lecturer at University of Canterbury. He has progressed up the ranks, and now holds a Professorship in the School of Mathematics and Statistics.

Rick's field of research is in approximation theory/numerical analysis and his work is of enviable international reputation (including 4947 citations on Google Scholar at time of writing). Throughout his career, Rick has worked on challenging and interesting mathematical problems that have important technological and/or industrial applications. He is famous for his fast evaluation and iterative fitting techniques for many types of Radial Basis Functions (RBFs) which reduce the operations count for interpolating at N data points by orders of magnitude. This breakthrough allowed RBFs to be used for very much larger data fitting problems than was previously possible.

To provide a little detail of the above the problem is to find a radial basis function

$$s(x) = p(x) + \sum_{i=1}^{N} \lambda_i \Phi(x - x_i),$$

which interpolates to given values f_i at the nodes x_i. Here p is a low degree polynomial and Φ some suitable radial function. Fitting the interpolant reduces to solving the linear system

$$\begin{bmatrix} A & P \\ P^T & O \end{bmatrix} \begin{bmatrix} \lambda \\ a \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}.$$

Here $a_{ij} = \Phi(x_i - x_j)$ and $p_{ij} = q_j(x_i)$, where $\{q_1, \dots q_\ell\}$ is a basis for the space of low degree polynomials involved. In many cases Φ does not have compact support and therefore the $(N+\ell) \times (N+\ell)$ matrix in the linear system is full. Thus direct solution requires $\mathscr{O}(N^3)$ operations and $\mathscr{O}(N^2)$ storage. Rick, and coauthors, developed appropriate fast multipole methods to compute matrix vector products and coupled these with suitable domain decomposition preconditioned iterative methods. Taken together these reduced the computational costs to $\mathscr{O}(N(\log N)^2)$ operations and $\mathscr{O}(N)$ storage. This enabled the use of RBF methods on problems involving hundreds of thousands of points.

A distinguished feature of Rick's research is that he develops both theoretical aspects, and the algorithms to implement these theories. Indeed, his world-class applied mathematics marries deep, insightful and rigorous mathematical analysis with clever algorithms and programming skill to make tools that solve a plethora of scientific and engineering problems. As an example we will focus here on his work with Qui Bui on the technique of "implicit smoothing". One begins with a "noisy" data set. First, the data is interpolated with an RBF yielding an approximation $s(x) = p(x) + \sum_{i=1}^{n} \lambda_i \Phi(x - x_i)$. Then this initial approximation, s, is smoothed via convolution with a mollifier k. For some practically important basic functions Φ, a suitable choice of k yields a simple and easy to evaluate $\Psi = \Phi * k$. Then the smoothed approximation is $s * k(x) = p * k(x) + \sum_{i=1}^{N} \lambda_i \Psi(x - x_i)$. In practical terms, this substantially reduces the cost of implementing a smoothing process, as calculating the convolution is "free" once the function Ψ is known. This technique has been successfully applied to lidar and laser scanners in his consulting work with ARANZ (Applied Research Associates New Zealand Limited), a technological firm in Christchurch. In this application, a natural choice for Φ is $\Phi(x) = |x|, x \in \mathbb{R}^3$, the biharmonic spline basic function in $\mathbb{R}^3$, and a judicious choice of k results in a simple mollification formula $\Psi = \Phi * k$, with $\Psi(x) = \sqrt{|x|^2 + c^2}$, where $c > 0$ is a parameter related to the "scale" and "error bounds" of the smoothing process. The mathematics underlying this smoothing process was done in a collaboration with Qui Bui at Canterbury, where they also established analogous formulas for a number of other important RBF basic functions. The mathematical tools used include results from Schwartz distribution theory, harmonic analysis and complex variables. This is a testimony to Rick's versatility and willingness to work with people from other branches of mathematics.

Rick has had many international collaborators over the years. These include a number of eminent mathematicians, such as (the late) Mike Powell and (the late) Will Light (UK), Charles Chui (USA), Nira Dyn (Israel). He has been invited to be a plenary speaker at a number of large international conferences in his fields. For his collaboration with industry, he was awarded the Innovation Medal by the University of Canterbury in 2015.

Finally, Rick is an excellent colleague. He is an inspiring and effective teacher, with a particular penchant for ensuring the smooth running of exceedingly large enrolment undergraduate courses ("Instructions to Lecturers" have been know to include an elaborate incentive system for minimising errors in tutorial sheets, facilitated by exponential growth in the provision of chocolate bars). He has served on the NZMS Council, organised Colloquia, and is a diligent contributor to departmental life.

Qui Bui, Rua Murray and Miguel Moyers