2021 Manawatū–Wellington Applied Mathematics Conference

1 December 2021, Online

`https://massey.zoom.us/j/85027636506`

Contact information: `r.mclachlan@massey.ac.nz`

9.00 am **Keynote address**
Vivien Kirk, University of Auckland
Mathematics from physiology?

The language and tools of mathematics have been used for decades to help investigate and explain physiological phenomena. Many ODE or PDE models have been constructed, fit to data, and then, in some cases, used to make suggestions and predictions for new experiments that have led to a better understanding of the physiology of interest.

This interaction between mathematics and physiology has not been one way. Efforts to explain physiological observations have frequently led to the development of new mathematical tools and theories, and have sometimes provoked mathematical excursions that have little to do with understanding physiological phenomena.

This talk will discuss some mathematical issues that have arisen in recent work on intracellular calcium dynamics, describing the types of questions that are of interest and the tools that are helpful.

9.45 am Break

10.00 am **Carlo Laing, Massey University**
Periodic solutions of a theta neuron subject to delayed self-feedback

We consider a single theta neuron with delayed self-feedback in the form of a Dirac delta function in time. Because the dynamics of an uncoupled neuron can be solved explicitly we can derive algebraic expressions for the existence and stability of periodic solutions. We can also give explicit expressions for the location in parameter space of saddle-node bifurcations of these orbits, giving a complete description of these types of solution. The generic nature of the model means that our results give a basis for understanding similar dynamics observed in other excitable systems subject to delayed self-coupling.

10.20 am **Sishu Muni, Massey University**
Globally resonant homoclinic tangencies

Attractors in dynamical systems govern its typical long term behavior. The presence of many attractors for a fixed set of parameters is an exotic phenomenon known as multistability. I will describe a bifurcation phenomenon in planar maps in which an infinite number of attractors coexist, and explore the unfolding of this bifurcation as the parameters of the system are varied.

10.40 am **Brandon Jones, Massey University**
Mathematical models of microbial growth

Models can be used to describe and explore the growth of populations of a microbial species, and so understand communities that consist of multiple co-existing populations. In this talk we are concerned with models that describe the substrate limited growth of anaerobic microbes. We want a model that contains thermodynamic inhibition which predicts a non-zero threshold substrate concentration and can be extended to multiple growth-limiting substrates. We consider four established models with a single growth limiting substrate, note their strengths and weaknesses, and derive new connections among them, before looking at how they may be extended to model situations with multiple limiting substrates.

11.00 am Break

11.20 am **John Butcher**
Isomeric trees with applications to Runge-Kutta methods

Classical Runge–Kutta methods, up to the work of Nyström, were derived to achieve a required order for scalar problems, even though they often retained the same asymptotic accuracy for high-dimensional problems. By writing trees in terms of products of what will be referred to as atomic factors, the distinction between scalar and vector conditions is seen to be equivalent to identifying permuted products with the same factors. These interrelated trees are said to be isomeric.

The number of conditions for order p, in the scalar and vector cases, are shown in the table below. Also shown are the number of free parameters for an s stage method, where s corresponds to the minimum number of stages to achieve order p.

p	scalar	vector	s	parameters
1	1	1	1	1
2	2	2	2	3
3	4	4	3	6
4	8	8	4	10
5	16	17	6	21
6	31	37	7	28

For $p = 5$, methods exist which have this order for a scalar problem, but not for a vector problem. The first method of order 6, derived by A. Huťa, used $s = 8$ stages, so that there would be 36 parameters, which were expected, at that time, to be necessary to satisfy the 31 conditions for a scalar problem.

11.40 am **Christina Lin, University of Auckland**
Modelling housing feature impacts on sale values in newly developed suburbs

There is a recent trend of entire new suburbs that support a local community being built to solve the shortage of affordable housing all around the world. The aim of this study is to estimate the value of housing features in suburbs that are in the planning stage, but no sales have been recorded yet. For this purpose, we are separating price movements over time from individual housing features to analyze their impact on final sale prices in recently developed suburbs. To generate insights on housing features that can be directly interpreted by developers, we propose modelling house prices relative to a standard house representative of local preferences. The proposed model is successfully evaluated on newly developed suburbs in Auckland, New Zealand. The case study on the newly developed suburbs of Fairview Heights, Oteha, and Stonefields demonstrates that the proposed modelling approach effectively captured the complex relationship between housing features and sale price relative to a standard house (R-squared 93%). The proposed model generalizes to a reasonable extent to house prices in Auckland's new suburb Hobsonville (R-squared 68%) without using any historical sale records in this suburb. This indicates that the insights on housing features relative to the standard house are applicable to other new suburbs still in the planning stage and therefore has the potential to support future suburb developments. It is expected that the concept of valuating housing features relative to a locally specific standard house translates to the international market.

12.00 pm **Winston Sweatman, Massey University**
Symmetric families of periodic orbits with four- and five-bodies

We consider systems of four and five masses with a rotational symmetry (Caledonian problems) and study some families of periodic orbits.

12.20 am Break

1.00 pm **Keynote address**
Mark McGuinness, Victoria University of Wellington
Under pressure: Volcanic bombs that don't explode

During Surtseyan volcanic eruptions, hot bubbly magma bombs containing pockets of liquid water fly from the volcano, trailing white steam behind. Many bombs are fist-sized or smaller, while some are the size of a car. Each bomb is like a pressure cooker, boiling the water inside to make steam. The puzzle is, how do most bombs survive without exploding due to steam pressure buildup inside? We solve the puzzle, using mathematical equations describing the rates of change of pressure and temperature inside a bomb, to show that steam escape saves the bomb from exploding.

This work is in collaboration with Emma Greenbank and Ian Schipper, and has recently appeared in the New York Times and Proc Roy Soc London A.

1.45 pm Break

2.00 pm **Nicholas Witte, Victoria University of Wellington**
Beyond Toeplitz matrices: Modulated or slanted systems

Toeplitz matrices are fundamental in many areas of physics: the spin-spin correlations of the planar Ising model, the density matrix for 1-dimensional quantum Bose gases and random unitary matrices under Haar measure. The latter application has important implications for number theory, and a recent extension for the orthogonal and symplectic groups arising in number theory has motivated looking beyond the Toeplitz structure. We reveal what lies ahead.
Joint work with R Gharakhloo, Colorado State University

2.20 pm **Howard Cohl, NIST**
Special values for continuous q-Jacobi polynomials and applications

In q-calculus, which is a difference calculus, the $q \to 1$ limit connects with standard differential calculus. In this talk we describe a q-calculus treatment of orthogonal polynomials in the q-Askey scheme of hypergeometric orthogonal polynomials which are closely connected with Jacobi polynomials. These were introduced by Dick Askey and are called continuous q-Jacobi polynomials. We are able to compute special values for the continuous q-Jacobi polynomials in terms of q-Racah polynomials. Then by starting with Gasper and Rahman's Poisson kernel for these polynomials and the special values, we compute new generating functions and summation expressions for these polynomials and for orthogonal polynomials in their subfamilies, such as for q-ultraspherical polynomials and in the q to 1 limit, for Jacobi polynomials. We then show how one can use these special values by utilization with the Poisson kernel for continuous q-Jacobi polynomials to obtain a 4x4 grid of transformation formulas for single nonterminating basic hypergeometric functions.

2.40 pm **Graham Weir, Massey University**
Power loss in soft magnetic materials

This paper develops a parametric model describing power loss per unit volume, on unbiased minor loops, in the low frequency limit, in magnetic materials. After reviewing permanent magnetism and hysteretic losses in magnetic materials, we use dimensional analysis to identify three loss regimes, and obtain parametric estimates for the corresponding losses. The parameters used include remanence, coercivity, saturated induction field value and power loss per unit volume, from the major loop. The parametric models are assessed against published data. Both measured and modelled values can contain large uncertainties

3.00 pm Break

3.20 pm **Brendan Harding, Victoria University of Wellington**
Inertial migration of spherical particles in curved ducts having a trapezoidal cross-section

Finite size particles suspended in flow through micro-scale ducts are known to migrate across streamlines and focus towards stable equilibria whose location depends on a variety of factors. This has a variety of practical applications involving the separation and isolation of cells. Building on some earlier work on modelling particle migration in curved ducts with a rectangular cross-section, and motivated by experiments involving trapezoidal cross-sections, I have been investigating the migration dynamics of particles suspended in flow through two families of trapezoidal cross-sections. I will briefly describe this problem in more detail and present some recent results.

3.40 pm **Dimitrios Mitsotakis, Victoria University of Wellington**
Recent advances in the theory of nonlinear and dispersive water waves

A class of weakly nonlinear and weakly dispersive systems of partial differential equations of Boussinesq-type is presented. These mathematically justified equations describe the propagation of small amplitude, long water waves (such as tsunamis) in the ocean and approximate basic equations of fluid mechanics. Contrary to other Boussinesq systems, the new systems can be used in bounded domains with physically and mathematically justified boundary conditions.

4.00 pm **Robert McLachlan, Massey University**
Functional equivariance of numerical integrators

Preservation of linear and quadratic invariants by numerical integrators has been well studied. However, many systems have linear or quadratic observables that are not invariant, but which satisfy evolution equations (e.g. of mass or momentum) expressing important properties of the system.

This talk introduces the concept of *functional equivariance*, a natural sense in which a numerical integrator may preserve the dynamics satisfied by certain classes of observables, whether or not they are invariant. After developing the general framework, we use it to obtain results on methods preserving local conservation laws in PDEs.