*CENTREFOLD*

## Gaven Martin

Gaven Martin is the first mathematician to be awarded the James Cook Fellowship---according to their website the most prestigious award of the Royal Society of New Zealand. This event marks an important step in the recognition of mathematics as a field of research for New Zealand scientists. At the same time the choice of Gaven Martin confirms the lofty status of the Cook fellowship, since he is a world-renowned mathematician who is already recognized by the international mathematical community as one of the leading researchers in a wide variety of fields.

Gaven grew up in Rotorua and moved to Auckland just before starting high school. He completed his undergraduate workat the University of Auckland. Throughout high school and university, Gaven worked mostafternoons and weekends at a local vineyard(mostly washing and then filling bottles) and has since never lost hisaffection for the grape. Indeed he andDianne (his later to be wife) worked together there for many years. At Auckland he was attracted to analysis bythe “demanding but beautiful” lectures of Ramankutty and then toquasiconformal mappings by lectures of Gauld andVamanamurthy, both of whom had recently been to Michigan. So off he wentto the University of Michigan in 1981with a Fullbright fellowship. While there he obtained an inaugural AlfredP. SloanDissertation Fellowship (awarded annually to the top 12 PhD math studentsin the US). His PhD thesis, writtenunder the supervision of Fred Gehring, was awarded the Sumner MyersPrize: the best PhD thesisat Michigan in 1986 (a prize he shares with the Uni-bomber!)

Very quickly Gaven became a leading member of the “quasiconformal community.” He had post-doctoral experience at MSRI (Berkeley) and Yale (Gibbs instructor) during which time he held several grants from the NSF and the FinnishAcademy of Sciences. At the insistence of David Gauld, Gaven was given a lectureship at theUniversity of Auckland and returned to New Zealand for a year in 1988/9 where he and Dianne had their first child. Then the family spent 1990 at the Mittag-Leffler Institut of the Swedish Academy of Sciences and the Institute des Hautes Etudes Scientifiques (IHES), France, which Gaven says “was probably the most important and productive period of my career”.

After returning to NZ in 1991, he was offered a professorship at the University of Sydney and a similar position in the Institute at the Australian National University. Subsequently Auckland matched these offers and in February 1992 he was given a Personal Chair at the University of Auckland, becoming the youngest full professor in New Zealand. Gaven spent the next four years jointly between ANU and Auckland.

There are two major themes to Gaven’s research. First, since 1991, Gaven’s collaboration with Tadeusz Iwaniec, following their seminal 1993 *Acta Mathematica* paper on quasiregular mappings, has hugely influenced several branches of analysis. In particular the elliptic regularity theory for the PDEs of conformal geometry; the generalised Beltrami systems

$$D^t f(x)\, H\, (f(x))\, D\, f(x) = J_f^{2/n}(x)\; G(x)G(x).$$

Here $G(x)$ and $H(x)$ are measurable conformal structures (matrix fields) on the domain and target space of a mapping $f$ in a suitable Sobolev class. They showed the regularity theory was completely determined, in even dimensions, by the $L^p$ -norms of the Hilbert transform on differential forms (a singular integral operator generalising the Spin and Dirac operators of conformal geometry). Beltrami systems play an important role in harmonic analysis, non-linear potential theory, non-linear elasticity theory and topological index theory. The surprising fact is that the solutions of these highly non-linear equations poses additional smoothness analogous to solutions of linear Laplace or Schrodinger equations. From this fact a number of spectacular results were established, including the complete solution (in even dimensions) of the Liouville problem (1850) on the solutions to Cauchy-Rieman systems ($G = H =$

*Id*) and the Painleve problem on the structure of the singular sets for solutions to this and related systems. Since Dennis Sullivan showed that an arbitrary topological $n$-manifold ($n = 4$) admits measurable conformal structures, topological invariants can be picked up by studying the solutions to these equations. For instance a measurable Yang-Mills theory provides the analogous Donaldson invariants, and recently found their way into non-commutative geometry where Connes-Sullivan-Teleman used these equations to develop a more general theory of characteristic classes and the Novikov conjecture.

A few years later Gaven discovered another surprising connection between this equation and Hilbert's fifth problem on Lie groups. He proved that *every locally compact group which acts quasiconformally on a Riemannian manifold is necessarily a Lie group*. This led to uniqueness theorems for homeomorphic solutions to to generalised Beltrami systems for which there had been no earlier (non-trivial) results at all. Recently these results have been jazzed up to include odd dimensions as well and the degenerate elliptic setting. But the results there are not so precise. All this work has been encapsulated in the recent book Gaven wrote with Tadeusz for Oxford University Press.

Another major theme of Gaven's work has been the collaboration with Fred Gehring and others (notably Colin Maclachlan) on the geometry of discrete groups, and their connection with low dimensional topology and geometry (recall Thurston's Uniformisation programme which asserts that the canonical geometry of 3 dimensions is hyperbolic). A recent focus has been on the arithmetic properties of these groups and their orbit spaces (hyperbolic 3-manifolds) and recently good progress has been made in an attempt to classify the two generator groups, analogous to the hyperbolic triangle groups of 2 dimensions. Another problem is the higher dimension problem of Hurwitz: what is the minimal volume hyperbolic 3--manifold? Applications include generalisations of the famous $84g$-84 theorem on the maximal number of automorphisms of a Riemann surface to 3 dimensions. To date the best known results are due to Gaven and his coworkers. Early on in his collaboration with Gehring, they introduced the concept of convergence groups. These groups are close to Gromov's hyperbolic groups, and encapsulated the essence of the theory of Fuchsian groups. They were famously used to solve the Seifert Fibre space conjecture concerning foliations of 3-manifolds by circles. By earlier work (of many people including Gaven) it was known the conjecture was equivalent to showing that every action of a convergence group on the circle was topologically conjugate to the action of a Fuchsian group. This was the problem finally solved by Gabai and Casson-Jungreiss.

For the two years before taking up the Cook fellowship he served the University of Auckland as an excellent and highly-respected head of department, a job he did not have to hesitate about giving up.

To finish this note, we include a few statistics: Professor Gaven Martin has been awarded 20 special grants and fellowships, in addition to 21 travel grants all over the world; he has held 16 visiting or temporary positions; he has had 4 administrative appointments at the University of Auckland; and most important of all, he has (at last count!) published 75 papers, many in top-notch international mathematical journals. \par

We hope that the inspiring example of Professor Gaven Martin as a brilliant researcher, a fair and honest administrator and a wonderful colleague will attract more talented students to mathematics.

*Norm Levenberg, Boris Pavlov*