CENTREFOLD

Geoff Whittle

In 2001, Geoffrey Peter Whittle was promoted to a personal chair in Mathematics at Victoria University of Wellington less than ten years after his initial appointment there as a Lecturer. During Geoff's decade at Vic, his career has flourished. He was awarded the New Zealand Mathematical Society's Research Award in 1996 and he has become internationally recognized as a world leader in discrete mathematics, particularly matroid theory. Geoff is also a fine teacher being both a stimulating and energetic lecturer, and an enthusiastic and successful postgraduate supervisor with one of his Ph.D. students having won the RSNZ's Hatherton Award in 1998. One of Geoff's former M.Sc. students is now pursuing her doctorate in Oxford and a second one will follow her to Oxford later this year. In this article, we review some aspects of Geoff's life and describe the significance of his research and why it has attracted such international attention.

Geoff Whittle was born in 1950 in Launceston, a city in northern Tasmania whose famous sons include David Boon and Ricky Ponting. Geoff completed high school in Launceston in 1968 and then spent a year as a mine worker in Tasmania, Western Australia, and the Northern Territory. The following year, he travelled throughout Asia and Europe. Among the most memorable incidents on these travels was having an infected tooth removed in Baghdad without the benefit of anaethesia! In 1971, he began a degree in Philosophy and Mathematics at the University of Tasmania graduating with a B.A. in 1973. Two Mathematics courses that he took in this period were particularly influential. The first was a course in point-set topology, which was taught by Howard Cook using the Moore method. This method, pioneered by the famous University of Texas topologist R.L. Moore, requires the students to prove all of the theorems. It was Geoff's introduction to the life of a mathematician. Howard Cook, who was one of Moore's more than fifty Ph.D. students, was Professor of Pure Mathematics at the University of Tasmania for two years beginning in 1972 before returning to his former post at the University of Houston. The other undergraduate course that profoundly influenced Geoff was a course on projective geometry taught by Don Row, who was to become, nearly ten years later, Geoff's Ph.D. supervisor. A constant theme in Geoff's research has been the successful harnessing of his considerable geometric insight.

After two years as a high school teacher of Mathematics and Science, Geoff returned to the University of Tasmania in 1976 to complete his Honours degree in Philosophy and Mathematics. For many years, Don Row taught an honours course in matroid theory that followed on from his third-year projective geometry course. This course influenced several students including Geoff, James Oxley, and Dirk Vertigan (both now at Louisiana State University) to pursue the study of matroids. In Geoff's case, this pursuit was interrupted by three years as a Lecturer in Teacher Education in Tasmania and then two years as a Mathematics Lecturer at the University of the South Pacific in Fiji. Geoff returned to his studies at the University of Tasmania in 1982 and, while working as a Tutor in Mathematics, completed a Ph.D. in 1984. He stayed on as a Tutor after receiving his doctorate and then, from 1989 until 1991, was a Research Fellow in Mathematics.

Geoff had wanted to stay in Tasmania but, when it became clear that a permanent job there was not on the horizon, he applied for a Lectureship in Wellington and he moved to New Zealand in 1992. He was promoted to Senior Lecturer in 1994, to Reader in 1997, and to a personal chair last year. In 1998, Geoff spent a term as a Visiting Research Fellow at Merton College, Oxford with Dominic Welsh. He has made ten short-term research visits to Louisiana State to collaborate with Oxley and Vertigan. In the late 1990s, Geoff began collaborating with another Australian matroid theorist, Jim Geelen of the University of Waterloo. This collaboration has taken Geoff to Waterloo four times and brought Jim to Wellington for six months from September, 2001.

What is matroid theory and what are Geoff's contributions to the subject? The rest of this article will deal briefly with these questions. Matroids were introduced by Hassler Whitney in his 1935 paper "On the abstract properties of linear dependence". Independence is a core concept in mathematics and, in defining matroids, Whitney attempted to capture the fundamental aspects of independence that are common in graph theory and linear algebra. Subsequent work has shown that Whitney's definition is quite robust and also encompasses several other natural notions of independence including algebraic independence over a field. Matroids arise naturally in timetabling problems and indeed play a central role in combinatorial optimization since they are exactly the structures for which a locally greedy stategy can be guaranteed to produce a global maximum. Whitney called a set of columns of a matrix *independent* if it is linearly independent, while a set of edges of a graph is *independent* if it is a forest, that is, it does not contain the edges of any closed path. A *matroid* consists of a finite set and a collection of its subsets called independent sets that behave both like the sets of linearly independent columns of a matrix and like the forests in a graph. Specifically, the collection of independent sets is non-empty, it is closed under taking subsets, and if one independent set is larger than another, then the larger set contains an element not in the smaller one that can be added to the smaller one to produce another independent set. A matroid that arises from a matrix over a field F is called *F-representable*, while a matroid that arises from a graph is *graphic*. Every graphic matroid is F-representable for all fields F, but there are matroids, the smallest with eight elements, that are not F-representable for any F.

In Geoff's Ph.D. thesis and his early papers, he worked on Crapo and Rota's "critical problem".The critical exponent of a matroid representable over $GF(q)$ is a parameter that generalizes the chromatic and flow numbers of a graph, and the redundancy of a linear code. The matroids that are, in a certain strong sense, minimal having a fixed critical exponent are called tangential blocks. Geoff solved a number of problems of Dominic Welsh by providing a series of general constructions for tangential blocks, which revealed that such structures were far more numerous than had previously been believed. In the early 1990s, Geoff decided to begin attacking some of the many notoriously difficult unsolved problems in matroid representability. This was a very wise decision for the success of Geoff's work in this area led to his NZMS Research Award and has resulted in a series of important advances in an area that, only a decade ago, had looked totally intractable. The Research Award was given for Geoff's work on ternary matroids, those representable over the 3-element field. Every binary matroid M is representable over all fields of characteristic two. If M is representable over some additional field, then it follows from fundamental work of Tutte from the 1950s that M is representable over $GF(3)$. Geoff proved the beautiful result that if a ternary matroid is representable over some field of characteristic other than three, then it must be representable over one of $GF(2)$, $GF(4)$, $GF(5)$, $GF(7)$, or $GF(8)$.

Geoff's current research programme has two closely related strands. The first is to extend Neil Robertson and Paul Seymour's fundamental theory of graph minors to matroids representable over finite fields. Robertson and Seymour proved that, in every infinite sequence of graphs, there is one that is a minor of another, where H is a minor of G if H can be obtained from G by deleting edges or vertices or contracting edges. It is well-known that this theorem does not extend to the class of all matroids. Whether the theorem extends to the class of matroids representable over a fixed finite field $GF(q)$ is the major unsolved problem of this strand. The second strand of Geoff's current programme is the pursuit of Rota's 1971 conjecture that, for every prime power q, the set of minor-minimal matroids not representable over $GF(q)$ is finite. In collaboration with Bert Gerards, Geoff and Jim Geelen showed that the Robertson-Seymour theorem extends to the $GF(q)$-representable matroids of bounded branch-width, loosely speaking, those that are not too tightly connected. In addition, Geoff and Jim have proved that there are only finitely many excluded minors for $GF(q)$-representability whose branch-width is less than some fixed bound. These are important and exciting developments and Geoff has firmly established himself as a world leader in matroid theory. Those of us who work in this area hope that the very fruitful last decade that Geoff enjoyed will be exceeded by an even more productive next decade.

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