Abstracts

Plenary talks

Jörg Frauendiener

(University of Otago , joergf@maths.otago.ac.nz)

Scattering of gravitational waves, a geometric perspective

Scattering is described in physics by the relations between asymptotically ingoing and outgoing states. The corresponding notions in Einstein's theory of gravity are the past and future light-like infinities as introduced by Roger Penrose. They rely on the conformal structure of the Lorentz manifold describing the system. In this talk we will discuss how conformal methods can be used to describe the scattering of gravitational waves geometrically and numerically. (ThA8)

Graeme Hocking

(Murdoch University, G.Hocking@murdoch.edu.au)

Theory and Practice of withdrawal from stratified reservoirs

Water contained in storage reservoirs in temperate climates is stratified in density for most of the year due to variations in temperature and salinity. When water is withdrawn for irrigation, town supply and reservoir management its origin depends on a number of factors. This problem has been of interest since the early 1900s and the first papers on the practical application appeared in the late 1940s. This was followed by a period of intense research activity involving both experimental and theoretical work that provided an outline of the general behaviour of such flows, but as always threw up a number of further questions. The details for nonlinear stratifications were difficult to obtain and so in most cases a linear approximation was used for 'real' modelling of lakes and reservoirs. Fast computers and better algorithms now allow a much more accurate representation of these flows including nonlinearity and complicated geometries, yet there remain a number of unsolved questions. A history of the study and understanding of this problem and the techniques employed to examine it will be given, including some interesting mathematical and numerical problems that have arisen. There will be movies! Some important recently obtained results will be given. (TA6)

Rachel Kuske (ANZIAM Speaker)

(University of British Columbia, rachel@math.ubc.ca)

Tipping and Fat Tails: Stochastic models in climate dynamics

Understanding variability in climate dynamics naturally motivates better understanding of stochastic modeling and nonlinear effects. This talk will cover two different areas of climate modeling where there are many mathematical and statistical questions still unanswered. We compare the effects of stochastic and periodic variability on transitions near tipping points, motivated by reduced models of arctic sea ice extent. By applying multiple scales and WKB-type approximations we consider the relative effects of noise, periodic variability, and slowly varying control parameter. We also propose novel averaging methods leading to reduced models for systems with Levy noise forcing. The work is motivated both by recent data analysis that indicates the presence of non-Gaussian stochastic processes in climate proxies and by certain intermediate models with correlated additive and multiplicative noise that can lead to this behaviour. This is joint work with Jielin Zhu and William Thompson (UBC), and Adam Monahan (U. Victoria). (WA5)

Said Sidki

(University of Brasilia, ssidki@gmail.com)

Automorphisms of Trees and Automata

Constructions in the early 1970's of groups by finite automata progressed quickly in the last three decades to reveal many new phenomena in infinite Group Theory with applications to Probability and Dynamical Systems. We introduce input-output automata, their growth and their interpretation as actions on 1-rooted regular trees. Then we explore the Group of Finite Automata, its subgroup of automata whose growth is of polynomial type, as well as recent results on the n-adding machine and on decidability questions. (ThA1)

Tom ter Elst (NZMS Speaker)

(The University of Auckland, terelst@math.auckland.ac.nz)

Does diffusion determine the drum?

The famous question of Kac is whether one can hear the shape of a drum. Or more precisely, whether all eigen frequencies of a drum determine the drum. In general the answer to the latter question is negative. The eigen frequencies are equal if and only if there exists a unitary operator which maps the Laplacian on the first drum onto the Laplacian on the second drum. Equivalently, this unitary operator intertwines between the two heat semigroups on the drums. In this talk we discuss what happens if the unitary operator is replaced by an order isomorphism, i.e., if it maps positive functions to positive functions. Then that order isomorphism maps positive solution of the heat equation on the first drum onto positive solutions on the second drum. For this problem the answer is positive and the drums are congruent if such an order isomorphism exists. Thus diffusion determines the drum. (TA1)

Matt Visser
(Victoria University of Wellington, matt.visser@msor.vuw.ac.nz)

Energy conditions in Einstein's general relativity

Einstein's equations of general relativity relate the Ricci curvature of spacetime to the distribution of matter. However, unless one places some sort of constraint on the class of matter distributions one is willing to call "physically acceptable", the Einstein equations become an empty tautology – everything becomes allowable. Typically this is dealt with by imposing some sort of "energy condition", some combination of energy densities and pressures is assumed positive; or at least not too far negative once one allows for quantum physics effects. The physics community is still arguing over exactly which "energy condition" is "best". I will give a gentle introduction to these ideas, and describe the state of the art. (WA1)

Contributed talks

Abdullahi Rashid Adem

(North-West University, Abdullahi.R.Adem@gmail.com)

New exact solutions and conservation laws of a coupled Kadomtsev-Petviashvili system

In this talk we obtain exact solutions of a new coupled Kadomtsev-Petviashvili system, which arises in the analysis of various problems in fluid mechanics, theoretical physics and many scientific applications. Lie symmetry method along with the multiple-exponential method is employed to find the travelling wave solutions of the underlying system. In addition, we derive the conservation laws of the coupled Kadomtsev-Petviashvili system using the multiplier method. (ThA5)

Khadijo Rashid Adem

(North-West University, Khadijo.R.Adem@gmail.com)

Exact solutions of a generalized Camassa-Holm Kadomtsev-Petviashvili equation

In this talk we obtain the solutions of a generalized Camassa-Holm Kadomtsev-Petviashvili (gCH-KP) equation, which is used to understand the role of dispersion in the formation of patterns in liquid drops. The Lie group analysis is used to carry out the integration of this equation. Furthermore, we employ the Exp-function and simplest equation methods to obtain more exact solutions. (TA7)

Amjad Ali

(Massey University, Auckland, a.ali.1@massey.ac.nz)

Steady groundwater flow in multi-layered aquifers containing wells

The soil properties, e.g. permeability, porosity etc. in “naturally stratified” formations vary from layer to layer. Within each layer, properties can be assumed constant at each horizontal point, but they are allowed to vary horizontally. The fluid flow and hence pollutant transport models are simplified for such aquifer discretised into naturally occurring sub-layers. The physical thickness of the nearly horizontal sub-layers is allowed to vary horizontally. The fluid flow in such schematised aquifer is governed by a set of coupled PDEs each describing hydraulic head or dynamic pressure in a sub-layer of the aquifer based on the conservation of mass of fluid.

Fluid flow models for both the confined and unconfined multi-layered aquifers are developed and solved for few cases for illustration. The method is useful both for the point fluid source/sink in any of the sub-layers (like a pumping or recharge well) or a distributed source/sink (like a stream or lake of fluid in top sub-layer). For demonstration purpose, the examples discussed are for the two-dimensional aquifers where sub-layer thicknesses and all other properties vary only in one horizontal direction. (TA9)

Astrid an Huef

(Univeristy of Otago, astrid@maths.otago.ac.nz)

Operator algebras associated to transformation groups

A transformation group consists of a locally compact group G acting continuously on a space X. The construction of the transformation-group C^*-algebra generalises the construction of the group C^*-algebra. Many important C^*-algebras arise from transformation groups and their representation theory is reflected in the dynamics of (G, X). I'll survey some of these results using examples. (ThC3)

Andrea Babylon

(Massey University, A.Babylon@massey.ac.nz)

Modelling leptospirosis in rats

Leptospirosis is an infectious disease resulting from a bacterial infection. It is an occupational disease that is often transmitted from livestock such as sheep and cattle to humans. Transmission occurs when contaminated material, such as water polluted with the urine of an infected animal, comes into contact with broken skin, such as cuts or scrapes, on a person or is ingested internally, such as by a rat drinking contaminated water. Blood vessels are penetrated by the bacteria which then reproduce and infect several organs. In humans, symptoms are usually flu like, but the disease can damage the liver and result in renal failure. Six out of the 250 known serovars are present in New Zealand. As the disease can not only damage our agriculture but can also be fatal in humans it is important to study the multispecies dynamics of the disease and the impact of vaccinating against it. A simple mathematical model showing the dynamics of the spread of the disease in rats in Tanzania, which will be used as a building block for future work, is proposed and used to predict conditions under which the disease will persist in the population. (WA3)

Igor Boglaev

(Massey University, i.boglaev@massey.ac.nz)

Monotone alternating direction implicit (ADI) scheme for nonlinear parabolic problems

The talk deals with numerical solving nonlinear parabolic problems based on a nonlinear ADI method. The convergence of the nonlinear ADI method to the continuous solution is established. For solving the nonlinear ADI method, a monotone convergent ADI scheme, based on the method of upper and lower solutions, is constructed. This monotone convergence leads to the existence and uniqueness of a solution of the nonlinear ADI method. An analysis of convergence of the monotone ADI scheme to the solution of the nonlinear ADI method on the whole time interval is given. Numerical experiments are presented. (ThC5)

Nicholas Brettell

(University of Canterbury, nick.brettell@pg.canterbury.ac.nz)

Matroid connectivity: constructing a k-tree

Connectivity has become an increasingly important tool in the understanding and eventual solution of many problems in matroid theory. In this talk, we shall give a quick introduction to matroid connectivity, show how a certain labelled tree, known as a k-tree, can describe the connectivity structure of a matroid, and touch on some recent research regarding a polynomial-time algorithm for constructing such a k-tree. (TD8)

Kevin Broughan

(University of Waikato, kab@waikato.ac.nz)

The product and quotient of Euler's totient and the sum of divisors functions

This is joint work with Daniel Delbourgo, Kevin Ford, Florian Luca and Qizhi Zhou (Zoe). We have been exploring the mysterious connections between $\phi(n)$, Euler's totient, and $\sigma(n)$, the sum of divisors function. For example they have asymptotically the same average order and size of the value set. I will describe how often the product of the two functions is a square, and an associated tantalizing density problem. The issue of when $\phi(n)$ evenly divides $\sigma(n)$ is considered. This is related to the infinitude, or otherwize of Mersenne primes $p = 2^q - 1$. We used the product compactification topology of $\mathbb{N}$ to show that to obtain an infinite number of divisors, $\Omega(n)$, the total number of prime divisors including multiplicity, must be unbounded. Good progress has been made when the number of distinct prime divisors of n is three or less, leaving one case yet to be solved.

[1] Kevin Broughan and Daniel Delbourgo, On the ratio of the sum of divisors and Eulers totient function I, Journal of Integer Sequences, vol 16 (2013) A 13.8.8

[2] Kevin Broughan, Kevin Ford and Florian Luca, On square values of the product of the Euler totient function and the sum of divisors function, Colloquium Mathematicum, vol 130 (2013) 127–137. (TB4)

John Butcher

(University of Auckland, butcher@math.auckland.ac.nz)

Counting trees and rooted trees with applications

Rooted trees and related structures, such as Hopf algebras and B-series, play a central role in the theory of numerical methods for differential equations. For canonical methods, which conserve quadratic invariants, trees, rather than rooted trees, are equally significant. We will recall classical combinatorial results on trees and rooted trees and outline some applications to numerical analysis. (ThC4)

Pengxing Cao

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On the role of stochastic properties of inositol trisphosphate receptors in calcium oscillations in airway smooth muscle cells

Calcium models with deterministic formulations of the inositol trisphosphate receptor (IP3R) have been used to explain and predict relevant physiological processes for over 20 years, but their unrealistic assumptions were recently challenged by experimental data and stochastic simulations, and so were their validities for model predictions. In this talk, by comparing the deterministic and stochastic modeling approaches, we investigated the role of stochasticity of IP3R in airway smooth muscle calcium oscillations. We found that the termination of each oscillation is primarily modulated by a rapid calcium-inhibition in a deterministic way, whereas the generation of individual oscillations is significantly affected by stochastic openings of IP3Rs, which could be a source of failure for the deterministic models to predict a correct frequency change. (TB7)

Alveen Chand

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Axiomatic Analysis via Value Quantale

Analysis began with the notion of metric spaces by Frechet in 1906. From then on, countless mathematicians have contributed to the advancement of this field and to generalization of metric spaces. These generalizations have furnished a deeper understanding of the role of the real numbers in Analysis. This presentation focuses on a special type of generalization, called continuity spaces (or V-spaces), introduced by Flagg in the early 1990's. We present a completion construction for V-spaces that satisfy a weak form of symmetry. This construction generalizes the standard completion of ordinary (symmetric!) metric spaces. We then discuss the subtleties of completion when no form of symmetry is present, and we ouline a new approach that unifies various existing constructions. (ThB5)

Marston Conder

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The Intersection Condition for Regular Polytopes

The automorphism group of a regular polytope is a smooth quotient of a string Coxeter group (with a linear Dynkin diagram). Conversely, any finite smooth quotient of such a group is the automorphism group of a regular polytope, provided that it satisfies the so-called 'intersection condition'. I will describe some recent discoveries about the intersection condition, especially in ranks 3 and 4, and its application to find the smallest regular polytopes of any given rank. (ThB2)

Daniel Delbourgo

(Waikato University, delbourg@waikato.ac.nz)

Number theory and map colourings

We discuss some number theoretic properties exhibited by the chromatic polynomial (which interpolates the number of proper colourings of a graph). We then explain how tools from ramification theory can be used to determine both the algebraic nature of chromatic roots, and the Galois groups associated to their splitting fields. (ThB4)

Tan Do

(The University of Auckland, tdo006@aucklanduni.ac.nz)

Core Properties for Degenerate Elliptic Operators

Let $\Sigma_\theta := \{re^{i\alpha} : r \geq 0, |\alpha| \leq \theta\}$ be a sector of angle $\theta \in [0, \pi)$. Let $c \in W^{1,\infty}(\mathbb{R})$ be sectorial. That is, the function c takes values in Σ_θ. We would like to investigate core properties for the operator

$$T = -\frac{d}{dx}\left(c\frac{d}{dx}\right).$$

The question of our interest is: "When is $C_c^\infty(\mathbb{R})$ a core for T?" By definition, a subspace D of the domain $D(T)$ is called a core for T if D is dense in $D(T)$ for the graph norm $\|u\|_{D(T)} := \|u\| + \|Tu\|$. We will give a characterisation for this core property. (TB10)

Joshua Duley

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A Mathematical Model of Volcanic Plumes

Volcanic plumes and the resultant tephra fallout are of significant concern not only to those living nearby the erupting volcano where entire communities are often blanketed in ashfall. Several recent large-scale eruptions have caused such disruption to air traffic that huge proportions of European commerce have been severely compromised.

Currently, separate models do exist for plume dynamics and the atmospheric transport of particles, with a mixture of qualitative and quantitative results. In this thesis we develop a mathematical model with some similarities and some differences to those already in use.

The model has its core in the conservation equations of mass, momentum and energy for the plume's driving gases and suspended particles. While these equations are non-linear and difficult (if not impossible) to solve analytically, we can solve the equations numerically using a discretisation of the plume along its axis.

Initially equations are provided with full time-dependency. However, the numerical results are limited to steady-flow simulations of varying complexity. (TA3)

Leon Escobar Diaz

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The Cauchy problem in some cosmological models

The standard model of cosmology based on Einstein's theory of gravity is built on the assumption of the so called cosmological principle, which states that our universe is homogeneous and isotropic. The assumption of homogeneity and isotropy is just a first approximation introduced to simplify the system of partial differential equations (PDEs) derived from Einsteins equations, leading to a system of ODEs. However, such homogeneous cosmological models are still an incomplete description of the universe and therefore more general mathematical models are needed. This is our motivation to study the system of PDEs which come from certain inhomogeneous and anisotropic cosmological models. Particularly, we are interested in the question of whether an early rapid expansion of the universe is able to homogenise and isotropise certain cosmological models despite its early inhomogeneous and anisotropic state. We address this question by investigating the Cauchy problem in some of those models using a combination of analytical and numerical techniques. (WC3)

Steven Galbraith

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Isogeny graphs of elliptic curves

The isogeny graph of elliptic curves has vertices corresponding to isomorphism classes of elliptic curves and edges corresponding to isogenies (group homomorphisms). This graph is mainly of interest for elliptic curves over finite fields. The isogeny graphs for ordinary and supersingular curves have rather a different flavour (e.g., the former is regular and the latter is usually is not). Both graphs have a number of applications in computational number theory and cryptography, and have been studied for a long time. A natural algorithmic problem (the isogeny problem) is to compute a path in the isogeny graph between two vertices. Up to now, this problem has been solved fully in the ordinary case, but the supersingular case has been more awkward. In this talk I will survey the subject. I will also report on recent joint work with Christina Delfs (Oldenburg) where we study the "subgraph" of supersingular elliptic curves over F_p, and present a new algorithm for the supersingular isogeny problem. (TB2)

David Gauld

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Selections and metrisability of manifolds

I am collecting topological conditions which superficially look different from one another yet become equivalent when restricted to topological manifolds: my current list contains well over 100 items.

Recently I have been studying selections in the context of manifolds. For families $\mathfrak{A}$ and $\mathfrak{B}$ of subsets of some set consider the *selection principle*:

$\mathsf{S}_1(\mathfrak{A}, \mathfrak{B})$: for each sequence $\langle \mathcal{A}_n \rangle$ from $\mathfrak{A}$ there is a sequence $\langle B_n \rangle$ with $B_n \in \mathcal{A}_n$ for each n and $\{B_n \ / \ n \in \omega\} \in \mathfrak{B}$.

We have 'selected' B_n from $\mathcal{A}_n$ so that when we gather all B_n together we get a member of $\mathfrak{B}$.

More explicitly, let X be a topological space. Then $\mathfrak{A}$ and $\mathfrak{B}$ might each be some family of open covers of X: for example the family $\mathfrak{O}$ of all open covers of X or the family $\mathfrak{K}$ of *k-covers* of X (those open covers $\mathcal{U}$ for which $X \notin \mathcal{U}$ and every compactum is contained in some member of $\mathcal{U}$).

> *A manifold is metrisable if and only if it satisfies any one of the selection principles* $\mathsf{S}_1(\mathfrak{K}, \mathfrak{O})$, $\mathsf{S}_1(\mathfrak{K}, \Lambda)$, $\mathsf{S}_1(\mathfrak{K}, \Omega)$ and $\mathsf{S}_1(\mathfrak{K}, \mathfrak{K})$.

I shall discuss the proof of this theorem, explain why $\mathsf{S}_1(\mathfrak{O}, \mathfrak{K})$ does not appear on the list, present some related results and ask some questions. (TD3)

Shishay Gebregiyorgis

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Elliptic Curve Discrete Logarithm Problem

We give a survey of index calculus algorithms for the discrete logarithm problem, in particular for elliptic curves over finite fields. (TB3)

Emily Harvey

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Analysing a mathematical model of the division of labour in metabolic pathways

In naturally occurring microbial ecosystems it is often observed that the metabolic steps in the metabolism of a single substrate are divided between distinct subpopulations within the microbial community. This ‘division of labour has also been repeatedly observed to evolve from monocultures in experimental settings, and these ‘consortia are found to have increased productivity and robustness when compared to a monoculture.

In this work, through analysis of a mathematical model of a prototypical system, we compare the productivity (biomass production) of a consortium where the metabolic pathways have been divided between two distinct populations of microbes, to a single super microbe which has the same metabolic capacity. We prove that if there is no change in the growth rate or yield of these pathways when the metabolism is specialised into separate microbes, the biomass (productivity) of a consortia system is always less than that of a monoculture. To explain the observed increased productivity of the consortium, we conclude that some adaptation must have occurred, and using a specific example of *Escherichia coli* on a glucose substrate we are able identify the metabolic changes that would have the strongest effect on increasing the productivity. (ThA4)

Ian Hawthorn

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Physics on a curved manifold with local symmetry $so(2,3)$

For sufficiently large r, the group $SO(2,3)$ acts on the 4-D surface $\lambda^2+t^2-x^2-y^2-z^2=r^2$ near the point $\lambda = r$ in a manner which is locally indistinguishable from the Poincaré group. Hence this group is a possible alternative symmetry group for the universe. In this talk we look at the possible advantages of using $so(2,3)$ and consider a curved model which uses $so(2,3)$ to describe local symmetry. The curvature in such a model naturally separates into components and these are constrained by equations arising purely from the geometry which appear gravitational and electromagnetic in nature. (TC10)

Jörg Hennig

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A generalisation of the Taub-NUT cosmological model in general relativity

The Taub solution, discovered in 1951, is a general relativistic model for a spatially homogeneous universe. This cosmological model is regular between some point in the past and some point in the future. It is also well-known that the Taub solution can be extended beyond this regular domain into a region in which so-called closed causal curves are present. The existence of such extensions, which are called Taub-NUT solutions, implies the strange possibility of a time travel backwards in time. Clearly, this is a violation of causality. The interesting question is whether more general (i.e. less symmetric) cosmological solutions to Einstein's field equations can also suffer from such defects. In this talk, we derive and discuss a three-parametric, spatially inhomogeneous family of exact solutions that generalises the two-parametric Taub solution and contains it as a special case. In particular, we show that for a special choice of the parameters, this spacetime contains a curvature singularity with directional behaviour. For other parameter choices, the maximal globally hyperbolic region is singularity-free. But again there are extensions that allow access into domains with closed causal curves. (WC2)

Ali Jaballah

(University of Sharjah, ajaballah@sharjah.ac.ae)

Non-Valuation Domains Whose Overrings are Valuation

We study in this paper maximal non-valuation subrings of a field, i.e. non-valuation domains R whose overrings are valuation. We establish several new characterizations of maximal non-valuation subrings of a field involving several concepts of commutative algebra related to the set of prime ideals and the set of overrings. For example we show that an integral domain R of finite dimension d is a maximal non-valuation subring of a field if, and only if R is either integrally closed with a set of overrings isomorphic to a kite-graph of dimension $d+1$, or is non-integrally closed with a chained set of overrings of dimension $d+1$. We also establish several characterizations independent of whether R is integrally closed or not. Finally we notice several similarities in the behavior of prime ideals and overrings for maximal non-valuation subrings of a field. (ThC6)

Sebastian Jambor

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Word maps on finite simple groups

For a group G and a word w on n letters, define the word map $G\times\cdots G \to G$, $(x_1, \ldots, x_n) \mapsto w(x_1, \ldots, x_n)$ by evaluation. Word maps have been widely studied, with a particular focus on the surjectivity of these maps. For instance, the Ore conjecture (formulated in 1951 and proved in 2012 by Liebeck, O'Brien, Shalev, and Tiep) states that every element in a finite non-abelian simple group is a commutator ($w = [x_1, x_2]$). Shalev conjectured that every word w which is not a proper power defines a surjective word map on $\mathrm{PSL}(2, q)$ for all sufficiently large q. In recent work with Liebeck and O'Brien we gave a counterexample to Shalev's conjecture. We exhibit explicit non-power words which yield non-surjective word maps for infinitely many groups $\mathrm{PSL}(2, q)$. In this talk I will highlight some of the results in the theory of word maps and explain our counterexample to Shalev's conjecture. (ThB3)

Andrew Keane

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Bifurcation analysis of a model for the El Niño southern oscillation

Climate models can take many different forms, all the way from very detailed highly computational models with hundreds of thousands of variables, down to more phenomenological models of only a few variables that are designed to investigate fundamental relationships in the climate system. Important ingredients in these models are the periodic forcing by the seasons, as well as global transport phenomena of quantities such as air or ocean temperature and salinity.

We consider a phenomenological model for the El Niño southern oscillation system, where the delayed effects of oceanic waves are incorporated explicitly into the model, which gives a description by a delay differential equation. We conduct a bifurcation analysis of the model in two parameters by means of dedicated continuation software. It explains some previous results, but also uncovers surprisingly complicated behaviour concerning the interplay between seasonal forcing and delay-induced dynamics. (TA5)

Chaudry Masood Khalique

(North-West University, Mafikeng Campus, Masood.Khalique@nwu.ac.za)

On the exact solutions and conservation laws for a (3+1)-dimensional generalized B-type Kadomtsev-Petviashvili equation

In this talk we study a (3+1)-dimensional generalized B-type Kadomtsev-Petviashvili (BKP) equation. This equation is an extension of the well-known Kadomtsev-Petviashvili equation, which describes weakly dispersive and small amplitude waves propagating in a quasi-two-dimensional media. We first obtain exact solutions of the BKP equation using (G'/G)-expansion method. Furthermore, the conservation laws for the BKP equation are constructed by using the multiplier method. (ThA7)

Igor Klep

(The University of Auckland, igor.klep@auckland.ac.nz)

Convex hulls of semialgebraic sets

We discuss a noncommutative analog of real algebraic geometry – the study of polynomial inequalities and equations over the reals – focusing on matrix convex sets C and their projections $\overline{C}$. Given a polynomial p, a free basic semialgebraic set $D_p = \{X : p(X) \succeq 0\}$ which is convex can be represented as the solution set of a Linear Matrix Inequality (LMI), suggesting that convex free semialgebraic sets are rare. Further, Tarski's transfer principle fails in the free setting: The projection of a free convex semialgebraic set need not be free semialgebraic.

In this talk I will present the construction of *a sequence $C^{(d)}$ of LMI domains in larger and larger spaces whose projections $\overline{C}^{(d)}$ close down on the convex hull of D_p.* It is based on free analogs of moments and Hankel matrices. Such an approximation scheme is likely the best that can be done in general. As evidence, we study the nonconvex free TV screen $p = 1 - x^2 - y^4$.

We remark that solutions sets of LMIs play a prominent role in many areas: control systems, mathematical optimization, statistics, quantum theory, etc. (ThC2)

Carlo Laing

(Massey University, c.r.laing@massey.ac.nz)

Twisted states in phase oscillator arrays

We consider a one-dimensional array of phase oscillators with non-local coupling and a Lorenztian distribution of natural frequencies. The primary objects of interest are partially coherent states that are uniformly "twisted" in space. To analyze these we take the continuum limit, perform an Ott/Antonsen reduction, integrate over the natural frequencies and study the resulting spatio-temporal system on an unbounded domain. We show that these twisted states and their stability can be calculated explicitly. We find that stable twisted states with different wave numbers appear for increasing coupling strength in the well-known Eckhaus scenario. (ThB6)

Michael Lockyer

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Mountains, paths, and generalised inverse limits

About 10 years ago, people began studying generalised inverse limits, which are constructed from upper semicontinuous set valued functions instead of the usual continuous functions. Many theorems regarding inverse limits do not carry over to the generalised setting. A property that inverse limits with continuous functions have is that they are either homeomorphic to an arc, or not path connected. This is not true for generalised inverse limits, leading to the question: 'under what conditions will a generalised inverse limit be path connected?'.

The problem of whether a finite segment of a generalised inverse limit is path connected can be reduced to the famous 'mountain climbing problem'. This problem informally asks: 'Under what conditions can two people climbing the same mountain from different sides reach the top, while maintaining the same altitude as each other during the entire journey?', or more formally: 'if $f, g : [0, 1] \to [0, 1]$ are continuous, with $f(0) = g(0)$ and $f(1) = g(1)$, under what conditions do there exist continuous $h, j : [0, 1] \to [0, 1]$ such that $f \circ j = g \circ h$?'.

In this talk, I will introduce the mountain climbing problem and generalised inverse limits, and show how results on the mountain climbing problem relate to the question of path connectedness in finite sections of a generalised inverse limit. (TD2)

Gabriel Magalakwe

(North West University, Mafikeng campus, magalakwe@webmail.co.za)

Exact solutions and conservation laws of the generalized double sinh-Gordon equation

This work aims to study the generalized double sinh-Gordon equation, which appears in several physical phenomena such as integrable quantum field theory, kink dynamics and fluid dynamics. Lie symmetry analysis along with the exponential function method is used to obtain travelling wave solutions for the generalized double sinh-Gordon equation. Thereafter, conservation laws for the generalized double sinh-Gordon equation are obtained by employing the direct method. (TA8)

Karen McCulloch

(Massey University, k.mcculloch@massey.ac.nz)

Epidemics on networks

The dynamics of an SIR (Susceptible-Infected-Recovered) epidemic process on small networks with different topological structures are investigated in order to better understand how the structure of a contact network impacts the transmission of infection throughout a population.

With a network of N nodes, the state space for an SIR model consists of 3^N possible states. These states are lumped together based on symmetries of the network, in the most simple case of a complete network we can lump states together that have the same number of S, I and R nodes. For networks which are not complete we must look at the topology and the number of nodes in each infection state before determining which system states can be lumped together. Differential equations which describe the transition of the network between the lumped states are derived. The individual final size probabilities are found analytically and a final size distribution produced for each of the small networks. The final size is the total number of individuals that were infected at some point during the epidemic. (WA2)

Mark McGuinness

(Victoria University of Wellington, Mark.McGuinness@vuw.ac.nz)

Erupting Dusts

Volcanoes provide endless fascination and sometimes awesome power and danger. Mathematics can be used to help understand how expanding gases break rock and cause hot rock and ash to erupt violently from earth to air in huge black clouds. Previous work has successfully explained why competent but porous rock breaks up into slab structures, when high pressure gas is allowed to destructively exit the porous rock structure.

I will describe recent mathematical modelling that was inspired by laboratory observations of erupting dusts, with smaller overpressures but with the same slab structures initially observed at the initiation of eruption. I will again use conservation of mass and momentum equations to provide an explanation for the way the dust is observed to lift off in layers with a typical lengthscale, at the onset of fluidisation. (TA2)

Robert McKibbin

(Massey University, R.McKibbin@massey.ac.nz)

Estimating airflow turbulence scales from gas tracer data

Gas concentration data collected downwind of tracer releases are used to estimate the dominant length scale L of the near-ground atmospheric turbulence. Recorded concentrations are compared with those predicted by a simplified mathematical model. Here a tracer gas is released steadily from a fixed height into a wind with a near-uniform speed w but which veers (changes direction) during the experiments. The turbulence near the ground is assumed to be isotropic; the dispersion coefficient in the model is written $D = wL$ where L is the dominant length scale associated with the turbulence. The formula used for comparison with the experimental data is the steady-state solution of an advection-dispersion equation for a uniform wind over an impervious surface.

Results show that the computed values for the dominant turbulence length scale fall in a relatively small range. Considering the variety of wind speeds and day-time/night-time experiment times, this gives a closely-constrained estimate for L. The effects of the assumption of a uniform wind are also investigated. Differences in computed concentration values between the uniform and non-uniform wind cases are only a few percent. The assumption of a uniform wind seems justified.

[A poster with the same title is available.] (ThA2)

Maarten McKubre-Jordens

(University of Canterbury, maarten.jordens@canterbury.ac.nz)

A fresh look at the continuum

In this talk on paraconsistent mathematics, we begin investigations into an entirely new approach to the continuum.

Paraconsistent logics are often derided as being too weak to study mathematics in-depth. In a recent paper, Weber and McK-J showed that this is not the case; using a suitable paraconsistent logic, reconstruction of proofs – or creation of new proofs altogether – is possible for most of the desired theorems from classical real analysis. This showed that real analysis is *possible* when using a paraconsistent logic.

In our approach we find that there is more to a continuum than meets the classical eye – or, indeed, any mathematical eye thus far considered. The approach is top-down, starting with the fundamental intuition of continuity, and working down to basic properties. Along the way we meet approaches reminiscent of Peirce, Cauchy, Leibniz and, of course, Dedekind. This begins to show that, in the end, real analysis is indeed very *rich* when done paraconsistently. (TD10)

Mohd Hafiz Mohd

(University of Canterbury, hafiz.mohd@pg.canterbury.ac.nz, mohdhafizmohd@gmail.com)

Coexistence and invasibility of competing species

Competition between similar species has long been perceived as a major factor shaping community structure and diversity within ecology. In the main, the study of population dynamics involving competitive interactions has received great attention in ecology due to the influential role that interspecific competition played in the development of ecological theory. One way to try to understand this role is by using mathematical models. We study a simple model of biotic interactions consists of interspecific competition and carrying capacity terms. We explore the influence of competitive interactions on the distribution of species, particularly the effects varying some of the parameters has on the coexistence, exclusion and invasibility of species. The occurrence of local bifurcations within the model has ecological implications which will be discussed. (TB8)

Antoine Nectoux

(University of Auckland, antoine.nec@gmail.com)

The essential rank of the alternating group

Two subsets of a Sylow p-subgroup P of a finite group G may be conjugate in G without being conjugate in P. A conjugation family for G is a family of subgroups of G whose normalizers control this “fusion” of subsets of P. Alperin and Goldschmidt proved that a family of subgroups of G is a conjugation family if and only if it contains P and intersects non-trivially with every conjugacy class of essential subgroups of G. In this talk I will explain how to obtain the essential subgroups of the alternating group and their conjugacy classes. (TB5)

Graeme O’Brien

(Massey University, qe@ihug.co.nz)

Why life without discrete groups is of measure zero: counting the groups

When people discover that I am researching Pure Mathematics, there is almost invariably (and understandably) a question about my subject matter. The usual reaction is a blank look for terms like “Möbius transformation” but often an “aah, I know what discrete means”. Unfortunately they don’t, and I have to continually revise my response in an attempt to explain the why and the importance.

This talk is essentially a crash course on hyperbolic geometry, quotient spaces, n-dimensional manifolds, discrete groups, geometrisation theorems, Möbius transformation matrix representation, a couple of theorems by Jrgensen and Klein, isometric circle concepts, and finally some results of my own on quantification of discrete groups. Even a theorem or two, and all in a few minutes. (WB2)

Catherine O'Byrne

(University of Auckland, kateobyrne87@hotmail.com)

An analogy between airway smooth muscle and complex fluids

Complex fluids, such as toothpaste, are composed of a suspension of particles which have the ability to cluster or break apart according to the level of stress applied to them. The way in which they fluidize following imposed strain is similar to the response of airway smooth muscle (ASM) under the same circumstances. We want to know if we can use the transition between solid and fluid phases exhibited by complex fluids theory as a way of combining our active and passive models.

We have previously combined our models for active and passive ASM contraction in parallel to capture several experimental results. Now we use a constitutive model formulation as a novel way of combining the models to determine whether similar results can be obtained. (TC2)

Dion O'Neale

(University of Auckland, d.oneale@auckland.ac.nz)

Will we always be poor; what can complex systems analysis teach us about our economic prospects?

There is strong evidence that the productivity *per capita* of cities and regions increases with population. While satisfactory models underlying this increased productivity are still lacking, the most likely explanations tend to involve increased innovation within regions, made possible by bringing together individuals and organisations with diverse, specialised capabilities.

We have used the REGPAT patent database to construct a bipartite network of geographic regions and the patent classes for which those regions display a revealed comparative advantage. By analysing this network, we can infer relationships between different types of patent classes, and hence the structure of (patentable) technology. We investigate measures such as the diversity and ubiquity of innovations within regions and find that diversity (resp. ubiquity) is positively (resp. negatively) correlated with population. We also find evidence of a nested structure for technical innovation. That is, specialised innovations tend to occur only when other more general innovations are already present.

These findings have implications for regional economic development, especially for countries with small, isolated populations or for rural regions. (WB3)

Roberto Panai

(University of Auckland, robertopanai@sardus.it)

Nearly Káhler geometry and the $(2,3,5)$*-distribution via Cartan* G_2^**-holonomy*

Nearly Kähler geometries are one of the most important classes in the celebrated Gray-Hervella classification of almost Hermitian geometries; in particular these have a non-trivial role in dimension 6. On the other hand the Cartan's $(2,3,5)$-distribution is the geometry arising from a maximally non-degenerate distribution of 2-planes in the tangent bundle of a 5-manifold. Our aim in this work is to expose and study a beautiful convergence of these structures mediated by projective differential geometry. This is a joint work with Rod Gover and Travis Willse. (TD4)

Iain Raeburn

(University of Otago, iraeburn@maths.otago.ac.nz)

Equilibrium states

States are special linear functionals on algebras of operators. In physical models, equilibrium states have proved to be an especially interesting family. In famous work of Kubo, Martin and Schwinger in the 1950's, they were characterised by a commutation relation now known as the KMS relation. This has allowed us to transport the concept to other areas of mathematics, and equilibrium states have proved be very interesting in situations of purely mathematical origin. (As an extreme example, even in number theory.)

We will discuss a family of algebras associated to finite directed graphs, and a rather improbable relationship between the equilibrium states and the Perron-Frobenius theory of the vertex matrix. (TB9)

Charles Semple

(University of Canterbury, charles.semple@canterbury.ac.nz)

How many characters to capture an evolutionary tree?

Evolutionary trees illustrate the ancestral relationships of present-day species. Characters describe attributes of the species and are the data typically used to infer such trees. If we are to recover the correct tree, amongst the exponentially many possible evolutionary trees, then it is necessary that the character data is consistent with just a single tree, that is, the characters capture a tree. How many characters does it take to capture an evolutionary tree? (TD9)

Katie Sharp

(University of Auckland, ktsharp@xtra.co.nz)

Cystic Fibrosis: A Steady State Model of Fluid Flow

The genetic disease cystic fibrosis (CF) is currently incurable, with typical symptoms including a build up of mucus in the lung, and analysed samples have provided conflicting information as to the reason why this occurs. One theory is that the submucosal glands secrete much less serosal fluid in lungs affected by CF. This is due to the genetic failure of a specific chloride ion channel found in abundance in the glands.

The mathematical model is constructed as a time-independent boundary value problem, with ionic concentrations and volumetric water fluxes from submucosal gland secretions as boundary conditions. The equations are derived using a reaction-diffusion system to govern concentrations of particular ion species and fluid flow. The height of the fluid above the cells is predicted by satisfying the boundary conditions, due to an imposed extra condition.

In cystic fibrosis conditions, the predicted height value decreases and absorbed fluid through the cell increases. as is seen experimentally. By changing the boundary conditions, we are able to rehydrate the fluid in CF conditions, which could be reproduced experimentally and implies a change in gland secretion *in vivo* could reduce mucus build up in the lungs of cystic fibrosis patients. (TA10)

David Simpson

(Massey University, d.j.w.simpson@massey.ac.nz)

Probabilistic forward evolution through singularities of discontinuous vector fields

Phenomena with switches or impacts are often modelled by ODEs that are discontinuous on one or more surfaces. Generically, such surfaces may have points at which the vector field is tangent to the surface on both sides. These points, called two-folds, have recently been identified in models of electrical and mechanical systems. However, forward evolution from a two-fold is undefined. In this talk I will describe how forward evolution from two-folds may be defined probabilistically for a two-fold in a two-dimensional vector field. Results are obtained by considering arbitrarily small perturbations by hysteresis, time-delay and white noise. (ThB7)

Callum Sleigh

(University of Auckland, callumsleigh@gmail.com)

Tractor calculus and group cohomology

The tractor calculus is the intrinsic calculus of conformal differential geometry; playing the same role that tensor calculus plays for Riemannian differential geometry. We will show how the tractor calculus can be used to study the cohomology (with coefficients) of hyperbolic manifolds. Time permitting, we will discuss the relation of these techniques to a nonvanishing theorem of Millson/Lafontaine: the first cohomology group of a hyperbolic manifold M with coefficients in $\mathbb{R}^{n+1}$ vanishes if M contains a totally geodesic hypersurface. (WB4)

Gunter Steinke

(University of Canterbury, gunter.steinke@canterbury.ac.nz)

The topology of 3-interpolating systems on the 2-sphere

A 3-interpolating system $\mathcal{C}$ on the 2-sphere is a collection of simply closed curves such that any three points on the 2-sphere are on exactly one member of $\mathcal{C}$. Geometrically such a 3-interpolating system is known as a spherical circle plane. For example, if S is a strictly convex closed surface in Euclidean 3-space, then S is a topological 2-sphere and the plane sections of S having more than one point yield a 3-interpolating system on S.

$\mathcal{C}$ carries a natural topology, and the question is what topological manifold it is homeomorphic to. In case that $\mathcal{C}$ is even kind of a Hermite 3-interpolating system one obtains geometrically a spherical Möbius plane, and the answer had been given by K. Strambach in 1974: the circle space $\mathcal{C}$ is homeomorphic to the circle space of the classical Möbius plane obtained as above when S is the unit 2-sphere.

In this talk I present joint work with Rainer Löwen and describe a method of parametrising $\mathcal{C}$ that works without the tangency condition Strambach assumed. It then shown that the circle space of every spherical circle plane is homeomorphic to the real projective 3-space minus a point. (TD5)

Chris Stevens

(University of Otago, cstevens@maths.otago.ac.nz)

The Friedrich-Nagy gauge for colliding plane gravitational waves

In many numerical calculations of initial value problems in General Relativity, artificial boundaries are introduced so that the computational domain becomes finite This however, introduces several problems. How does one choose physically meaningful boundary conditions? Will this choice lead to a well-posed problem and propagate the constraints? The Friedrich-Nagy gauge is one of only two formulations of the Einstein's field equations that gives rise to stable implementations of boundary conditions for a well-posed IBVP. The aim of this project was to implement this gauge numerically, using the problem of colliding impulsive plane gravitational waves as a simple application. We did this by creating a "gravitational plane wave collider" using the Python package COFFEE (ConFormal Field Equation Evolver) developed by the relativity group at the University of Otago. We checked the correctness of our code and then proceeded to reproduce the already known analytic solution of Khan and Penrose, which describes the mutual scattering of colliding impulsive plane gravitational waves. This is work with Jörg Frauendiener. (TC9)

Tim Stokes

(University of Waikato, stokes@waikato.ac.nz)

Rings with kernel inclusion

Building on previous work for semigroups of functions and binary relations, we axiomatize structures consisting of endomorphisms of abelian groups equipped with composition, the usual pointwise operations, and the quasi-order of kernel inclusion. The resulting structures are associative rings enriched by a quasi-order satisfying a finite set of laws. More generally, we axiomatize the kernel inclusion quasi-order on the ring R induced by a right R-module, and we call the resulting abstract structures rings with ker-order. A characterisation of the possible ker-orders on a fixed ring is given, in terms of certain families of its right ideals. (ThC7)

Prof Winfried Stute

(Justus Liebig University Giessen, ginnyn@msor.vuw.ac.nz)

Univariate And Multivariate Volterra Equations In Nonparametric Statistics

In the statistical analysis of lifetime data it is quite common that, due to censorship effects, the available information is incomplete. To obtain efficient estimators one needs to solve appropriate Volterra equations. The solution depends on the cumulative hazard function. In the multivariate case, however, the hazard function does not uniquely determine the distribution. In our talk we discuss a way out of the dilemma and present the solution of a long-standing open problem, the multivariate extension of the Kaplan-Meier estimator. (TC8)

Winston Sweatman

(Massey University, w.sweatman@massey.ac.nz)

Orbits close by equal-mass four-body central configurations

Masses moving under gravity can remain in a central configuration indefinitely. Neighbouring orbits are approximated as a perturbation from the central configuration orbit. The equal-mass four-body case proceeds in a similar fashion to previous cases. Several terms appear in the perturbation. The dominant perturbation term determines how the system evolves as it leaves the neighbourhood of the quadruple central configuration. (TC7)

Stephen Taylor

(The University of Auckland, s.taylor@auckland.ac.nz)

Stabilisation of the Korteweg De Vries Equation

The Korteweg de Vries (KdV) Equation is one of the most important partial differential equations in applied mathematics and, starting with shallow water waves in the 1800s, is still finding applications in many areas of physics and mathematics. Our recent work on long waves for swirling flow through a pipe has shed light on previously unknown properties of the solutions of initialboundary value problems for the KdV equation. This talk will report on some of these results. (WA4)

Timm Treskatis

(University of Canterbury, timm@treskatis.net.nz)

Trust-region SQP methods for numerical simulations of viscoplastic flows

What do toothpaste, pizza dough and paint have in common? Not only are there numerous blank spots on the map of their underlying theory. Also on the numerical side, existing algorithms for the simulation of such viscoplastic fluids suffer from poor convergence properties or proneness to ill-conditioning.

After an introduction to non-Newtonian fluid mechanics, this talk will present first results that have been achieved during the initial stage of this PhD project. This includes an analysis of the governing equations for so-called Bingham and Herschel-Bulkley fluids, from which a new formulation in terms of an infinite-dimensional, linearly constrained optimisation problem can be derived. Furthermore, we will discuss specifically adapted methods from the trust-region framework in order to achieve fast global convergence, even inspite of singularities in gradients and Hessians.

An outlook to future extensions and hurdles on that way there shall conclude this presentation. (TD7)

Attique Ur Rehman

(University of Auckland, attique.ur-rehman@auckland.ac.nz)

Coupled Orbital and Thermal Evolution of Major Uranian Satellites

We have developed a model of the orbital and thermal evolution of the five major Uranian satellites over millions of years. The model consists of detailed ordinary differential equations for the orbital evolution coupled to the one-dimensional heat equation for the thermal evolution. We present preliminary results that show how the different terms in the orbital equations such as the oblateness of Uranus affect the orbital semi-major axis and eccentricity of the satellites. (TA4)

Bruce van Brunt

(Massey University, b.vanbrunt@massey.ac.nz)

Eigenfunction expansions for solutions to a nonlocal PDE from a cell growth model

A simple model for cell growth and division into $\alpha > 1$ daughter cells is given by the equation

$$\frac{\partial}{\partial t}n(x,t) + g\frac{\partial}{\partial x}n(x,t) + bn(x,t) = b\alpha^2 n(\alpha x,t).$$

Here, n denotes the number density of cells of size x at time t , g is the growth rate, and b is the division rate. ("Size" is usually measured by DNA content.) A notable feature of this differential equation is the αx argument. The differential equation is supplemented by the condition

$$n(x,0) = n_0(x),$$

where n_0 is the initial cell size distribution, which may be regarded as a probability density function. Solutions are also required to satisfy the boundary condition $n(0,t) = 0$, and this leads to an initial-boundary value problem.

Substantial work has been done on steady size distribution solutions. These are the separable solutions to the differential equation and they play an important rôle in the long term asymptotic behaviour of any solution to the problem. There is, however, no general solution method for an arbitrary initial cell distribution n_0. In this talk we will look at a class of eigenfunctions defined by Dirichlet series that can be used to construct solutions to the problem. (ThA3)

Graeme Wake

(Massey University at Albany, g.c.wake@massey.ac.nz)

Counting our gene structures

Despite the vast amount of information known about the sequencing of our genome, the nucleus is still a mysterious place. DNA molecules undergo many levels of organization resulting in discrete nuclear environments. This organization is dynamic and responsive as cells produce proteins, responding to their environment and dividing. All cells contain the same DNA content; however the activity of particular genes changes during development leading to specialized cells and tissues. Growing evidence indicates that the spatial organization of the genome, i,e. the three-dimensional configuration and localization of genes within the nucleus, plays an important role in the systems that control gene activity and expression (the phenotype). These principles govern genome organization in space and time. Specifically: what proteins are involved in establishing and maintaining this organization; how changes occur during cellular development and disease; and how the structure of the nucleus and regulation of gene activity are linked in mammalian cells. This involves determining the number of configurations of n oriented fragments (line segments) whose endpoints are free, joined to the endpoints of other fragments or together as a loop. This number satisfies a solvable recursive relation for the number of different arrangements, giving important statistical information. (TC3)

Graham Weir

(Callaghan Innovation, graham.weir@callaghaninnovation.govt.nz)

Causality in Configuration space

The Dirac equations for n fermions are a linear system of 4^n equations, amongst 4^n variables, in one time and n independent 3D Cartesian spatial coordinates. We show that the natural scalar equation in this system is a linear equation of order 2^n, which admits travelling wave solutions, with a velocity typically exceeding the speed of light. However, when this velocity is projected into any set of the 3D Cartesian coordinates, the corresponding wave speed is exactly the speed of light. The group of affine transformations which preserve the wave equation is highly degenerate. (WC4)

Geoff Whittle

(Victoria University of Wellington, geoff.whittle@vuw.ac.nz)

Rota's Conjecture

Recently Jim Geelen, Bert Gerards and I announced that we had a proof of Rota's Conjecture. This is, perhaps, the most well known conjecture in matroid theory. Our proof is long and complex and it will be several years before it is fully written up. The talk will be very general. I will try to give insight into Rota's Conjecture and why it is interesting. I will also try to give a high-level overview of our proof strategy. (WD3)

Mark Wilson

(University of Auckland, mcw@cs.auckland.ac.nz)

Diagonal asymptotics for products of combinatorial classes

We generalize and improve recent results by Bna and Knopfmacher and by Banderier and Hitczenko concerning the joint distribution of the sum and number of parts in tuples of restricted compositions. Specifically, we generalize the problem to general combinatorial classes and relax the requirement that the sizes of the compositions be equal. We extend the main explicit results to enumeration problems whose counting sequences are Riordan arrays. In this framework, we give an alternative method for computing asymptotics in the supercritical case of Flajolet and Sedgewick, avoiding explicit diagonal extraction. We claim that this method is more computationally efficient. (WD2)

Phillip Wilson

(University of Canterbury, phillip.wilson@canterbury.ac.nz)

When does a rocking body lift off from Mars?

Fluid flows past a body lying at rest or rocking on a solid and fixed horizontal surface. Motivated by recent observations on Martian dust movement we investigate under which conditions hydrodynamic forces remove the body from the surface, or instead keep the body rocking. Other small- and large-scale applications include situations in industrial cleaning, biomedical processes, sporting events, and geological erosion. Set within a broader context of recent study of the role of added mass in this type of momentum- and pressure-dominated fluid-body interaction, the non-linear theory here has unsteady motion of an inviscid fluid interacting with a thin body free to move on a solid and fixed horizontal surface. A combination of asymptotic and numerical analysis is used to address various body shapes and a range of initial conditions. Evolution and body shape couple dynamically with scaled mass, moment of inertia, and gravity effects to create a subtle parameter space. Body lift-off generally cannot occur without fluid flow, but it can occur either immediately or within a finite time after the fluid starts to flow. Parameters for lift-off are found and comparisons made with Martian observations. This is joint work with F.T. Smith (UCL). (ThA6)

Ali Ashher Zaidi

(Massey University, a.a.zaidi@massey.ac.nz)

A Size Structured Cell Growth Model

We present a model that describes the growth, division and death of a cell population structured by size. The model is an extension of that studied by Hall and Wake (1989) and incorporates the asymmetric division of cells. We consider first the case of a cell of size ξ dividing into α cells of size x. We then consider the case of binary asymmetrical splitting in which a cell of size ξ divides into daughter cells of different sizes. (TC4)

Raziyeh Zarredooghabadi
(Massey University, r.zarre@massey.ac.nz)

Image registration

Image registration is one of the main areas in image processing which gives information about images for instance, how an image has been evolved to another with time. One of the images is referred to as source and the others are referred to as the target images. Image registration involves spatially registering the target image(s) to align with the reference image. It has many applications in medical eld, object-class detection, etc.

There are some factors in image registration, For example: (1) Transformation which is determined by relating positions in one image to corresponding positions in one or more other images. There are different groups of dependencies in transformation: whole diffeomorphism group and its finite dimensional subgroups, (2) Measure distance, (3) initial guess to start optimization, etc. In this talk I will explain how to align target to the source in finite dimension group. (TC5)

Posters

Alona Ben-Tal (Massey University), Ioannis Kevrekidis (Princeton University), Joshua Duley (Massey University)

(a.ben-tal@massey.ac.nz)

Coarse-graining of the dynamics observed in neural networks

A system of bursting neurons often exhibits complex dynamics which consist of multiple states and chaotic behaviour. However, in certain applications, the exact details of this dynamics may not be important and one would like to use a simplified model that captures the dynamics roughly. We have developed a computational approach that maps between the variables of a bursting neural network (for which the equations are known) and the variables of a simplified model (for which the equations are unknown). By simulating the neural network for short periods of time we can estimate the dynamics the simplified model should retain. Using this approach we created "coarse" bifurcation diagrams for 2D simplified models representing networks of 1 to 50 cells and compared them with bifurcation diagrams of the detailed networks. Our calculations show that a 2D model can capture the dynamical behavior of the neural system roughly for certain parameter values and identify the domain of parameters for which the 2D simplification is invalid.

Annalisa Conversano (Massey University), Anand Pillay (University of Notre Dame)

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Connected components of definable groups

We give the first known example of a definable group whose model-theoretic connected components differ.

Jennifer L. Creaser, Bernd Krauskopf, Hinke M. Osinga (University of Auckland)

(j.creaser@auckland.ac.nz)

alpha-flips and T-points in the Lorenz system

The Lorenz system is a well-known chaotic system. It is a system of three ordinary differential equations that was introduced by Edward Lorenz in 1963 as a simplified model of thermal convection. We present and characterise a phenomenon, which we call an *alpha*-flip that is associated with how the overall dynamics is organised. There are infinitely many *alpha*-flips and they are connected to infinitely many so-called T-points, which are organising points for chaotic dynamics.

Frederic Effenberger (University of Waikato), Yuri E. Litvinenko (University of Waikato)
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Solving the telegraph equation for solar energetic particle intensity and anisotropy profiles

The diffusion approximation to the Fokker-Planck transport equation is commonly used to model the transport of energetic particles originating at the Sun. Yet this approximation is expected to break down at early times during a solar energetic particle event. In this study, we compare exact analytical predictions of a higher order telegraph approximation for particle transport and stochastic numerical solutions of the full Fokker-Planck equation. We also investigate the focusing effect of a spatially varying magnetic field on an evolving anisotropic particle distribution. We compute spatial and temporal profiles of the particle intensity and anisotropy, and we use them to argue that the telegraph approximation has some significant advantages in cases where the focusing is strong. Furthermore, the telegraph approximation resolves the well known causality problem of the diffusion model.

Jung Min Han (University of Auckland)
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A Mathematical Model of Saliva Secretion

Oscillations in free intracellular calcium (Ca^{2+}) concentrations are known to act as signals in various cell types, including salivary acinar and duct cells. Specifically, a rise in free cytosolic Ca^{2+} concentration plays an important role during early stages of saliva secretion. It is believed that these Ca^{2+} signals are controlled by the opening and closing of the inositol 1,4,5-triphosphate receptors (IP_3Rs) on the membrane of the endoplasmic reticulum (ER) or the sarcoplasmic reticulum (SR). We construct a mathematical model of oscillations in Ca^{2+} concentration and the messenger, inositol (1,4,5)-triphosphate (IP_3), in a salivary duct cell line. A set of ODEs is included to portray IP_3R kinetics. The model captures some of the experimentally observed salivary cell activities. From this model, we can postulate essential features of the salivary gland at the cellular level.

Ragheb Hasan (University of Auckland)
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Moving from square-wave to pseudo-plateau bursting

Many biological phenomena have the property that some variables evolve much faster than others. For example, neuronal cells produce different bursting patterns due to a difference in timescales between changes in voltage across the cell membrane and changes in calcium concentration inside the cell. We focus on two bursting patterns that appear very similar but are mathematically different in the mechanism by which they are generated; these are so-called square-wave and pseudo-plateau bursting. The aim of our study is to find a direct transition between those two patterns and show that they are not only experimentally but also mathematically similar.

Peter Langfield, Bernd Krauskopf and Hinke Osinga (University of Auckland)
(p.langfield@auckland.ac.nz)

The interactions of forward- and backward-time isochrons

An isochron is the set of all points in the basin of an attracting periodic orbit that converge to the periodic orbit in forward time with the same asymptotic phase. We generalise this definition to repelling periodic orbits by defining backward-time isochrons as the set of all points in the basin of a repelling periodic orbit that converge to the periodic orbit in backward time with the same asymptotic phase. We consider planar dynamical systems that have both an attracting and a repelling periodic orbit and study the intersections between the corresponding forward- and backward-time isochrons. We identify a tangency transition between the two sets of isochrons, which we relate to the development of phase sensitivity in the systems. It is shown to exist in two concrete examples: a normal-form-type model and the slow-fast Fitzhugh-Nagumo model.

Karen McCulloch (Massey University), Mick Roberts (Massey University), Carlo Laing (Massey University)
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Epidemics on networks

The dynamics of an SIR (Susceptible-Infected-Recovered) epidemic process on small networks with different topological structures are investigated in order to better understand how the structure of a contact network impacts the transmission of infection throughout a population.

With a network of N nodes, the state space for an SIR model consists of 3^N possible states. These states are lumped together based on symmetries of the network, in the most simple case of a complete network we can lump states together that have the same number of S, I and R nodes. For networks which are not complete we must look at the topology and the number of nodes in each infection state before determining which system states can be lumped together. Differential equations which describe the transition of the network between the lumped states are derived. The individual final size probabilities are found analytically and a final size distribution produced for each of the small networks. The final size is the total number of individuals that were infected at some point during the epidemic.

Robert McKibbin (Massey University), Aimee Harris (Massey University)

(R.McKibbin@massey.ac.nz)

Estimating airflow turbulence scales from gas tracer data

Gas concentration data [1] collected downwind of tracer releases were used to estimate the dominant length scale L of the near-ground atmospheric turbulence, by comparing recorded concentrations with those predicted by a simplified mathematical model. Here a tracer gas is released steadily from a fixed height into a wind with a near-uniform speed w but which veers (changes direction) during the experiments. The turbulence near the ground is assumed to be isotropic; the dispersion coefficient in the model is written D = wL where L is the dominant length scale associated with the turbulence. The formula used for comparison with the experimental data is the steady-state solution of an advection-dispersion equation over an impervious surface. To the authors' knowledge, this approach is new. Results show that the computed values for the dominant turbulence length scale fall in the range L = 0.03 0.24 m with a mean of about 0.13 m and a standard deviation of about 0.06 m. Considering the variety of wind speeds and day-time/night-time experiment times, this is a closely-constrained estimate for what represents the average of the wide range of length scales that pertain to local turbulence in the wind.

[1] $\texttt{http : //www.jsirwin.com/Tracer}_{\texttt{D}}\texttt{ata.html}$

Pingyu Nan (University of Auckland)

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Three time scale oscillations and their analysis

Many physiological systems have the property that some processes evolve much faster than others, and mathematical models can be constructed to reflect this property. Methods for the analysis of models with two distinct time scales are now well-established, but little is known about the case of three or more time scales. In this poster, we will show the progress that has been made on understanding three time scale oscillations in a biologically motivated model.

Dion O'Neale (University of Auckland), Shaun Hendy (University of Auckland)

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Using network science to explore innovation

We live in a world where scientific and technical advances require increasingly specialised knowledge while drawing expertise from ever more diverse technical areas. In an effort to better understand the relationships between different areas of innovation, and the role of specialisation, diversity, and ubiquity in national and regional economies, we have mined several million patent records from the European Patent Office, along with their classification codes, and used them to construct a network of "patent-space". We identify when individual geographic regions have a "revealed comparative advantage" (RCA) for particular technical areas, or codes, by comparing the patent outputs from each region-code pair against the global output for that code. Using co-occurrence relationships in the RCA data, we construct a proximity network for the different classes of patentable innovation. We find that patent-space is heterogeneous and highly structured. By overlaying data for particular regions on the patent-space base map we are able to explore temporal and regional trends in particular how the innovation systems of different countries has produced quite different areas of specialisation.

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Global effects of Pyragas time-delayed feedback control

Pyragas time-delayed feedback control is designed to stabilize an unstable periodic orbit. The delay in the feedback term is set to the period of the target periodic orbit and, hence, this control scheme is non-invasive. We consider here Pyragas control applied to the widely considered normal form of a subcritical Hopf bifurcation. Previous work focused primarily on the mechanism of stabilization. Here we present a global picture of the dynamics induced by the time-delayed feedback. In particular, we find infinitely many delay-induced Hopf bifurcation curves that move as the 2π-periodic feedback phase is varied. Furthermore, we discuss the effect of choosing a delay term that is close but not equal to the period of the target orbit.

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Cystic Fibrosis: The Mathematics of Fluid Recycling

The genetic disease cystic fibrosis (CF) is currently incurable with symptoms including a build up of mucus in the lungs. CF causes the failure of a chloride ion channel (CFTR), and subsequent depletion of serous fluid located beneath the mucus layer. Analysed samples have provided conflicting information as to the reason why this occurs. One theory is that the submucosal glands, which are known to secrete serosal and mucosal fluid, secrete much less serous fluid in lungs affected by CF. Another theory is that functioning CFTR inhibits a sodium channel (ENaC), and that with CFTR malfunction, ENaC activity increases, which results in excess fluid absorption.

The mathematical model is constructed as a time-independent boundary value problem, with ionic concentrations and volumetric water fluxes from submucosal gland secretions as boundary conditions. The height of the serous fluid is predicted by satisfying the boundary conditions, which is imposed by an extra condition.

We show that a *combination of both hypotheses predicts a more accurate height value that is seen experimentally. Therefore we conclude that in human bronchial epithelia, it is likely that a reduction in gland flux combined with an over-active sodium channel results in the depletion of serous fluid.*

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Mathematics-in-industry study group (MISG) projects in Australia and New Zealand

Mathematics-in-Industry Study Group (MISG) workshops run annually in Australia and New Zealand. They are instructive and fun – so attend one if you have the chance! This poster relates to some projects from the last decade. Industries involved include those of steel, electricity and agriculture.